A New Model of Trend Inflation

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The views expressed are not necessarily those of Federal Reserve Bank of New York or the Federal Reserve System

Plan of talk

- Many recent time series models of US inflation imply inflation expectations are I(1) – unmoored
- Develop a new model of trend inflation where long-run inflation expectations are contained
- Estimation of model uses a variety of new/special algorithms
- Compare estimated model with those in the the current literature
  - New Model has superior in-sample performance
  - In real time forecasting exercise performs well
  - Earlier version of model useful in interpreting market based inflation expectations

Definition of underlying inflation

Observed inflation sum of two components

$$\pi_t = \tau_t + c_t,$$

1. Trend or Underlying rate of inflation τ_t
2. Deviations from underlying rate, c_t

Properties of trend inflation

$$\pi_t = \tau_t + c_t,$$

- Central Bank is targeting trend inflation such that actual inflation converges to it in expectation
  - $E_t\left[\pi_{t+j}\right] \longrightarrow E_t\left[\tau_{t+j}\right]$ as j increases
  - Transitory component goes to zero in expectation $E_t\left[c_{t+j}\right] \longrightarrow 0$.
- Many time series models assume trend inflation has property:
  - $E_t\left[\tau_{t+j}\right] = \tau_t$
  - Thus medium to long-term expectations/forecasts build in random walk type property globally
- In new model $\tau_t \in [a, b]$, where the interval $[a, b]$ is related to the price stability objective of the central bank

Linear unobserved components models

$$\begin{aligned} \tau_t &= \tau_{t-1} + \varepsilon_t^{\tau} \\ c_t &= \varepsilon_t \exp\left(\frac{h_t}{2}\right) \\ h_t &= h_{t-1} + \varepsilon_t^{h} \end{aligned} \quad , \tag{1}$$

where $\varepsilon_t^{\tau} \sim N\left(0, \sigma_{\tau}^2\right)$, $\varepsilon_t \sim N\left(0, 1\right)$ and $\varepsilon_t^{h} \sim N\left(0, \sigma_h^2\right)$. These errors are assumed to be independent of one another and at all leads and lags.

- Use of stochastic volatility in transitory component to capture important features of the data
- IMA(1,1) representation, MA coefficient varies with h_t / σ_{τ}^2

Stock Watson Model

$$\begin{aligned}
\varepsilon_t^{\tau} &\sim N\left(0, \exp(g_t)\right), \\
g_t &= g_{t-1} + \varepsilon_t^g \\
\varepsilon_t^g &\sim N0, \sigma_g^2)
\end{aligned}$$

Instantaneous moving average coefficient varies with h_t / g_t

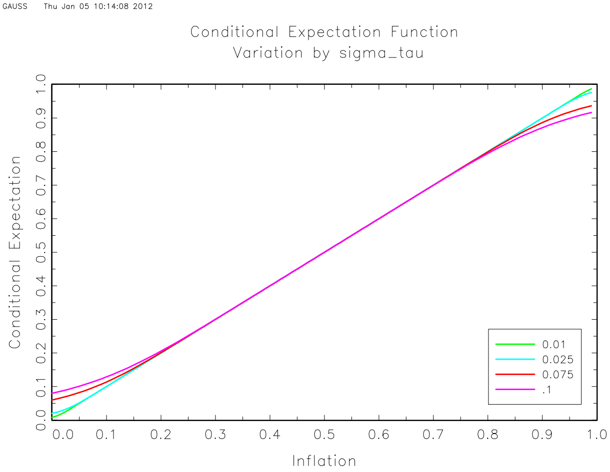
Model for Trend component

$$\begin{aligned} \tau_t &= \tau_{t-1} + \varepsilon_t^\tau, \\ \varepsilon_t^\tau &\sim \text{Trunc Norm}(a - \tau_{t-1}, b - \tau_{t-1}; 0, \sigma_\tau^2) \end{aligned}$$

$$E_{t-1}[\tau_t] = \tau_{t-1} + \sigma_\tau \left[\frac{\phi\left(\frac{a-\tau_{t-1}}{\sigma_\tau}\right) - \phi\left(\frac{b-\tau_{t-1}}{\sigma_\tau}\right)}{\Phi\left(\frac{b-\tau_{t-1}}{\sigma_\tau}\right) - \Phi\left(\frac{a-\tau_t - 1}{\sigma_\tau}\right)} \right] \text{ if } a \leqslant \tau_{t-1} \leqslant b$$

Both underlying inflation and inflation expectations are contained in $[a, b]$

One period expectations mean revert close to bounds, approximately random walk further inside bounds


GAUSS Thu Jan 05 10:14:08 2012
Conditional Expectation Function
Variation by sigma_tau
Conditional Expectation
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Inflation
0.01
0.025
0.075
.1

Transitory Component

Assume some of the short-term dynamics driven by bounded time-varying persistence in the transitory component

$$
\begin{aligned}
c_t &= \rho_t c_{t-1} + \exp(\frac{h_t}{2}), \\
\rho_t &= \rho_{t-1} + \varepsilon_t^{\rho} \\
\varepsilon_t^{\rho} &\sim \text{Trunc Norm}(a_\rho - \rho_{t-1}, b_\rho - \tau_{t-1}; 0, \sigma_\rho^2)
\end{aligned}
$$

Competing Models

Table: A list of competing models.

Model	Description
Trend-SV	Inflation trend model as in Stock and Watson
Trend	**SV** only in Transitory Component
Trend-bound	Same as **Trend** but $\tau_t \in (0, 5)$
AR-trend	$\tau_t \in R, \rho_t \in R$ (No Bounds)
AR-trend-bound	$\tau_t \in (a, b)$ and $\rho_t \in (0, 1)$

Prior on Initial Conditions

The state equations for τ_t, ρ_t and h_t are initialized with

$$\tau_1 \sim TN(a, b; \tau_0, \omega_\tau^2),$$
$$\rho_1 \sim TN(0, 1; \rho_0, \omega_\rho^2),$$
$$h_1 \sim N(h_0, \omega_h^2),$$

where τ_0, ω_τ^2, h_0, ω_h^2, ρ_0 and ω_ρ^2 are known constants. In particular we set $\tau_0 = h_0 = \rho_0 = 0$, $\omega_\tau^2 = \omega_h^2 = 5$ and $\omega_\rho^2 = 1$. The prior variances are set to be relatively large, so that the initial distributions for the states are proper yet relatively non-informative.

Prior on Parameters

$p(\theta) = p(a, b)p(\sigma_h^2)p(\sigma_\rho^2)p(\sigma_\tau^2)$ where:

1. $a = 0$ and $b = 5$ or uniform $[0, 1.5], [3.5, 5]$
2. $\sigma_\tau^2, \sigma_\rho^2, \sigma_h^2 \sim IG(\underline{\nu}_{\tau,\rho,h}, \underline{S}_{\tau,\rho,h})$.

Degrees of freedom parameters: $\underline{\nu}_\tau = \underline{\nu}_\rho = \underline{\nu}_h = 10$.
Scale $\underline{S}_\tau = 0.18, \underline{S}_\rho = 0.009$ and $\underline{S}_h = 0.45$.

Prior Means $\sqrt{E\left(\sigma_\tau^2\right)} = 0.141$, $\sqrt{E\left(\sigma_\rho^2\right)} = 0.0316$, and $\sqrt{E\left(\sigma_h^2\right)} = 0.224$).

Prior Predictive Analysis (based on Geweke 2010)

- Initialize with CPI in 1947Q2
- Draw from prior of models
- Generate time series using prior draw and initial condition
- Repeat 10,000 times
- Compare prior predictive CDFs with observed statistics in the observed CPI sample
  - Include MA coefficient estimated by MLE
- Form "Bayes Factors" from the prior predictive analysis

Prior CDF Evaluation (close to 0.5 is good)

Table: Prior cdfs of features.

Feature	Trend-SV	Trend	Trend-bound	AR-trend	AR-trend-bound
16%-tilde	0.833	0.856	0.734	0.767	0.757
median	0.678	0.889	0.816	0.754	0.801
84%-tilde	0.503	0.827	0.815	0.499	0.753
variance	0.205	0.690	0.707	0.348	0.635
fraction of $\pi_t < 0$	0.133	0.175	0.423	0.246	0.370
fraction of $\pi_t > 10$	0.464	0.812	0.794	0.465	0.731
lag 1 autocorrelation	0.315	0.771	0.814	0.615	0.540
lag 4 autocorrelation	0.227	0.638	0.687	0.300	0.550
MA coefficient	0.497	0.941	0.949	0.648	0.492

Log Bayes Factors from Prior Predictive Analysis

Table: Log Bayes factors in favor of each model over the trend model.

Feature	**Trend-SV**	**Trend**	**Trend-bound**	**AR-trend**	**AR-trend-bound**
Quantile	-12.640	6.008	6.820	-654.581	6.832
Spread and Drift	-11.474	3.027	2.876	-∞	4.881
Dynamics	-0.319	-2.957	-2.414	-0.709	2.083
All	-23.584	4.308	2.713	-∞	13.307

Blocking of the Sampler

We develop an MCMC algorithm which sequentially draws from:

1. $p(\tau \,|\, y, h, \rho, \theta)$
2. $p(h \,|\, y, \tau, \rho, \theta)$
3. $p(\rho \,|\, y, \tau, h, \theta)$
4. $p(a \,|\, y, \tau, h, \rho, \sigma_h^2, \sigma_\rho^2, \sigma_\tau^2, b)$
5. $p(b \,|\, y, \tau, h, \rho, \sigma_h^2, \sigma_\rho^2, \sigma_\tau^2, a)$
6. $p(\sigma_h^2, \sigma_\rho^2, \sigma_\tau^2 \,|\, y, \tau, h, \rho, a, b)$ using the conditional independence, separate draws

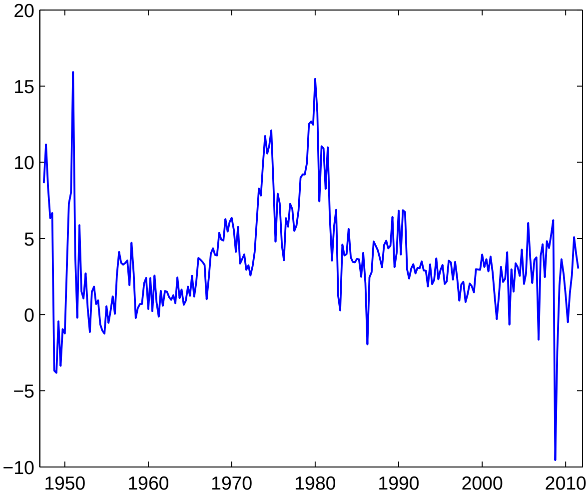
Drawing the bounded sequences

- $p(\tau \mid y, h, \rho, \theta)$ and $p(\rho \mid y, \tau, h, \theta)$ are non-standard and conventional methods of inference in state space models cannot be used
  - Koop and Potter 2011 explains why a simple accept-reject algorithm is incorrect
- Chan and Strachan (2012) Gaussian approximation to $p(\tau \mid y, h, \rho, \theta)$. based on precision based algorithm adapted from Chan and Jeliazkov (2009).
  - Gaussian approximation is proposal density for an accept-reject Metropolis-Hasting (ARMH) step
- $p(\sigma_\rho^2 \mid y, \tau, h, \rho, a, b)$ and $p(\sigma_\tau^2 \mid y, \tau, h, \rho, a, b)$ are also non-standard densities, use an independence-chain MH algorithm.
- Bounds estimated using griddy gibbs

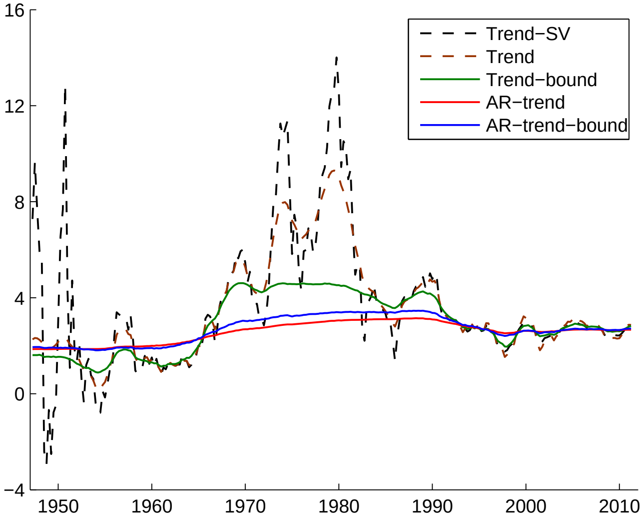
Data

- Focus on CPI data since 1947
  - We use the quarterly average of the CPI index
- Similar results for
  - GDP deflator
  - PCE deflator
  - Annual CPI over longer period
  - Monthly CPI data

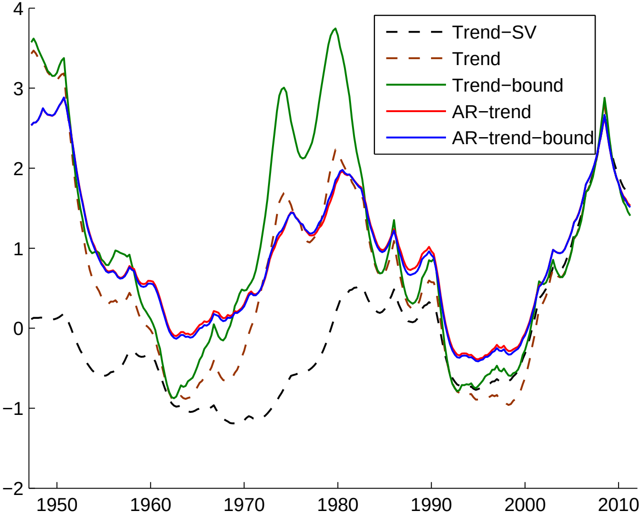
Quarterly CPI



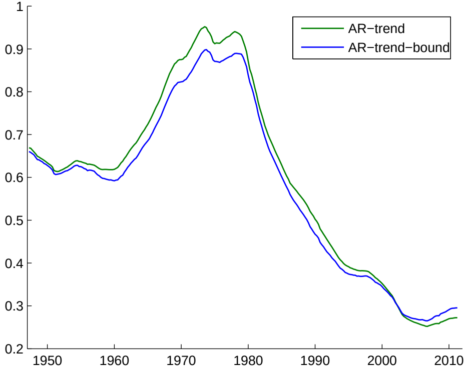
Estimates of Trend



Estimates of Volatility in Transitory Component



Estimates of Time Varying Persistence in Transitory Component



Forecasting Exercise

- Bounded models require simulation techniques to produce multi-step ahead forecasts
- Use "efficiency" of algorithm to recursively estimate the various bounded models
- Evaluation Period Runs from 1975Q1 to 2011Q3
    - CPI is only mildly revised for new seasonal factors, thus close to real time forecasting
- Add in time varying AR model that did well in Clark and Doh study

Root Mean Loss Results

Table: RMSFEs for forecasting quarterly CPI.

	$k = 1$	$k = 4$	$k = 8$	$k = 12$	$k = 16$
Trend-SV	2.168	2.644	3.290	3.592	3.636
Trend	2.332	2.703	3.112	3.354	3.412
Trend-bound	3.032	3.067	3.079	3.148	3.140
AR-Trend	2.139	2.866	4.686	10.536	26.945
AR-trend-bound	2.089	2.430	2.916	3.116	3.168
TVP-AR	2.156	2.826	4.464	6.761	11.637

Log Predictive Likelihood Results

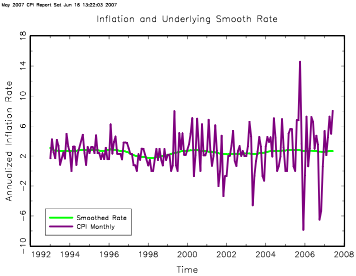
Table: Average log predictive likelihood for forecasting quarterly CPI.

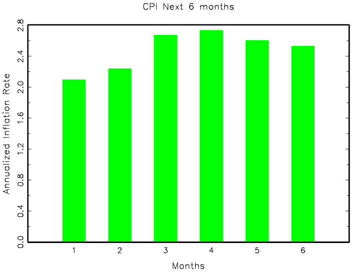
	$k=1$	$k=4$	$k=8$	$k=12$	$k=16$
Trend-SV	-2.052	-2.323	-2.494	-2.562	-2.624
Trend	-2.088	-2.332	-2.490	-2.548	-2.592
Trend-bound	-2.221	-2.341	-2.395	-2.434	-2.425
AR-Trend	-2.041	-2.264	-2.426	-2.471	-2.531
AR-trend-bound	-2.025	-2.214	-2.339	-2.358	-2.404
TVP-AR	-2.040	-2.250	-2.394	-2.413	-2.472

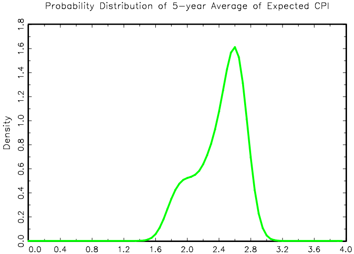
Practical Application

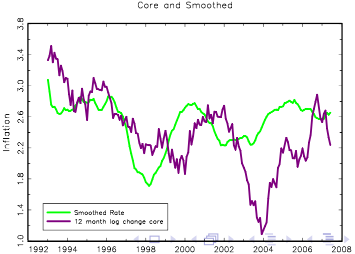
- Version of model without time varying persistence used internally since 2004 at FRBNY to evaluate anchoring of inflation expectations
- Used $a = 1, b = 3.5$
- Market based estimates of forward inflation expectations appear to exhibit containment – a crucial feature of the model (see Jochmann, Koop and Potter, 2010 Jn of Emp Finance)

Earlier Version Example Following May 2007 CPI Report

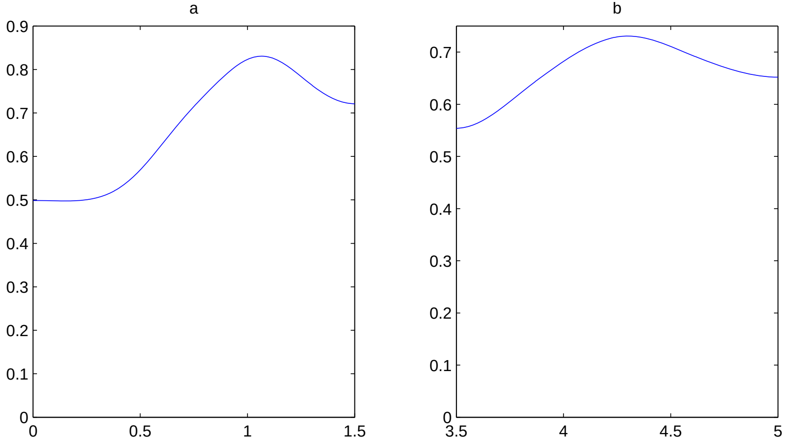








Posterior of Bounds in AR Trend Bound Model



Summary

- Developed a new model for trend inflation
- Competitive with existing models without the implications that inflation expectations are unmoored
- Modern computational techniques allow practical implementation of the model