LIQUIDITY REQUIREMENTS AND PAYMENT DELAYS

PARTICIPANT TYPE DEPENDENT PREFERENCES

by Christian Schulz

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In 2011 all ECB publications feature a motif taken from the €100 banknote.

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CONTENTS

Abstract:

The paper presents an analysis of the trade-offs of participants of different type between payment delay and liquidity requirement on the basis of synthetically generated data. The generation of the synthetic transaction data set for a simple RTGS system is described and calibrated using real world parameters. The payment system is simulated for various liquidity levels and it is shown that participants of different size in terms of transaction volume and value will have different optimal liquidity requirements, as the payment delays they face for each liquidity level will be different. This is shown using indifference curves between payment delay and liquidity requirements.

JEL classification: C15, C5, E58, L14, L41, L51.

Key words: Payment system, simulation, data generation, competition, oversight.

1.   Introduction

Since the start of the current financial crisis and in the context of initiatives aimed at enhancing macro-prudential supervision of the financial system such as the European Systemic Risk Board (ESRB), the interest of monetary policy decision makers in quantitative analysis of inter-bank large value payment flows for the purpose of financial stability analysis has increased substantially. Payment systems data may be considered one of the key sources of quantitative analysis for macro prudential purposes as this data is available almost in real-time and in high quality. However, research into this data is relatively recent and the contributions of such research have been limited to the understanding of only a few financial markets, such as the overnight money market. Market infrastructures and in particular large value payment systems have long been an important focus of central bank activities, because of their importance for any well-functioning financial system and for the implementation of the monetary policy. Yet even for the more narrow applications within the oversight and operation of payment systems, there remains much room for the establishment of new quantitative tools. Simulations of payment systems are a very promising way of gaining a deeper understanding of the payments data and are therefore of interest for payment systems operations and oversight specifically and for the quantitative analysis of payments data in general.[1]

While publications on payment system simulations are relatively recent, there is a growing body of literature on the topic. Starting with seminal papers on systemic risk in payment systems, such as Angelini, Maresca and Russo (1996), publications of quantitative analyses have evolved. Among the first major contributions in the field of quantitative simulations, which established this field and led to many other papers using similar tools and methodology were Koponen and Soramäki (1998) and Leinonen and Soramäki (1999). The tools which were developed have since been extended and enriched. They also form the basis of the analytical parts of this paper.

The contribution of this paper is twofold: first, we describe a set-up for the analysis of payment system issues based on synthetically generated transaction and participant data and simulation tools. To our knowledge, there are currently no simulation publications based on fully synthetically generated data. Generated data allows the researcher to determine the data generating process and may even include behavioural assumptions. Publications on transaction data generation for payment systems simulations are very recent, with Docherty (2010) being the only available dedicated contribution so far. The findings in this study are partly based on the fact that the composition of the participant community and their inter-linkages can be controlled via the data generation process.

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[1] The link between payment systems oversight and wider financial stability analysis becomes also clear from the mandate of the oversight function. As stated in the Eurosystem Oversight Report 2009 (p. 9), "oversight aims at ensuring the safety and efficiency of the overseen systems in order to contribute to financial stability and maintain public confidence in the currency".

Second, using the established framework, we identify how decisions regarding the design and rules of a payment system may influence the competitive situation of banks participating in a payment system. This serves as an example for the fact that simulations can be used in order to gain information for solidly quantitative assessments. Concretely, the decision analysed in this paper is changing the liquidity requirements in a simple real-time gross settlement system. Such liquidity requirements are important in systems with liquidity saving mechanisms which may achieve significant liquidity savings on the one hand, but also introduce payment delays and therefore credit or liquidity risks to the system and its participants on the other. In fact, there is in all payment systems a trade-off between liquidity requirement and immediacy of payment settlement or payment delay. Simulations[2] reveal that different types of participants are likely to have different preferences regarding this trade-off. Large participants will prefer low liquidity requirements as they will be able to minimise payment delays by liquidity recycling due to their higher number of payments. Smaller participants will hardly benefit from lower liquidity requirements as they will in many cases need to fully fund their few daily transactions by raising their account balances or pledging collateral in order to avoid damaging payment delays, due to the fact that they only have few transactions per day. The steepness of the trade-off between liquidity need and payment delay depends on the pattern of payments among and between the different groups of similarly sized participants. Different types of participants will have different optimal levels for liquidity requirements, derived in this paper with the help of curves of indifference between combinations of liquidity requirement and payment delay. There may be intentionally or unintentionally hidden, unfair discrimination against certain types of participants inherent in the choices made with respect to certain design features of a payment system, just like a non-tariff trade barrier may be a hidden inhibition to international trade. The knowledge and quantification of such effects may alter the assessment of the overseer compared to an assessment which does not make use of simulation techniques.

In the course of this paper, a number of research questions related to the use of simulations for oversight and operational purposes will be answered, including which purposes payment systems simulations can serve, which payments data is available and what are advantages of using artificially generated data for simulations. Regarding the latter, the paper highlights which key parameters need to be calibrated in the data generation process and how close the generated data comes to real data. Furthermore, it is an important question how the effects of system design changes on payment system participants can be illustrated and measured, why large and small payment system participants can have very different preferences for the design of the system and why should this be of concern to an overseer of the system. In section 2 of the remainder of this paper, the purposes of simulations are presented and the literature on payment system simulation reviewed. Section 3 describes the data generation and calibration of the process. Section 4 presents the results of the various simulations and discussion of the findings before section 5 concludes.

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[2] For the simulations, we use the Payment Systems Simulator 2 developed by the Bank of Finland (BoF PSS2).

2.   Purposes of simulations and literature review

The aim of quantitative analyses in the field of payment systems is to gain insight into their functioning, the inherent risks and the efficiency. Electronic payment systems store huge amounts of data in efficient data warehouses. These databases should produce a favourable environment for quantitative analyses. Yet, while the use of payments data employing statistical methods and tools is well-established, analytical tools regarding the behaviour and preferences of participants and system owners are scarce. This is because the characteristics of large value payment system are very diverse. Most payment systems exhibit a large number of transactions among a community of participants, which can vary from very large to small. Especially large participant communities will exhibit a high degree of heterogeneity. Participants may be large or small banks, public or private banks, wholesale or retail banks, non-bank financial intermediaries such as insurances, central securities deposits, central counter parties, central banks and a wide variety of ancillary systems. Participants may hold one or more accounts which they may or may not manage centrally. The transactions between the participants can vary in volume and value, in their intraday pattern or over time. Systems will have different liquidity provision mechanism, collateral policies, intraday credit limit arrangements, bilateral limit facilities, multilateral limit facilities, liquidity bridges or liquidity management tools. Systems will vary in their ways of achieving settlement of payment transactions and in which asset they settle transactions, commercial bank money or central bank money. And finally, the pricing structure and access criteria may be very different from one system to the other.

Modern large value payment systems come in two major varieties, real-time gross settlement (RTGS) systems and deferred net settlement (DNS) systems.[3] Since the 1980s, for their wholesale large value fund transfer systems, or inter-bank payment systems, most countries have introduced RTGS systems. These systems mostly settle in central bank money and all payments are settled immediately upon submission to the payment system. However, to avoid situations of gridlock and to optimise liquidity usage, all major systems have additional features. Many systems have additional settlement algorithms such as queuing or splitting algorithms, but also different submission or ending algorithms, intraday net-settlement and settlement of ancillary systems.

The present paper builds on several strands of the existing literature on large-value payment systems. The main strands, which are described below, are **(1)** literature formally modelling payment transactions, **(2)** applied quantitative literature including simulations on real historical data, and **(3)** the limited literature on payments data generation.

Regarding **(1)**, the literature which formally models payment transactions and systems including publications by Edward Green, such as Fujiki, Green and Yamazaki (2008), or Neil Wallace, such as Wallace (2000). Such models allow, for example, modelling the behaviour of participants in a payment system.

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[3] Between RTGS and DNS systems, many hybrids are possible, such as Continuous Net Settlement (CNS) Systems, which do not only settle net at a specific point in time but whenever new payment entries create possibilities to settle payments via netting, which is very similar to what queuing achieves in RTGS systems.

This paper follows a different strand of the literature, referred to above under (2), which uses simulation tools which are able to replicate the design of existing or fictitious payment systems. The plethora of data and parameters make analytical solutions for more complex systems and data sets difficult.[4] In such circumstances, numerical simulations can help to perform quantitative analyses. Besides the payments data, a tool is required, which can replicate the settlement process of a payment system as closely as possible and which allows, ideally, for the introduction of the various design features of payment systems. A widely accepted tool for this purpose is the Payment System Simulator 2 developed by the Bank of Finland. It allows simulating most types of large value payment systems and many other payment systems, also in combinations of several systems or with securities settlement systems.

Simulations can be of interest for all parties concerned by payment systems, i. e. providers/operators, participants and oversight authorities. From their different perspectives all three will essentially be interested in the safety and the efficiency of each payment system. For example, payment system providers might be interested in the effects of changes in the pricing structure, in the enhancement of business continuity and crisis management or the support of the functional evolution and settlement efficiency.

Before the 1990s, most payment systems settled daily, so the intraday perspective was widely covered in the literature as it is now.[5] Since the lack of an intraday perspective makes analytical solutions to the exposures between the participants rather simple, quantitative analyses and simulations were of less interest. In addition, information technology was not yet advanced enough to perform large-scale simulations. A wider body of literature with payment system simulation studies can be found from the mid-nineties onwards (Bowman, 1995 and McAndrews and Wasilyew, 1995). With the growing importance of RTGS systems since the early 1980s (in 1985 3 central banks had adopted RTGS systems, in 2006 already more than 90), these systems became the choice of central banks in developed countries.[6] There is an extensive and growing literature on analytical findings in the area of large-value payment systems. This literature covers, among other topics, the application of simulation tools, network analyses[7], behavioural studies[8] and experimental studies. Simulation tools, and in particular the Payment System Simulator 2 of the Bank of Finland (BoF-PSS2) have been applied to a range of topics, including the analysis of the effects of choices for the structure of a payment, scenario analyses of various critical situations or the studies on liquidity effects on the performance of the system and its participants (Leinonen and Soramäki, 2005). Other published applications include:

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[4] Deterministic analytical solutions, where the actions and preferences of participants could be represented in formulae, would be highly desirable as the major disadvantage of simulations is clearly that they cannot easily be used for optimisation purposes, as pointed out by Leinonen and Soramäki (2003).

[5] Manning et al. (2000)

[6] Bech and Hobijn (2006)

[7] There is a growing literature on the use of network topology as a tool to analyse financial networks. The aim in most studies (for a literature review see ECB Financial Stability Review 1 2010) is to identify systemically important nodes in the financial network concerned.

[8] Examples include agent-based modelling, such as in Arciero et al. (2008), which is designed to analyse the effects of behaviour of payment system participants, for example in response to a disruption at a large participant, where they might stop sending payments after a time of uncertainty.

- Analysis of operational failures, such as the outage of the most active transfer account or bank or group of banks for a certain period of time, such as a business day, as in Puhr and Schmitz (2009) or Bedford et al. (2004). Behavioural reactions to outages by other participants were studied by Ledrut (2007).

- Behavioural changes and their impact on the settlement outcomes have been analysed by Heijmans (2009). Such changes include changes in payment patterns and timing or changing payment values.

- Studies on liquidity focus on the trade-off between liquidity saving features and their effect on the settlement outcome and resulting credit risks (Leinonen and Soramäki (1999). This could be due to changing collateral values, for example.

- In Heijmans (2009) three scenarios of changing liquidity levels are simulated: (1) the lower bound, i. e. the final balance of the day can be settled, (2) the historical collateral availability and (3) the upper bound, i. e. all payments of the day can be settled immediately.

Depending on the objective of the simulation study, different indicators are being used for measuring the outcome, such as the number of unsettled payments at the end of the day, the value of unsettled payments, the number of banks with unsettled payments at the end of the day[9] or the participants who are not capable of paying end-of-day obligations because of insufficient capital.[10]

Focussing on the data used in the above-mentioned studies, most make use of actual historical transaction data collected from the payment system under inspection. In Europe, this has recently included the TARGET2 components of Finland (e. g. Leinonen and Soramäki (1999)), The Netherlands (e. g. Heijmans (2009)) and Austria (e. g. Puhr and Schmitz (2009)), Italy (Arciero and Impenna (2001)) as well as CHAPS in the UK (Bedford et al. (2004)), Kronos in Denmark (Danmarks Nationalbanken (2005)) and Norwegian (Enge and Øverli (2006)) large value payment systems. A major project on making pan-European TARGET2 data available to the oversight and operational functions in the ESCB for simulations has recently concluded and publications on this basis can be expected in the near future. The alternative, referred to above under (**3**), to using historical data is generating data based on assumptions regarding the data generation process. This road is followed in this paper and has only recently also been used by Docherty and Wang (2010). Synthetic data generation can also be performed by sampling from an existing "historical" data set, as was done, for example, by Koponen and Soramäki (1998). Using synthetically generated data as opposed to historical data has some advantages and some drawbacks, of which we list key ones here:

- One advantage of using artificially generated data is, according to Manning, Nier and Schanz (2009), the fact that behavioural changes of the payment system's participants can be modelled. This is not the case when exclusively using historical data. For example, removing

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[9] See Heijmans (2009), Schmitz and Puhr (2009).

[10] See Humphrey (1986).

one participant's outgoing payments from a data set, in order to simulate the effects on the balances of other participants, ignores the possibility that the other participants might stop sending payments to the removed participants after some time, or alter their behaviour otherwise (see Ledrut, 2009). In other words, generated data becomes particularly interesting when behavioural rules are built in.

- Another very important advantage of generated data is that no confidentiality rules need to be applied. In many payment systems, confidentiality rules apply to the use of the data. For TARGET2 this is regulated in Article 38 of the Harmonised Conditions, the contract between the payment system operators and the participants. While the use for oversight purposes is permitted, data has to be sanitised for other purposes and publications. The confidentiality argument has also been put forward in Docherty and Wang (2010).

- A third advantage of synthetically generated data is the fact that generated data will be free of "noise" due to events external to the payment system, such as financial markets events or public holidays.

- The disadvantage of using generated data, according to the Manning et al. (2009), is that the simulation results depend critically on the assumed data generation process. To mitigate this somewhat, the calibration of the data generation process should as much as possible take into account parameters estimated with historical data.

One deficiency of simulations, especially those using historical data is that they ignore possible behavioural reactions of participants to the simulated events. Behavioural reactions are not assumed in Heijmans (2009) or Puhr and Schmitz (2009). Possible reactions to an outage at a major participant could be stop-sending rules as in Ledrut (2007) or raising available collateral. However, other researchers have found that such behaviour is not observed empirically for short interruptions of less than one day so that for short term simulations such reactions may be ignored with some justification (Heijmans, 2009).

3.   Data, statistical analysis and network analysis

This paper makes use of synthetically generated data, because the available historical data is confidential and it is critical for the results of the study to have full information on the different types of participants. In the following we first describe the data generation process schematically, before proceeding with the calibration of the process using real historical payments data.

Starting with the schematic overview, each payment instruction is generated individually. For the transaction data set of the simulation software, the necessary information is (1) a unique identifier, (2) a date, (3) an introduction time, (4) a sender, (5) a recipient and (6) a value of the payment. The following figure gives a schematic overview of the data generation. For each piece of information it indicates the data generation process.

Figure 1 Data generation scheme
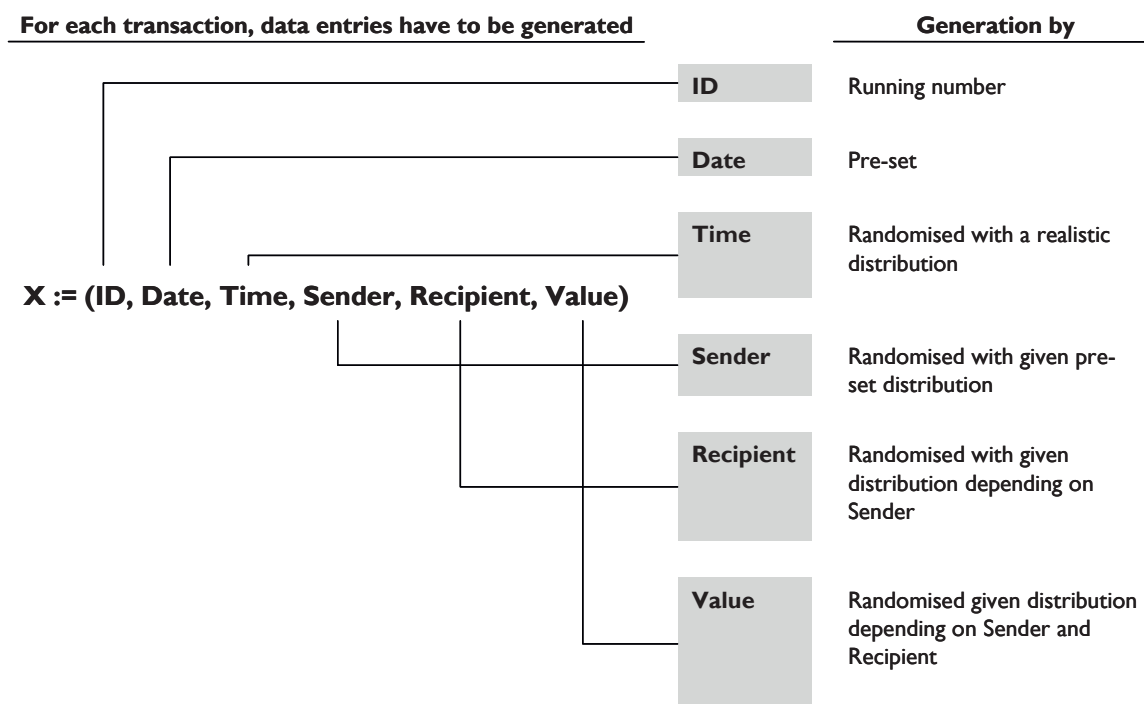
Source: Author's own representation

In the following, we describe the data generation process of the transaction data file step-by-step (see also (see also Figure 2). The process is based on the sender-receiver-volume matrix which contains the number of payments made between each pair of participants in a certain reference period, e. g. one day. The payment values are given by the sender-receiver-value matrix.

- Step 1 is the draw of the sending participant. The draw is based on the distribution of shares of the overall payment volume of each participant in the reference period, i. e. the vertical sum vector of the sender-receiver-volume matrix.

- Step 2 is the draw of the recipient participant, which depends on the previous draw of the sending participant. It is based on the share of payment transaction volume of the sending participant to each other participant, i. e. the row vector corresponding to the sending participant.

- Step 3 is the draw of the payment value. It depends on the previously drawn sending and receiving participants and is given by the value in the sender-receiver-value matrix designated by the pair of participants in the relationship.[11]

- Step 4 is the draw of the payment time. The distribution of payments over time can be determined at system-level or participant-level.

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[11] In practice, a (1,n)-row vector with a one in the place of the receiving participant and zeros otherwise is multiplied with the (n,n) sender-receiver-value matrix. The resulting (1,n)-row vector is multiplied with the (n,1)-column vector with a one in the place of the sending participant and zeros otherwise, resulting in a scalar with the payment value.

Figure 2 Data Generation in four steps

Sender / Recipient	Bank 1	Bank 2	Bank 3	Bank 4	Sum
Bank 1	-	1,000	1,000	100	2,100
Bank 2	1,000	-	100	10	1,110
Bank 3	1,000	100	-	10	1,110
Bank 4	100	10	10	-	120
Sum	2,100	1,110	1,110	120	4,440

1   Pre-set number and characteristics (large, small, medium) of participants

2   Select sender randomly from distribution of senders

3   Select recipient from conditional distribution of recipients

4   Value is given by sender and recipient

Source: Author's own representation

The data generation procedure in this paper is different and more demanding on technical resources than the one in the only available published paper which elaborates specifically on the payment system data generation process, Docherty and Wang (2010). Since their paper generates data for a DNS system, it only generates bilateral exposures and not individual payments, which greatly simplifies the generation since the number of generated items is much smaller. In fact, only the sender receiver matrix needs to be generated. Papers which sample data from historical data, on the other hand, like Koponen and Soramäki (1998), simplify the data generation process in that they only alter an existing sequence of transactions more or less significantly.

In the following second part, the calibration of the above-described data generation process with real historical data is described. When performing payment system simulations based on synthetically generated data, the assumptions of the data-generating process can be of high importance to the outcome of the simulation. As Manning et al. (2009, Box 3.3) propose, in order to calibrate the different parameters of the data generation, parameters from a real international large value payment system are observed and implemented in the data generation process. Docherty and Wang (2010) spend considerable effort on the calibration. As in this paper we are not calibrating the data to a specific system and participant community, we limit ourselves to a few realistic assumptions regarding key parameters, such as **(1)** the volume and **(2)** size of bilateral payments between the participants and **(3)** the overall timing of submitted payments.

The volume of payments **(1)** in bilateral relationships between banks is assumed to be correlated to their respective importance in the payment system. However, there are marked differences, which can be seen from the table below, with data from five typical examples of individual bank-to-bank

relationships taken from a European large value payment system during one week in December 2009. While the pair of large banks in domestic relationship 1 was transmitting thousands of payments from one to the other, numbers were much smaller between the pair of large banks in domestic relationships 2 and 3, respectively. Also when looking at cross-border payments, numbers was lower. The relationships between large banks and small banks show far fewer payments and small banks between each other seem to hardly make any payments at all. They may rather be using the more advanced cash management systems of larger banks (as correspondent banks) and only revert to direct payments for special reasons.

Table 1 Bilateral payment relationships between banks (one-way)

Bilateral payments between banks	Large to large	Large to large	Large to large	Large to large	Large to small
	Domestic relationship 1	Domestic relationship 2	Domestic relationship 3	International relationship	Domestic relationship 4
Average number of payments per day	3816	220	185	45	32
Average total value of payments per day	€5,963,385,702	€1,169,667,899	€305,566,450	€736,313,414	€37,007,861
Average value of payments	€1,562,814	€5,311,843	€1,651,711	€16,435,567	€1,142,218
Smallest payment value	€0	€30	€8	€5	€20
10-percentile	€123	€5,235	€342	€113	€370
25-percentile	€775	€36,000	€1,295	€1,502	€3,098
50-percentile (Median)	€4,890	€145,213	€11,110	€20,466	€29,750
75-percentile	€30,258	€378,948	€372,039	€538,428	€138,391
90 percentile	€266,249	€2,000,000	€1,802,874	€29,700,000	€875,706
Largest payment value	€1,275,000,000	€975,007,042	€400,000,000	€340,000,000	€50,000,000

The relationships between the banks in the week under consideration are evidently quite different in the total value transmitted, as can be seen from the table. What is more interesting is that the value of the payments is distributed quite differently in the selected examples. All distributions are skewed to the left, with the bulk of payments being rather small. The median is significantly smaller than the

average in all cases. While more than half of the payments between the two large banks in domestic relationship 1 are below € 5,000, in domestic large-to-large relationship 3 they are more than double that and in domestic large-to-large relationship 2 more than half of the payments have a value exceeding € 140,000. Also the cross-border payments are rather large compared to the domestic ones. The smallest payment value recorded in the sample is 5 cents between the two banks in the domestic large-to-large relationship 1; the largest – also between these two banks – is 1.275 billion. The following figure depicts the distribution of payments between banks. It can be seen that the distributions for the banks in domestic relationship 2, in the international relationship and in the large-to-small relationship (domestic relationship 4) are lognormal distributions peaking either around € 10,000 or € 100,000. The domestic relationship 3 banks' payment value distribution does not follow an approximate lognormal distribution in the period concerned. The above confirms findings by Soramäki et al. (2006) for Fedwire, where such lognormal distributions of the payments between participants were also found.

Figure 3 Distribution of bilateral payments across relationship types (one-way)
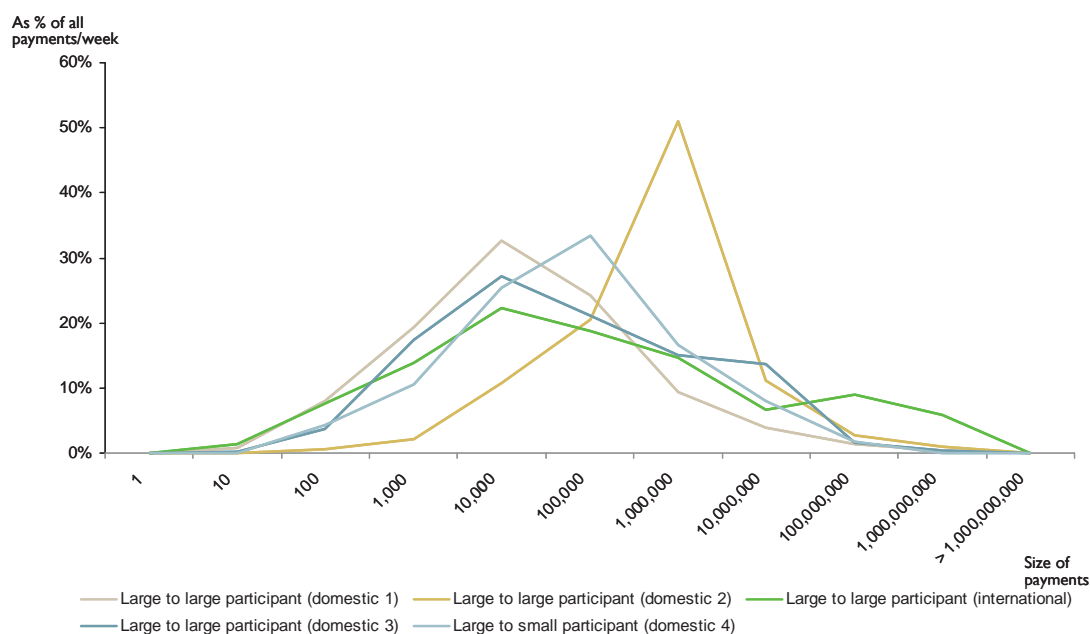
Note: Time range 14-18 December 2009

Source: Author's own representation

For this paper, in order to keep the data generation process manageable for ordinary computing power[12], we resort to rather simplified parameters for the volumes and values of bilateral payment relationships. Instead of taking account of different distributions, we resort to deterministic median payments between the participants according to the size of the sender and the receiver, respectively,

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[12] Standard Windows PC.

ignoring the high deviation of payments in reality. Given that the median transaction size between large participants in the typical examples above ranges from approximately € 5,000 to 145,000, we select a value at the lower end of the range, in particular since the payment system under consideration is much smaller than the one from which these typical relationships were taken. The remaining bilateral transaction sizes between the different types of participants are assumed to be smaller than the median payment between large participants.

Table 2 Sender-receiver-volume matrix

Median bilateral payment (no. of transactions)	Large participant	Medium participant	Small participant
Large participant	4,000	200	30
Medium participant	200	20	2
Small participant	20	2	1

Table 3 Sender-receiver-value matrix (€)

Median bilateral payment (€)	Large participant	Medium participant	Small participant
Large participant	10,000	8,000	6,000
Medium participant	8,000	4,000	3,000
Small participant	6,000	3,000	100

For the generated 20,000 transactions on one business day (7:00 – 19:00), the generated totals in terms of volume and value are reported in Table 4 and Table 5. The tables have the senders on the right and the recipients on the top. The network has 150 participants, of which 10 are large banks, 30 are medium sized banks and the remaining 110 are small banks. For simplification, there are no ancillary systems connected to the payment system.

Table 4 Total bilateral payments volume (No. of transactions)

| Total bilateral payments (no. of transactions)	Large participant	Medium participant	Small participant	Grand Total
Large participant	12,574	2,028	1,175	15,777
Medium participant	2050	578	220	2,848
Small Participant	760	223	392	1,375
Grand Total	15,384	2,829	1,787	20,000

Table 5 Total bilateral payments value (€)

Total bilateral payments (€)	Large participant	Medium participant	Small participant	Grand Total
Large participant	125,712,000	16,219,000	7,044,100	148,975,100
Medium participant	16,396,000	2,312,000	660,000	19,368,000
Small Participant	4,560,000	669,000	39,200	5,268,200
Grand Total	146,668,000	19,200,000	7,743,300	173,611,300

In comparison to international payment systems, the concentration ratio in the example system is rather high, both in terms of volume and value. Only the Continuous Linked System (CLS), the international foreign exchange settlement system, exhibits a higher concentration ratio, as can be seen in figure 04.[13]

Figure 4 Comparison of concentration ratios across different international systems
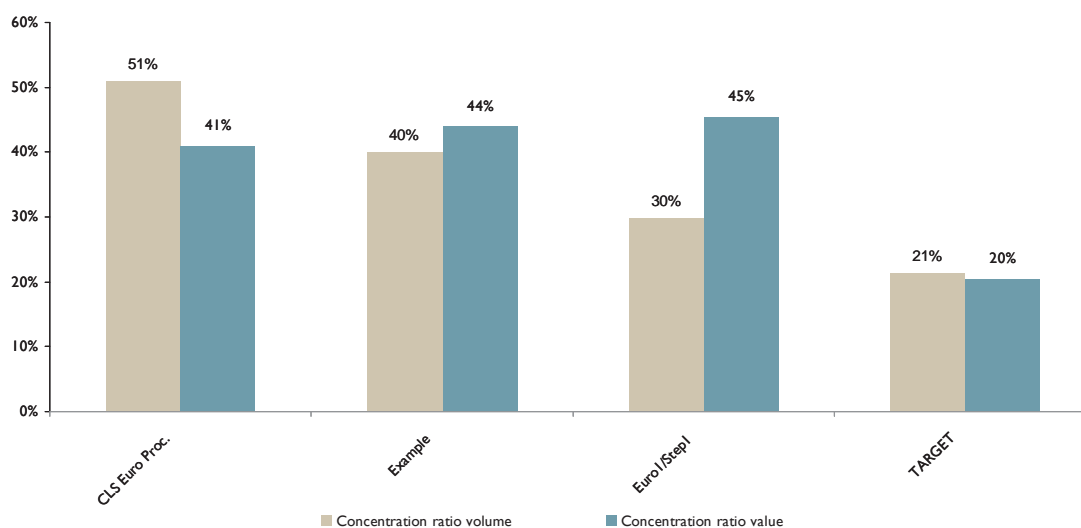
Note: Concentration ratio = share of five largest participants

Source: ECB (2008 data, 2007 for TARGET)

Another important parameter for the data generation is the distribution of payments over time. The literature reviewed reveals quite a variety of patterns across different payment systems.

- Van Oord and Lin (2005) find that in the Dutch System TOP, there were three peaks, at 07:00, 09:00-11:00 and 16:00 (opening hours 07:00 – 19:00).

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[13] See ECB Bluebook (2009)

- Arciero and Impenna (2001) find that in the Italian RTGS system BI-REL, there were two humps in mid-morning and at the end-of the day.

- In Japan, according to Bank of Japan (2001), about 50% of all transactions (in terms of volume) are settled in the first hour already.

- For Fedwire, Armantier et al. provide ample evidence of a late afternoon (15:00 – 16:00) maximum in the number and value of transactions.[14]

There are also systems in which the intraday payment pattern is specifically targeted by throughput rules. For example, in the CHAPS Sterling RTGS system in the United Kingdom, there are two throughput rules that stipulate that banks should make 50% of their payments by value by 12.00 and 75% of their payments by 14.30 on average each month, measured retrospectively (Buckle and Campbell, 2003). While none of the studies focuses on the variation of payment patterns across size classes or types of participants there is no reason to believe that such patterns are the same for all. Indeed, especially ancillary systems may have quite different patterns from other participants such as commercial banks. Advanced data generation could take account of such differences. However, in order to make the data generation manageable with ordinary computing power, we simplify by assuming the same distribution of payments across time for all participants. Following some of the studies, Table 6 in Appendix 1 shows that about half of the payments occur in the 4 morning hours, 20% in the 4 early afternoon hours and 30% in the 4 evening hours. The two humps in the morning and in the afternoon are clearly visible in Figure 5.

Figure 5 Distribution of payments over business day (value and volume)
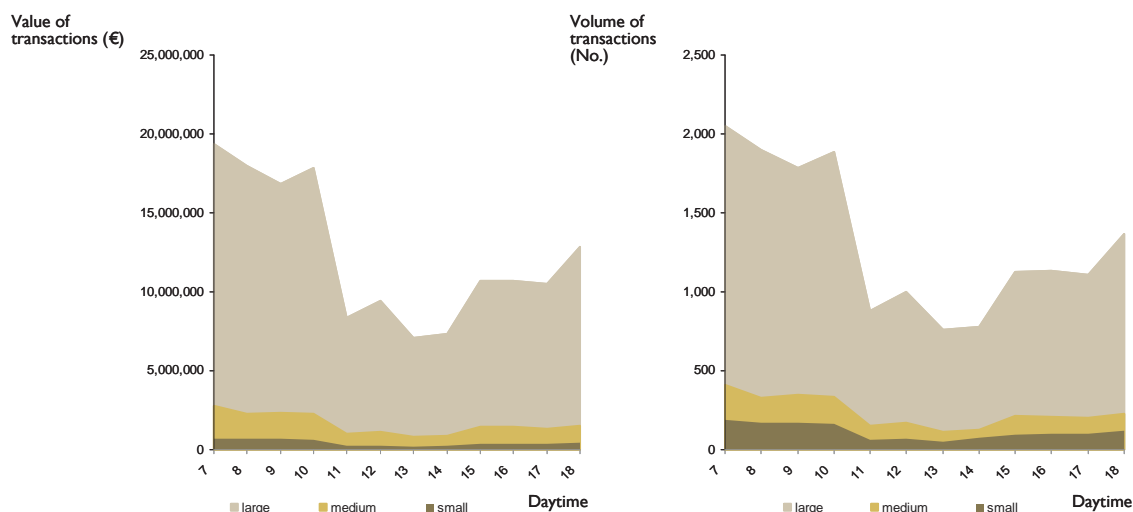
Source: Author's own representation

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[14] Especially the high-value payments are concentrated in the late afternoon.

In addition to the flow data or transaction data, another important component of payments data is the stock data. Besides (starting) account balances, this may include, for example, bilateral credit limits and collateral availability. For the sake of our analysis, we simplify to only two parameters, the starting balance and the intraday credit limits. All starting balances are set to zero, so the liquidity needs to be provided via intraday credit limits. There are two limiting levels of liquidity, the minimum and the maximum liquidity levels:

- Minimum liquidity = participant holds enough liquidity at the end of the day to settle all payments at the end of the day, net after receiving payments.

- Maximum liquidity = participant holds the full liquidity necessary to settle all transactions during the day immediately irrespective of incoming payments, i. e. the sum of all outgoing payments.[15]

Both levels are independent of the order of payments during the day and could in reality therefore quite easily be approximated ex-ante by the participants or the provider based on historical data. The assumption is made that participants do not actively manage their liquidity during the day by injecting or extracting liquidity on their accounts (see also Heijmans (2009)). Empirically, liquidity levels are influenced by several institutional and behavioural factors. The opportunity cost of holding liquidity on payment system accounts can depend on:

- Remuneration of liquidity holdings: the payment system provider may encourage participants to hold liquidity in the system, for example by offering outright intraday interest payments

- Collateral availability: availability and quality of eligible collateral on participants' books.

- Collateral requirements: restrictiveness of the payment system provider in accepting collateral. Possible is the application of haircut scales or certain credit quality thresholds.

- Convertibility of collateral: if the payment system provider offers other services than payment settlement to the participant, collateral may be flexibly used for the collateralisation of other service, such as longer term credit.

- Technical arrangements: payment system provider may offer participants to hold collateral/liquidity in other (ancillary) systems, yet offer liquidity bridges, i. e. quick redistribution of liquidity across systems. Data on these liquidity holdings may be separate from the data of the system concerned and consequently, in particular when using historical data, the true availability of liquidity may be underestimated.

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[15] The concept of "maximum liquidity" deviates from the "upper bound liquidity" which is used in most of the literature (e. g. Heijmans (2009). The upper bound liquidity in the literature is also the liquidity which is necessary to settle all transactions immediately, but takes into account incoming payments, so it depends on the order of payments. This upper bound liquidity will be harder to identify for payment system participants than the maximum liquidity, as the order of the payment flow will be difficult to predict. It is therefore likely that for example cautious participants will put the maximum liquidity on their account at the beginning of the day. Our concept of "minimum liquidity" does, however correspond to the "lower bound liquidity" referred to in the literature.

These factors may play a role in the fact that different studies seem to indicate very different average levels of liquidity holdings. For instance, Heijmans (2009) finds that in NL, historically available liquidity was far above upper bound liquidity at least for some participants, whereas Puhr and Schmitz (2009) report that for their sample in the Austrian Large Value Payment System Artis, liquidity holdings usually amount to only 50% of the total daily outgoing payments of participants.

4.   Simulations

The first step is the establishment of the benchmark simulation. For this, an RTGS system is defined with the following settings:

- Entry algorithm: the basic entry algorithm settles incoming payments upon receipt, if sufficient liquidity is available on the concerned account or if the intraday credit limit permits it. If the payment cannot be settled upon receipt, the payment is queued.

- Queue: payments which are queued are settled upon arrival of additional liquidity in first-in-first-out (FIFO) order.

- End: At the end of the day, yet unsettled payments are scheduled for the following day.

In all simulations, all participants have a starting balance on their account of 0. In the benchmark simulation, their intraday credit limit with the payment system is set to the minimum, that is, to the net outgoing payments or to zero if the net outgoing payments are negative. The latter is the case if the participant receives more payment value than it sends, as credit limits cannot be negative. The consequence is that while at the end of the business day all payments of all participants will be settled, payments will be queued during the day.

In the alternative scenarios, liquidity in the system is increased by setting the intraday credit limits with the payment system to different levels between the minimum and the maximum liquidity, that is, to the total sum of all outgoing payments.

$$L_P = L_{P(Min)} + \left(\alpha \cdot \left(L_{P(Max)} - L_{P(Min)}\right)\right), 0 \leq \alpha \leq 1$$

The variable α designates different liquidity requirement levels. These liquidity levels would not necessarily need to be implemented by this simple rule, which does not exist in reality, to our knowledge. Yet also in reality, liquidity requirements will depend in one way or another on the size of the participant.[16] It is assumed that the participants need to fully collateralise the increased credit limit so that participants will incur a cost. The increase in the liquidity requirement could be the consequence of a decision by the payment system provider or the overseer, for example, in order to
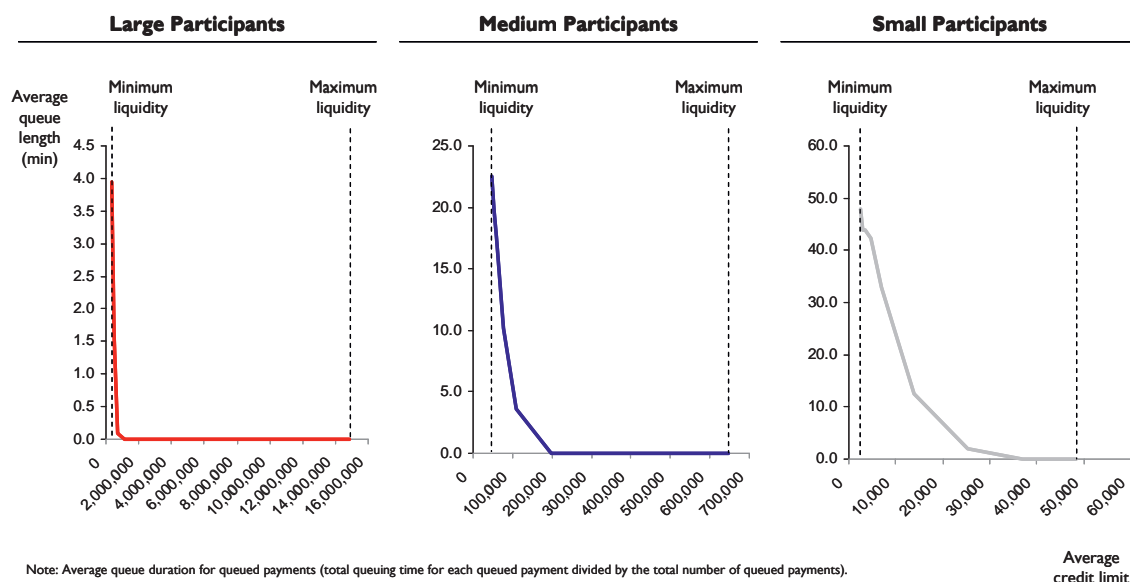
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[16] In real payment systems run by central banks, such as Fedwire or TARGET2, liquidity requirements may be derived, e. g. as composed of minimum reserve balances and clearing balance requirements (Afonso and Shin (2008)).

reduce liquidity risk in the system. Other ways of determining the amount of increased liquidity would be possible instead of increasing liquidity to a certain percentage between the minimum and the maximum liquidity, which is dependent on the payment traffic. For instance, all participants could be required to post the same amount of liquidity, irrespective of their importance, or they could be expected to raise liquidity related to specific payments, relative to their balance sheet, their risk properties or their capital share in the payment system if they are owners.

Figure 6 shows the trade-off curves between credit limits, i. e. liquidity requirements α, (on the x-axis) and queue lengths, i. e. payment delays, (on the y-axis) as averages for each group of participants. As measurement for the payment delay, we use the average queue duration for queued payments, which equals the total queuing time for each queued payment divided by the total number of queued payments.[17] The curve is much steeper for the large participants than for the small participants. The lines are based on the twelve simulations performed, namely at the 0%, 1%, 2%, 5%, 10%, 25%, 50%, 75%, 90%, 95%, 99% and 100% levels. They are not smooth, because the average payment delay can depend significantly on whether individual payments can be made or not. This is true in particular for the smaller participants since they only make a few payments, which can significantly alter the average payment delay of the group of small participants. Thus the lines can only indicate the rough direction of the trade-off curves.

Figure 6: Trade-off curves: Queue length as function of liquidity by participant group

Note: Average queue duration for queued payments (total queuing time for each queued payment divided by the total number of queued payments).

Source: Author's own representation

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[17] The intraday pattern of the queue is shown in Appendix 2.

The scales on both ordinate and abscissa are different in the figure above. For instance, for large participants, maximum liquidity equals approximately 16,000,000 while the minimum equals approximately 400,000, thus the maximum is 40 times the minimum. This ratio is far lower for the small participants at about 25. For better comparison, the axes are thus standardised in Figure 7. For each group of participants, each data point's coordinates are re-calculated as percentages of their respective maxima (for the credit limit on the x-axis as a percentage of the maximum of the interval between maximum and minimum). For convenience, the curves are coloured in the same way as in Figure 6 for each group.

Figure 7: Standardised trade-off curves between liquidity and payment delay
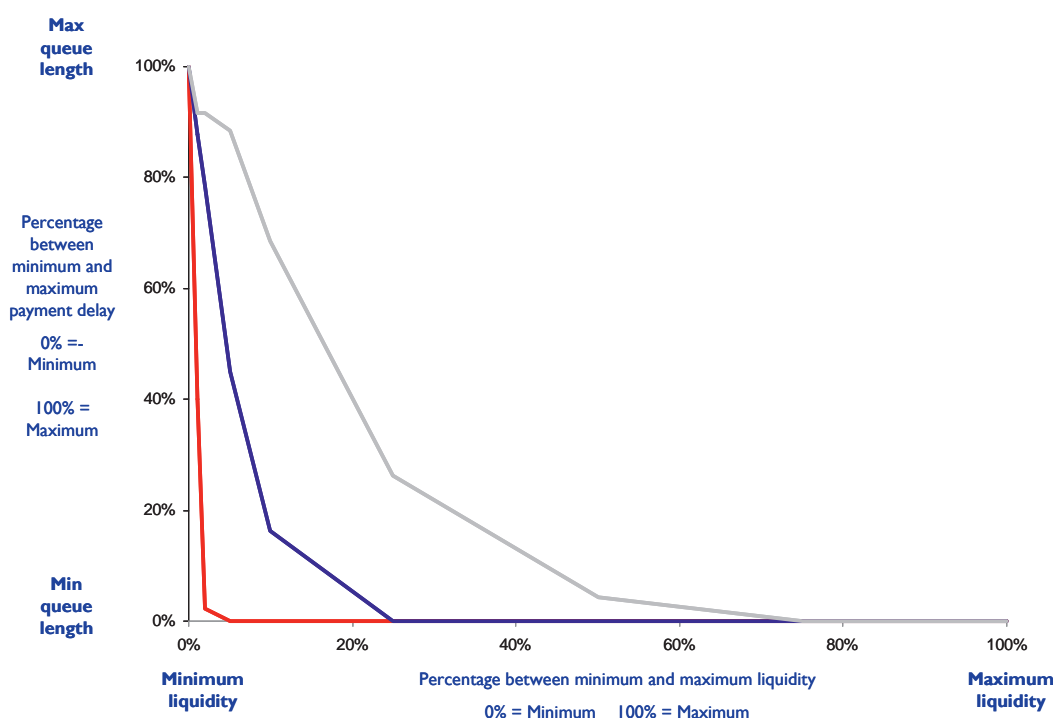
Source: Author's own representation

From Figure 7 it becomes clear that the effect of raising liquidity requirements (credit limits) will be different for each group of participants, depending on the interval, in which this raise take place. In an environment with very low liquidity (on the left side of the figure), a small increase in liquidity will strongly decrease the payment delay of large participants, but less so for the small and medium participants. If there is high liquidity already, for instance at the 40% point on the x-axis, a further increase will only reduce the payment delays incurred by the small participants, as the other two groups have already reduced their delays to 0. However, it needs to be kept in mind that the highest average payment delay incurred by large participants is less than four minutes (see Figure 6 left-hand), while for small participants it is about 50 minutes. Thus it is difficult to assess the overall effect of changing liquidity requirements by participant group, without assumptions about the utility functions.

In the remainder of this section, it will be shown that the utility participants derive from changes in the liquidity requirements will be different. Participants are faced with the choice between two negatives or "bads" (as opposed to "goods"): liquidity requirement and payment delay. As in traditional microeconomics, they will be indifferent between some combinations of these two "bads". The combinations from which participants derive the same utility form an iso-utility or indifference curve. In a two-dimensional representation with each of the "bads" on one axis, these indifference curves are assumed to be concave with respect to the origin as both payment delay and liquidity requirements are perceived as cost by the participants.

In Figure 8 the participants' stylised type-dependent indifference curves are derived, starting with a medium participant in the top-left quadrant. The marginal rate of substitution between the two "bads" (additional delay and additional liquidity requirement), the slope of the indifference curves is negative, while the slope of the marginal rate of substitution is positive,[18] which yields the concave form with respect to the origin, which is the optimum here. The scale of the axes is given by the minimum and maximum of payment delay and liquidity requirement, not the absolute values. This standardisation leads to differently shaped indifference curves. 100% on the y-axis correspond to 4 minutes for the large participants, 22 minutes for medium-sized participants and 50 minutes for small participants. Thus, even if the utility functions are the same for each group, the point with the coordinates 100% payment delay and 0% liquidity requirement (minimum liquidity), will be less beneficial for small participants than for large ones. Likewise, 100% on the x-axis mean 25 times minimum liquidity for small participants but 40 times the minimum for large participants. Thus large participants will derive lower utility from this combination than small ones.

- As a consequence, compared to the indifference curves of medium participants, those of small participants (top-right quadrant) will be rotated counter-clockwise and look flatter as they have a higher aversion to payment delay. The path of optima (dotted arrow) will be flatter, indicating that the effect of payment delay changes on utility will be higher than that of liquidity requirement changes.

- The indifference curves of large participants (bottom-left quadrant) will be rotated clockwise and look steeper, as they will have higher aversion towards additional liquidity requirements compared to the medium participants. The path of optima (dotted arrow) will be steeper, indicating that the effect of liquidity requirement changes on utility will be higher than that of payment delay changes.

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[18] The second derivative is negative. This implies that if the participant is facing very high liquidity requirement, even a small decrease will raise her utility more than the same decrease would if the participant were already at a very low level of liquidity requirement.

Figure 8: Indifference curves
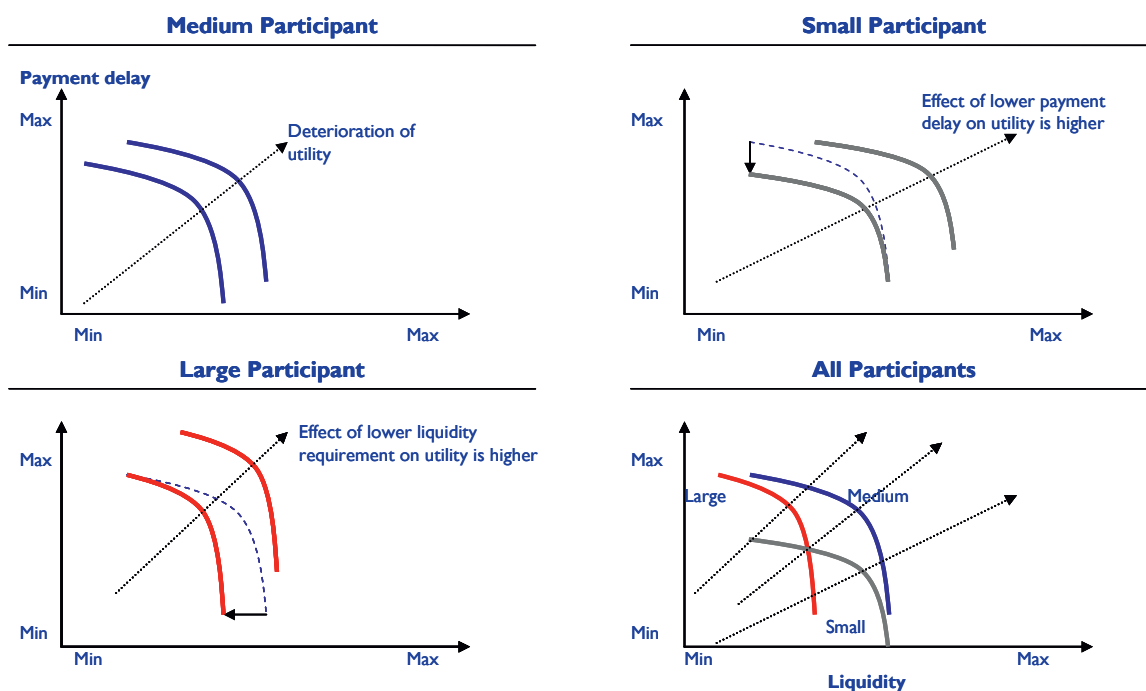
Source: Author's own representation

We combine the findings in Figure 7 and Figure 8 in Figure 9. If the liquidity holding decision is left to each participant, each will choose the combination which yields the highest utility. Graphically, this will be the point, where the trade-off curve is tangential to the indifference curve which is the closest to the origin. The graph reveals an interesting finding: As expected, large participants will be closest to the origin, i. e. the optimum with minimum liquidity and zero payment delay. This is due to economies of scale. However, the three types of participants will also favour *different* liquidity requirements (on the x-axis), as the large ones will prefer low requirements and the small ones are high requirements. The indifference curves will be tangential to the trade-off curves at different points, due to their rotation, but also due to the different shapes of the trade-off curves. A liquidity saving feature proposed by a payment system provider may thus have an unequal effect on the utility the payment system offers to different participant types, which should be taken into account by overseers assessing the consequences of proposed changes. The choice of the liquidity requirement may have an impact on the competitive position of the participant groups.

Figure 9 Trade-off between liquidity and queue length and indifference curves
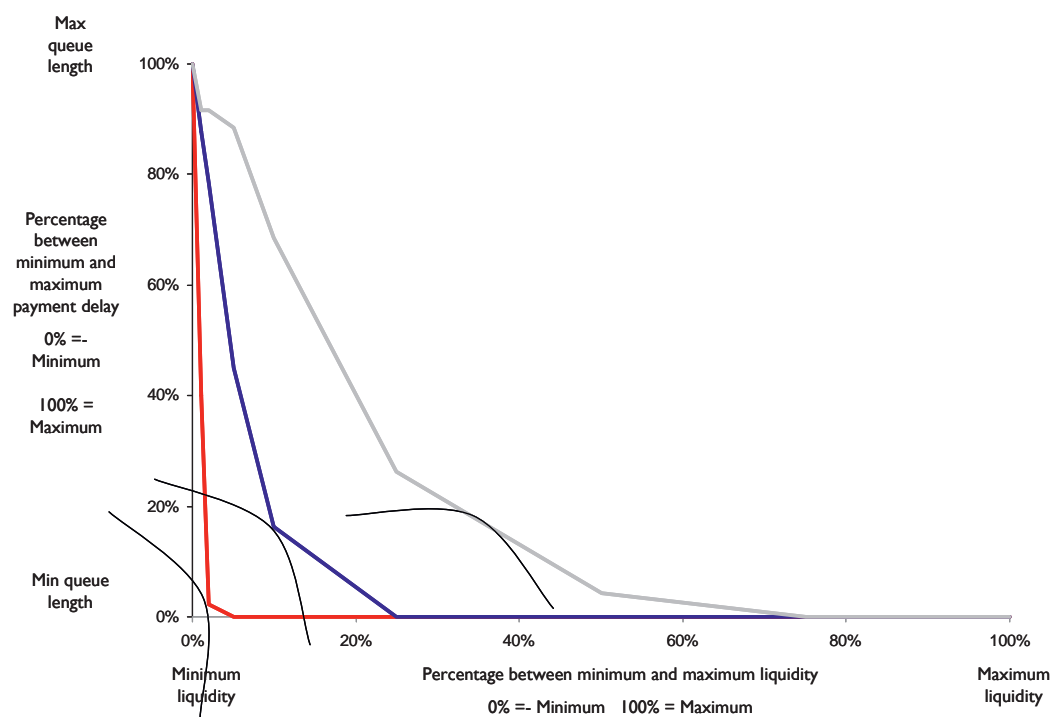
Source: Author's own representation

For the operator of the payment system, knowledge of this possibly asymmetric effect of the design change is important, not least because they may have to be taken account of in the pricing scheme. The design of the system will influence the behaviour of the participants and ultimately decide which type of entities will participate in the system (Fujiki et al. (2008)), making it vital from a business point of view to know about the consequences. For the oversight function, such analysis of the effects on the competitive situation of the participants is currently not foreseen in relevant frameworks, such as the Core Principles for systemically important payment systems of the of the Committee on Payment and Settlement Systems (CPSS). None of the CPSS (2001) core principles refers explicitly to the effects of payment system design on the competitive situation. Core principle IX, which calls for systems to have objective and publicly disclosed criteria for participation, which permit fair and open access, could be interpreted as opening a door for this kind of analysis, since certain design features might be interpreted as "hidden" obstacles to fair access. However, the current implementation guidelines do not foresee this and indeed competition issues could be left to the competent authorities, not central banks' payment system oversight functions. Yet the analysis shows that the recommendations and decisions of the oversight function itself may have an impact on the competitive situation. If the overseer decides to recommend a raise in the liquidity requirements in order to decrease liquidity risks in a system, this may well give an advantage to a certain group of participants over another. In the analysis above, small participants would be more likely to benefit from this, depending on what the starting point would be.

5. Conclusion

Simulations of payment systems help operators design and maintain a payment system in compliance with the targeted clients' needs and help overseers make informed assessments of the overseen infrastructure by enabling them to analyse the effect of changes in the complex technical environment. They can help compare across systems or analyse the effect of changes to a system, and enhance the general preparedness of overseers and operators as well as the foundations of decisions on concrete proposed measures of payment system providers, participants or overseers.

A major challenge for researchers in the field is the availability of transaction-level data, which is necessary to conduct the simulations. Access to historical data is mostly restricted and confidential and can therefore not be freely used for discussion and publication. The use synthetically generated data, as in this paper, avoids this problem. In addition, the data generation process is known to the researcher and does not include hidden behavioural assumptions or noise due to events outside the system, unless introduced by the researcher.

In the concrete case of this paper, the effects of changing liquidity requirements on different types of participants have been analysed. Besides the expected effect that more liquidity in the system will reduce payment delays and therefore liquidity and credit risk, there is an asymmetric effect on the participants. For example, over much of the relevant interval, large participants will receive no benefit from increasing liquidity requirements. If the liquidity requirement is higher than what would be necessary to ensure zero payment delay, the large participants would only have to post more liquidity/collateral. The small participants, on the other side, are very likely to benefit from the additional liquidity requirement, since their payment delays will still reduce over much of the relevant interval.

The simulations show that there may be an impact on the competitive situation of the participants by a change in the design of the system. This competitive impact could not be identified easily without the simulations. For operators, the assessment of the impact may be a business issue, but it may also have significant policy implications for the oversight function. Competitive consequences are not covered explicitly by any of the Committee on Payment and Settlement System's 2001 "Core Principles for Systemically Important Payment Systems". One might interpret core principle IX as relevant, because it could be seen as covering open discrimination against access of certain participants as well as implicit (or "hidden") disadvantages to certain participants within the system. This would be similar to international trade policy where the concepts of tariff and non-tariff barriers to trade are seen as inhibitions to trade.

The results of simulations depend critically on the employed data set, whether this is data of real transactions or synthetic data. In this paper, the data set consists of synthetically generated data, which has the advantage of having full information on the data-generating process, including any behavioural assumptions made. However, depending on the proposed analysis, the calibration of the data generation process will critically influence the results. The data generation process employs

assumptions which are realistic for many payment systems. This is true for the concentration ratio, the timing of payments, average size and volume of payments between different types of participants. On the other hand, there are less realistic assumptions. For instance, payments from one participant to another will not always have the same size and size may depend on the timing of the payment. Also, the timing may be different for the various participants.

Further research can immediately follow a number of directions, based on this paper:

- Refined calibration of the data generation process: The refinement of the calibration of the data generating process may be one of the avenues for future research. This includes both more analysis of the distributions of payment sizes, volumes and timing patterns between different types of participants. Variations of the data generation with different participant communities and behaviour should also follow.

- Determining alternative liquidity need vs. payment delay trade-off curves: With respect to the specific scenario of increasing liquidity, alternative requirements could be simulated, such as an increase related to the capital share or balance sheet size of the participants.

- Effects of pricing structure: The pricing structure may affect the payment patterns of participants. From the above analysis it is clear that participants with high volumes and low average payment values achieve higher utility levels, yet pricing structures may set different incentives.

- Studying effect of changes in the design of the system: The current paper uses a very simple RTGS system. More research on the effect of more complex structural characteristics of payment systems, such as settling algorithms or bilateral limits could be executed conveniently with the existing data sets.

- Application to real transaction data: The application of the analysis to real payment transaction data, would be reveal the real beneficiaries of the current and alternative structural settings of payment systems.

- Liquidity is assumed to be costly to participants since collateral has to be pledged. However, it would need to be further investigated why in some systems liquidity is so abundant that even the maximum liquidity level is exceeded.

Appendix 1: Intraday distribution of payments

Table 6 Distribution of payments over time

Introduction time (volume)	From large participants	From medium participants	From small participants	Grand Total
07:00 – 08:00	2,053	410	192	2,655
08:00 – 09:00	1,897	332	170	2,399
09:00 – 10:00	1,783	349	174	2,306
10:00 – 11:00	1,886	335	163	2,384
11:00 – 12:00	882	153	64	1,099
12:00 – 13:00	1,002	170	68	1,240
13:00 – 14:00	758	117	52	927
14:00 – 15:00	776	127	75	978
15:00 – 16:00	1,128	213	94	1,435
16:00 – 17:00	1,135	211	99	1,445
17:00 – 18:00	1,109	200	101	1,410
18:00 – 19:00	1,368	231	123	1,722
Grand Total	15,777	2,848	1,375	20,000

Appendix 2: Intraday queue pattern

For illustration, the following graph shows the development of submitted payments, settled payments and queued payments per hour, for the case of minimum liquidity. It can be seen that the queue is very large in the beginning of the day in value and in volume terms and decreases over the day. In the afternoon, when payment activity picks up again, the queue only builds up moderately as some participants have built up liquidity during the day.

Figure 10: Intraday change of queue
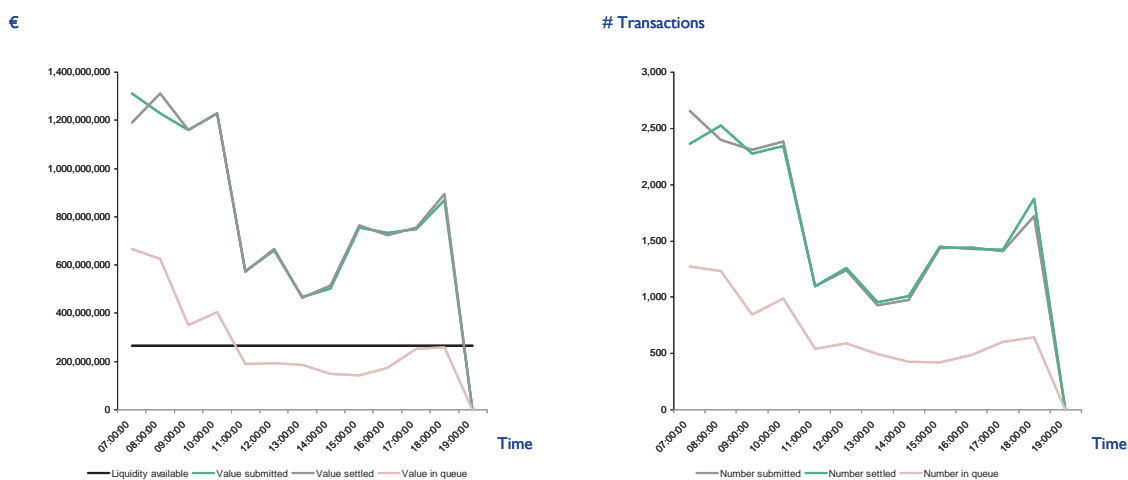
Source: Author's won representation

Bibliography

1. AFONSO, G. and SHIN, H. S. (2008) "Systemic Risk and Liquidity in Payment Systems", Federal Reserve Bank of New York Staff Reports No. 352, October 2008.

2. ARCIERO, L. and IMPENNA, C. (2001) "Time Pattern of Payments in the Italian RTGS System: The Effect of Institutional and Financial Variables", Banca d'Italia Market and Payment Systems Oversight Publication

3. ARCIERO, L., BIANCOTTI, C., D'AURIZIO, L. and IMPENNA, C. (2008) "An Agent-Based Model for Crisis Simulation in Payment Systems", Banca d'Italia Working Paper No. 686, 08/2008, Rome

4. ARMANTIER, O., ARNOLD, J. and MCANDREWS, J. (2008) "Changes in the Timing Distribution of Fedwire Funds Transfers", Economic Policy Review, September 2008, Vol. 14, No. 2

5. BAKŠYS, D. and SAKALAUSKAS, L. (2007) "Modelling, Simulation and Optimisation of Interbank Settlements", Kaunas University of Technology Panevėžys Institute, Discussion Paper presented at 5th Bank of Finland simulator workshop, Helsinki, 28-29 August 2007.

6. BANK OF JAPAN (2001) "Real-Time Gross Settlement (RTGS) in Japan: An Evaluation of the First Six Months", Bank of Japan Quarterly Bulletin November 2001, Tokyo

7. BECH, M. and HOBIJN, B. (2006) "Technology Diffusion within Central Banking: The Case of Real-Time Gross Settlement", Federal Reserve Bank of New York Staff Reports, September 2006, New York.

8. BEDFORD, P., MILLARD, S. and YANG, J. (2004) "Assessing Operational Risk in CHAPS Sterling: a Simulation Approach" in Bank of England (2004) "Financial Stability Report", No. 16, June 2004

9. BOWMAN, M. (1995) "Simulation Modelling Of Large Value Transfer Systems", paper prepared for Eurobanking 1995, Association For Payment Clearing Services.

10. BUCKLE, S. and CAMPBELL, E. (2003) "Settlement bank behaviour and throughput rules in an RTGS payment system with collateralised intraday credit", Bank of England Working Paper No. 209

11. COMMITTEE ON PAYMENT AND SETTLEMENT SYSTEMS (2001), "Core Principles for Systemically Important Payment Systems", Basel, January 2001

12. DANMARKS NATIONALBANKEN (2005) "Effect of Queue Facilities for Kronos Participants" Appendix C in Danmarks Nationalbanken "Payment Systems in Denmark", 2005

13. DOCHERTY, P. and WANG, G. (2010) "Using Synthetic Data to Evaluate the Impact of RTGS on Systemic Risk in the Australian Payments System", Journal of Financial Stability, Volume 6, Issue 2, June 2010, pp. 103-117.

14. ENGE, A. and ØVERLI, F. (2006) "Intraday Liquidity and the Settlement of Large Value Payments: a Simulation-Based Analysis" in Norges Bank Economic Bulletin 1/2006

15. EUROPEAN CENTRAL BANK (2009), "Eurosystem Oversight Report", Frankfurt, November 2009

16. FUJIKI, H., GREEN, E. J. and YAMAZAKI, A. (2008). "Incentive efficient risk sharing in a settlement mechanism" Journal of Economic Theory, vol. 142(1), pp. 178-195

17. FURFINE, C. (1999a) "The Pricing of Bank Lending and Borrowing: Evidence from the Federal Funds Market", Bank of International Settlements Working Paper, No. 62, 1999

18. FURFINE, C. (1999b) "Interbank Exposures: Quantifying the Risk of Contagion", Bank of International Settlements Working Paper, No. 70, 1999

19. GLASER, M. and HAENE, P. (2009) "Liquidity effects of a participant-level operational disruption in the Swiss Interbank Clearing System" in LEINONEN, H. (ed.) "Simulation analyses and stress testing of payment networks", Bank of Finland Scientific Monograph E:31(2005), pp. 297–320.

20. HEIJMANS, R. (2009) "Simulations in the Dutch Interbank Payment System: A Sensitivity Analysis", De Nederlandsche Bank Working Papers 199, January 2009

21. HUMPHREY, D. B. (1986) "Payments Finality and Risk of Settlement Failure", in: SAUNDERS, A. and WHITE, L. (eds., 1986) "Technology and the Regulation of Financial Markets", Lexington Books/Salomon Brothers Center Series on Financial Institutions and Markets, New York, 97-120

22. KOPONEN, R. and SORAMÄKI, K. (1998) "Intraday Liquidity Needs in a Modern Intraday Payment System – A Simulation Approach", Bank of Finland Studies E:14, 1998

23. LEINONEN, H. and SORAMÄKI, K. (1999) "Optimizing Liquidity Usage and Settlement Speed in Payment Systems", Bank of Finland Discussion Papers 16/99

24. LEINONEN, H. and SORAMÄKI, K (2003a) "Simulating Interbank Payment and Securities Settlement Mechanisms with the BoF-PSS2 Simulator", Bank of Finland Discussion Papers 23/2003

25. LEINONEN, H. and SORAMÄKI, K (2003b) "Simulation: A Powerful Research Tool in Payment and Settlement Systems", Journal of Financial Transformation, Vol. 12

26. LEDRUT, E. (2007) "Simulating Retaliation in Payment Systems: Can Banks Control Their Exposure to a Failing Participant", De Nederlandsche Bank Working Paper No. 133, March 2007

27. MANNING, M., NIER, E. and SCHANZ, J. (Eds.) (2009) "The Economics of Large-Value Payments and Settlement – Theory and Policy Issues for Central Banks", Oxford University Press, Oxford

28. MAZARS, E. and WOELFEL, G. (2005), "Analysis, by simulation, of the impact of a technical default of a payment system participant", in LEINONEN, H. (ed.) "Liquidity, Risks and Speed in Payment and Settlement Systems – A Simulation Approach", Bank of Finland Scientific Monograph E:31(2005), pp. 297–320.

29. MCANDREWS, J.J. and WASILYEW, G. (1995) "Simulations of Failure in a Payment System" Economic Research Division Working Paper 95-19, Federal Reserve Bank of Philadelphia

30. SCHMITZ, S. W. and PUHR, C. (2007) "Risk Concentration, Network Structure and the Assessment of Contagion in the Austrian Large Value System ARTIS", in LEINONEN, H. (ed.) "Simulation Studies of the Liquidity Needs, Risks and Efficiency in Payment Networks", Bank of Finland Scientific Monograph E:39 (2007), Helsinki, pp. 183-226

31. SCHMITZ, S. W. and PUHR, C. (2009) "Structure and Stability in Payment Networks – A Panel Data Analysis of ARTIS Simulations", in LEINONEN, H. (ed.) "Simulation Studies of the Liquidity Needs, Risks and Efficiency in Payment Networks", Bank of Finland Scientific Monograph E:42 (2009), Helsinki, pp. 145-188

32. SORAMÄKI, K., BECH, M. L., ARNOLD, J., GLASS, R. J., BEYELER, W. E. (2006) "The Topology of Interbank Payment Flows", Federal Reserve Bank of New York Staff Report, New York No. 243, March 2006

33. UPPER, C. (2007) "Using Counterfactual Simulations to Assess the Danger of Contagion in Interbank Markets", Bank of International Settlements Working Paper No. 234

34. VAN OORD, A. and Lin, H. (2005) "Modelling Inter- and Intraday Payment Flows", De Nederlandsche Bank Working Paper No. 74, Amsterdam

35. WALLACE, N. (2000) "Knowledge of Individual Histories and Optimal Payment Arrangements", Federal Reserve Bank of Minneapolis Quarterly Review, Vol. 24, No. 3, Summer 2000, pp. 11–21