Unveiling the Interplay between Central Bank Digital Currency and Bank Deposits

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Abstract

We analyze the risks to financial stability following the introduction of a central bank digital currency (CBDC). CBDC competes with commercial bank deposits as the household's source of liquidity. We revisit the result in the existing literature regarding the equivalence of payment systems by considering different degrees of substitutability between payment instruments and introducing a collateral constraint that banks must respect when borrowing from the central bank. When CBDC and deposits are perfect substitutes, the central bank can offer loans to banks that render the introduction of CBDC neutral to the real economy. Hence, there is no effect on financial stability. However, when CBDC and deposits are imperfect substitutes, the central bank cannot make the bank indifferent to the competition from CBDC. It follows that the introduction of CBDC has real effects on the economy. Our dynamic analysis shows that an increase in CBDC demand leads to a drop in bank profits but does not raise significant concerns about CBDC crowding out deposits.

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1 Introduction

Digital currencies have been around for a while, but their potential significance in the global economy has increased recently due to a growing demand for digital payment methods for retail purposes and the gradual decline of the use of cash for transactions in many economies. Besides the private digital means of payment currently in circulation, many central banks have been investigating the possibility of launching a central bank digital currency (CBDC). CBDCs are central bank liabilities denominated in an existing unit of account that serve as a medium of exchange and a store of value.[1] Were CBDCs to be issued to retail customers (i.e., households), they would likely be a digital form of cash that shares features with banknotes, as they are universally accessible but in a digital form, similar to central bank reserves.

Alongside an intense policy debate, a growing academic literature on the broader economic implications of CBDCs has emerged. A primary concern for central banks when considering the issuance of a CBDC is the risk to financial stability, intended as the risk of CBDC disintermediating the banking sector as the household substitutes CBDC for bank deposits, potentially leading to financial instability. Introducing a CBDC will likely alter the equilibrium in the real economy, as it will represent a novel payment option alternative to cash and commercial bank deposits. The macroeconomic consequences of introducing a CBDC will impact individuals and financial institutions. This paper analyses the implications of introducing a retail CBDC, particularly concerning its relationship with bank deposits.

The recent literature establishes an equivalence result between different payment systems. Brunnermeier and Niepelt (2019) consider a simplified scenario without reserves and resource cost of providing liquidity, while Niepelt (2022) includes a reserves layer and shows that introducing CBDC has no real consequences if the private and the public sectors are equally efficient in operating payment systems. For this to happen, the central bank must refinance the bank at a lending interest rate that supports the bank's original portfolio position so that central bank funding exactly replaces the lost deposits for the bank. Niepelt (2022) assumes central bank loans are extended against no collateral. However, the collateral requirement imposed by central banks when lending to commercial banks is potentially important for how introducing a CBDC may affect the banking sector and the real economy.[2] In practice, central banks lend to commercial banks (i.e., discount window lending) against collateral to support the liquidity and stability of the banking system. The liquidity provided by central banks helps financial institutions to manage their liquidity risks efficiently. These loans are

[1] As defined by the Committee on Payments and Market Infrastructures (CPMI) of the Bank of International Settlements.

[2] See, e.g., Burlon et al. (2022) and Williamson (2022).

issued at an administered discount rate and must be collateralized to the satisfaction of the issuing central bank. In the euro area, banks can make use of the marginal lending facility, which enables banks to obtain overnight liquidity from the European Central Bank against sufficient eligible assets. In the United States, the Federal Reserve offers different types of discount window credit, which must be collateralized. The discount window mechanism has become increasingly important after the Global Financial Crisis. Additionally, Niepelt (2022) equivalence analysis only considers CBDC and deposits that are perfect substitutes. However, CBDC will likely not be a perfect substitute for bank deposits [see, e.g., Bacchetta and Perazzi (2022)]. In this paper, we will revisit this equivalence result in the literature and explore its implication in terms of financial disintermediation by adding reality in two ways, (i) introducing a financial friction for central bank lending to banks (i.e., the collateral requirement), and (ii) considering different degrees of substitutability between CBDC and bank deposits (i.e., imperfect substitutability).

This work addresses the potential risk to financial stability following the introduction of a CBDC and investigates the impact of the substitutability between CBDC and bank deposits on this risk. Specifically, we investigate if the issuance of CBDC leads to disruptions in financial markets, thereby positing a risk to financial stability due to bank disintermediation rather than bank runs. To address this concern, we develop a model with a CBDC and deposits issued by a bank subject to a collateral requirement when borrowing from the central bank. The framework builds on Niepelt (2022) and is an extension of the model by Sidrauski (1967) that embeds a banking sector, bank deposits, government bonds, reserves, and a CBDC into the baseline real business cycle model. Households value goods, leisure, and the liquidity services that deposits and CBDC provide. Non-competitive banks invest in capital, reserves, and government bonds and fund themselves through either deposits or borrowing from the central bank. Firms produce using labour and physical capital. Finally, the consolidated government collects taxes, pays deposit subsidies, invests in capital, lends to banks against collateral, and issues CBDC and reserves.

In the first part of the paper, we revisit the result in the existing literature regarding the equivalence between payment systems when introducing a collateral constraint for central bank lending to banks. We study the problem under different degrees of substitutability between CBDC and bank deposits. Our findings reveal that when CBDC and deposits are perfect substitutes, the introduction of CBDC has no real consequences as long as (1) the resource cost per unit of effective real balances is the same for CBDC and deposits and (2) the central bank offers a loan rate that induces the non-competitive bank to maintain the same balance sheet positions as before the introduction of CBDC. Our equivalent loan rate is lower than the one obtained in Niepelt (2022) because of the collateral requirement the bank must respect when borrowing from the central bank. In particular, when the collateral constraint becomes more restrictive, the equivalent loan rate must be lower. However, when CBDC and deposits are imperfect substitutes, the central bank cannot make the bank indifferent to the competition from CBDC. This is because the central bank lending rate does not leave the bank's profits unchanged. A change in the bank's profitability implies that the new policy introducing CBDC does not guarantee the same equilibrium allocations as before, implying that the issuance of CBDC has real effects on the economy.

In the second part of the paper, we explore how an increase in the demand for CBDC affects the real economy and if it could potentially lead to the crowding out of deposits. To do so, we study the economy's responses to changes in the household's relative preferences for CBDC over bank deposits. We consider an economy where the household holds CBDC and deposits, and we assume that the payment instruments are imperfect substitutes. First, we look at how the economy reacts to a positive shock to the liquidity benefit of CBDC over deposits. We find that the demand for CBDC increases without crowding out deposits, but the bank's profit drops due to the lower market power of the bank. The same holds true when looking at the economy's reaction to a negative shock to the substitutability between CBDC and deposits. In this case, deposit demand drops mildly because of the reduced substitutability between payment instruments. The intuition is that when CBDC and deposits are less substitutable, it takes more of one to replace the other. This result relates to Bacchetta and Perazzi (2022), but in our case, the decrease in deposits is relatively small and should not raise significant concerns about CBDC crowding out deposits

Our work contributes to the recent literature examining the impact of the introduction of CBDC on commercial banks. For instance, Chiu et al. (2023) develop a micro-founded general equilibrium model calibrated to the U.S. economy and find that CBDC expands bank intermediation when the price of CBDC falls within a certain range while leading to disintermediation if its interest rate exceeds the upper limit of that range. Using a dynamic banking model, Whited, Wu, and Xiao (2023) assume that banks rely on deposits and wholesale funding and that the latter can potentially substitute deposit loss. Depending on whether CBDC pays interest or not, the synergies between deposits and lending can attenuate the impact of CBDC. In another study, Burlon et al. (2022) construct a quantitative euro area dynamic stochastic general equilibrium (DSGE) model, where banks must post government bonds as collateral to borrow from the central bank. They investigate the transmission channels of the issuance of CBDC to bank intermediation, finding a bank disintermediation effect with central bank financing replacing deposits, and government bonds displacing reserves and loans. Along similar lines, Assenmacher et al. (2021) use a DSGE model to investigate the macroeconomic effects of CBDC when the central bank administrates the CBDC rate and collateral and quantity requirements. Their findings indicate that a more ample supply of CBDC reduces bank deposits, while stricter collateral or quantitative constraints reduce welfare but can potentially contain bank disintermediation. The latter effect is particularly true when the elasticity of substitution between bank deposits and CBDC is low. Williamson (2022), on the other hand, explores the effects of the introduction of CBDC using a model of multiple means of payment. In his model, CBDC is a more efficient payment instrument than cash, but it lengthens the central bank's balance sheet, creating collateral scarcity in the economy. Differently from these works, our study investigates the implications of CBDC issuance on bank intermediation using a real business cycle model, building on Niepelt (2022). Specifically, we enrich the model with features that embed more reality (i.e. the collateral constraint for banks and imperfect substitutability between CBDC and deposits). Our findings indicate that increased demand for CBDC leads to a drop in the bank's profits, but without posing a significant risk of CBDC crowding out deposits.

Our study also contributes to the literature on the equivalence of payment systems. Existing work by Brunnermeier and Niepelt (2019) and Niepelt (2022) propose a compensation mechanism where the households' shift from deposits to CBDC can be offset by central bank lending to banks. However, these models abstract from the collateral constraint for central bank lending that is common in practice. Notably, Piazzesi and Schneider (2021) show that when banks are required to hold liquid assets to back their deposits and face asset management costs, the equivalence between alternative payment instruments breaks down, even if banks can be refinanced directly by the central bank. In light of this, we revisit the equivalence result by incorporating a collateral constraint for banks. We derive a new central bank lending rate that depends on the restrictiveness of the collateral requirement. Our findings reveal that the more restrictive the collateral constraint, the lower the loan rate the central bank must post.

Finally, we contribute to the literature on the relationship between CBDC and bank deposits by examining the effects of the introduction of CBDC considering different degrees of substitutability across means of payments [see, e.g., Bacchetta and Perazzi (2022), Barrdear and Kumhof (2022) and Kumhof and Noone (2021)]. For instance, Agur, Ari, and Dell'Ariccia (2022) consider CBDC, bank deposits and cash as imperfect substitutes with horizontal differentiation of means of payment and find that the degree to which introducing a CBDC brings bank disintermediation depends on how closely the CBDC competes with deposit [see also Andolfatto (2021)]. Also, Keister and Sanches (2022) consider a competitive market and show that a deposit-like CBDC tends to crowd out bank deposits but, at the same time, increases the aggregate stock of liquid assets in the economy, promoting more efficient levels of production and exchange and ultimately raising welfare. We find a result similar to Bacchetta and Perazzi (2022), such that when CBDC and deposits are less substitutable, it takes more of one to replace the other.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 revisits and discusses the analysis of the equivalence between operating payment systems. Section 4 characterizes the general equilibrium in which the household holds CBDC and deposits and discusses the dynamic effects of shifts in the household preference shocks. Section 5 concludes.

2 Model with CBDC and collateral-constrained banks

The model is based on Niepelt (2022) and describes an economy with a banking sector and CBDC in the absence of nominal rigidities. CBDC and deposits provide direct utility. We depart from that framework in two important dimensions: (i) we consider a collateral constraint for banks when borrowing from the central bank, and later (ii) we assume imperfect substitutability between CBDC and deposits in the liquidity function of households. There is a continuum of mass one of homogeneous infinitely-lived households who own a succession of two-period-lived banks and of one-period-lived firms. The consolidated government determines monetary and fiscal policy.

2.1 Households

The representative household wants to maximize the discounted felicity function $\mathscr{U}$, which is increasing, strictly concave and satisfies Inada conditions. Subject to their budget constraint, equation (1), they take prices, w_t and R_t^k; returns on asset i, R_t^i; profits, Π_t; and taxes, τ_t as given and solve

$$\max_{\{c_t, x_t, k_{t+1}^h, m_{t+1}, n_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mathscr{U}(c_t, x_t, z_{t+1})$$

$$\text{s.t.} \quad c_t + k_{t+1}^h + m_{t+1} + n_{t+1} + \tau_t = w_t(1 - x_t) + \Pi_t + k_t^h R_t^k + m_t R_t^m + n_t R_t^n, \quad (1)$$

$$k_{t+1}^h, m_{t+1}, n_{t+1} \geq 0,$$

where $\beta \in (0, 1)$ is the positive discount factor, c_t and x_t denote household consumption of the good and leisure at date t, respectively; k_{t+1}^h is capital at date $t+1$; and $z_{t+1} = z(m_{t+1}, n_{t+1})$ are effective real balances carried from date t to $t+1$. Effective real balances are a function of both CBDC, m_{t+1}, and bank deposits, n_{t+1}.[3] The household consumes, pays taxes, invests in capital, and has real balances, out of wage income, distributed profits and the gross return on the portfolio.

We focus on interior solutions for capital, CBDC, and deposits. To express the Euler equations for CBDC and deposits in a more compact form, we define the risk-free rate as

$$R^f_{t+1} = \frac{1}{\mathbb{E}_t \Lambda_{t+1}}, \tag{2}$$

where Λ_{t+1} is the household's stochastic discount factor:

$$\Lambda_{t+1} = \beta \frac{\mathscr{U}_c(c_{t+1}, x_{t+1}, z_{t+2})}{\mathscr{U}_c(c_t, x_t, z_{t+1})}. \tag{3}$$

Also, we define the liquidity premium, or interest spread, on asset i as

$$\chi^i_{t+1} = 1 - \frac{R^i_{t+1}}{R^f_{t+1}}, \qquad i \in \{m, n\}. \tag{4}$$

The spread on an asset i denotes the household's opportunity cost of holding said asset. A positive deposit spread, for instance, shows the interest return that the household forgoes by holding deposits. The household is willing to accept a lower return on deposits due to the liquidity service they provide. Assuming that the interest rates on CBDC and deposits are risk-free, we can summarize the first-order conditions as

$$x_t: \qquad \mathscr{U}_x(c_t, x_t, z_{t+1}) = \mathscr{U}_c(c_t, x_t, z_{t+1}) w_t, \tag{5}$$

$$k^h_{t+1}: \qquad 1 = \mathbb{E}_t R^k_{t+1} \Lambda_{t+1}, \tag{6}$$

$$m_{t+1}: \qquad \mathscr{U}_c(c_t, x_t, z_{t+1}) \chi^m_{t+1} = \mathscr{U}_z(c_t, x_t, z_{t+1}) z_m(m_{t+1}, n_{t+1}), \tag{7}$$

$$n_{t+1}: \qquad \mathscr{U}_c(c_t, x_t, z_{t+1}) \chi^n_{t+1} = \mathscr{U}_z(c_t, x_t, z_{t+1}) z_n(m_{t+1}, n_{t+1}), \tag{8}$$

The household's first-order conditions have standard interpretations. The leisure choice condition (5) equalizes the marginal benefit of leisure, $\mathscr{U}_x(c_t, x_t, z_{t+1})$, with its marginal cost in the form of reduced consumption due to less labour income, $\mathscr{U}_c(c_t, x_t, z_{t+1}) w_t$. The Euler equation for capital (6) dictates that the household saves in capital to the point where the marginal cost of saving in terms of consumption, $\mathscr{U}_c(c_t, x_t, z_{t+1})$, equals its expected discounted return, $\beta \mathbb{E}_t \mathscr{U}_c(c_{t+1}, x_{t+1}, z_{t+2}) R^k_{t+1}$. The first-order conditions for CBDC and deposits, equations (7) and (8) respectively, show that the household demands the liquid asset

[3] The household values liquidity, as suggested by the *money in the utility function* specification. In this setting, it only matters that the household demands liquidity services, not why they do.

$i \in \{m, n\}$ to the point where its marginal benefit $\mathscr{U}_z(c_t, x_t, z_{t+1})z_i(m_{t+1}, n_{t+1})/\mathscr{U}_c(c_t, x_t, z_{t+1})$ equals its opportunity cost in terms of foregone interest, χ^i_{t+1}. Combining equations (7) and (8) we also see how the household trades off CBDC and deposits:

$$z_m(m_{t+1}, n_{t+1})\chi^n_{t+1} = z_n(m_{t+1}, n_{t+1})\chi^m_{t+1}. \tag{9}$$

Equation (9) shows that the household allocates between CBDC and deposits so that the marginal rate of substitution, $z_m(m_{t+1}, n_{t+1})/z_n(m_{t+1}, n_{t+1})$, equals the relative price, $\chi^m_{t+1}/\chi^n_{t+1}$.

2.2 Banks

One of the often cited reasons in the literature for introducing CBDC is bank market power [see, e.g., Andolfatto (2021), Garratt, Yu, and Zhu (2022)]. Specifically, banks offer lower deposit rates to extract rents, and households are willing to accept this markdown as they value the liquidity service provided by deposits. A CBDC could compete with bank deposits, lowering the bank's market power. Our set-up follows Niepelt (2022) and assumes that each bank is a monopsonist in its regional deposit market, such that the household in a region can only access the regional bank.

A bank lives for two periods, and at date t issues deposits, n_{t+1}, borrows from the central bank, l_{t+1}, and collects government subsidies on deposit at rate θ_t. It invests in reserves, r_{t+1}, government bonds, b_{t+1}, and capital.[4] Without loss of generality, we abstract from bank equity. We follow Burlon et al. (2022) and assume that the bank is subjective to a collateral requirement such that the loans they get from the central bank must be lower than a fraction θ_b of government bond holdings. In this setting, government bonds are the only assets that can be pledged as collateral. For simplification, we abstract away from interbank loans with collateral. Holding government bonds gives liquidity benefits to the bank since they can use their holdings to obtain funding from the central bank. In other words, the bank is willing to forego a spread on the risk-free rate because of the collateral benefits of holding government bonds (i.e., convenience yield). The convenience yield of government bonds reflects the additional benefits the bank derives from holding these bonds beyond their financial yield. Therefore, government bonds are remunerated at a slightly lower rate than the risk-free rate.

The operating costs in the retail payment system, ν_t, are a negative function of the

[4] Bank's capital is defined as $k^b_{t+1} = n_{t+1} + l_{t+1} - r_{t+1} - b_{t+1}$. Alternatively, the bank can invest in loans to firms that eventually fund physical capital accumulation.

reserve-to-deposit ratio, ζ_{t+1}. This is analogous to a binding minimum reserves requirement, as larger reserves holdings relative to deposits lower the bank's operating costs. We also allow ν_t to vary with the stock of reserves and deposits of other banks, $\bar{\zeta}_{t+1}$, so as to capture positive externalities of reserve holdings.[5] To simplify the analysis, we make some assumptions which imply that in equilibrium $\zeta_{t+1} = \bar{\zeta}_{t+1}$, and reserves are strictly positive if and only if deposits are strictly positive: when a bank holds no deposits, its operating costs are null, and when all other banks have no deposits, the bank's operating costs are large but bounded. In this way, we rule out asymmetric equilibrium in the bank's deposits and other banks' deposits. Otherwise, the operating cost function, $\nu_t(\zeta_{t+1}, \bar{\zeta}_{t+1})$, is strictly decreasing in both arguments, strictly convex, and satisfies $\nu_{\zeta\bar{\zeta},t} = 0$ or $\nu_{\zeta\zeta,t} \geq \nu_{\bar{\zeta}\bar{\zeta},t}$, as well as $\lim_{\zeta_{t+1}\to 0} \nu_{\zeta,t} = \infty$.

The bank chooses the quantity of deposits and central bank loans subject to the deposit funding schedule of the household.[6] Since the bank acts as a monopsonist in its regional deposit market, it takes the deposit funding schedule (rather than the deposit and the central bank loan rates) as given. The program of the bank at date t reads

$$\max_{n_{t+1}, l_{t+1}, r_{t+1}, b_{t+1}} \Pi^b_{1,t} + \mathbb{E}_t \left[\Lambda_{t+1} \Pi^b_{2,t+1} \right]$$

$$\text{s.t.} \quad \Pi^b_{1,t} = -n_{t+1} \left(\nu_t(\zeta_{t+1}, \bar{\zeta}_{t+1}) - \theta_t \right), \tag{10}$$

$$\begin{aligned} \Pi^b_{2,t+1} = {} & (n_{t+1} + l_{t+1} - r_{t+1} - b_{t+1}) R^k_{t+1} \\ & + r_{t+1} R^r_{t+1} + b_{t+1} R^b_{t+1} - n_{t+1} R^n_{t+1} - l_{t+1} R^l_{t+1}, \end{aligned} \tag{11}$$

$$l_{t+1} \leq \theta_b \frac{b_{t+1}}{R^l_{t+1}}, \tag{12}$$

$$R^n_{t+1}, R^l_{t+1} \text{ perceived endogenous},$$

$$n_{t+1}, l_{t+1}, b_{t+1} \geq 0.$$

where

$$\zeta_{t+1} \equiv \frac{r_{t+1}}{n_{t+1}}, \text{ and } \bar{\zeta}_{t+1} \equiv \frac{\bar{r}_{t+1}}{\bar{n}_{t+1}},$$

and $\Pi^b_{1,t}$, $\Pi^b_{2,t+1}$ denote the cash flow generated in the first and second periods of the bank's operations, respectively.

We focus on interior solutions for deposits, loans, and government bonds, and we make use of the risk-free rate and the household's Euler first-order condition for capital, equations

[5] Niepelt (2022) uses a cost function in the form $\nu + \omega_t(\zeta_{t+1}, \bar{\zeta}_{t+1})$, where ν is the resource cost per unit of deposit funding, and ω represents the bank's resource costs of liquidity substitution.

[6] In the model, the central bank's loan funding schedule mirrors the deposit funding schedule. This assumption plays a crucial role in the context of the equivalence analysis.

(2) and (6), respectively. Also, we define the elasticity of the asset i with respect to the rate of returns on i as

$$\eta_{i,t+1} = \frac{\partial i_{t+1}}{\partial R^i_{t+1}} \frac{R^i_{t+1}}{i_{t+1}}, \qquad i \in \{n, l\},$$

and the liquidity premia on central bank loans, reserves and government bonds as in equation (4). The collateral constraint is binding in equilibrium:[7]

$$\mu_t > 0, \qquad l_{t+1} = \theta_b \frac{b_{t+1}}{R^l_{t+1}}.$$

We can write the bank's optimality conditions as

$$n_{t+1}: \qquad \chi^n_{t+1} - \Big(\nu_t(\zeta_{t+1}, \bar{\zeta}_{t+1}) - \theta_t - \nu_\zeta(\zeta_{t+1}, \bar{\zeta}_{t+1})\zeta_{t+1}\Big) = \frac{1}{\eta_{n,t+1}} \frac{R^n_{t+1}}{R^f_{t+1}}, \tag{13}$$

$$r_{t+1}: \qquad - \nu_\zeta(\zeta_{t+1}, \bar{\zeta}_{t+1}) = \chi^r_{t+1}, \tag{14}$$

$$l_{t+1}: \qquad \chi^l_{t+1} - \mu_t \left(1 + \frac{1}{\eta_{l,t+1}}\right) = \frac{1}{\eta_{l,t+1}} \frac{R^l_{t+1}}{R^f_{t+1}}, \tag{15}$$

$$b_{t+1}: \qquad \mu_t \frac{\theta_b}{R^l_{t+1}} = \chi^b_{t+1}. \tag{16}$$

We first comment on the liability side of the bank's balance sheet, starting with deposits. The left-hand side (LHS) of equation (13) represents the marginal profit from issuing deposits, which is given by the difference between the bank's gain from the positive deposit liquidity premium and the marginal cost associated with increased deposit issuance. The right-hand side (RHS) equals the profit loss of inframarginal deposits, as higher deposit issuance is associated with increased interest rates on deposits. Similarly, the condition for central bank loans, equation (15), states that the sum of the bank's marginal benefits of taking on more central bank loans and the gain coming from the positive loan liquidity premium should be equal to the profit loss from the marginal cost associated with central bank loans In fact, higher loan holdings are associated with an increase in the interest rate on the central bank loans. Turning now to the asset side of the bank's balance sheet, equation (14) equalizes the

[7] See Appendix A for the conditions under which the collateral constraint binds. The intuition is that with a non-binding collateral constraint in equilibrium:

$$\mu_t = 0, \qquad 0 \leq \theta_b \frac{b_{t+1}}{R^l_{t+1}} - l_{t+1}.$$

However, from the government bonds optimality condition, this violates the condition that $R^b_{t+1} < R^f_{t+1}$, so the collateral constraint must bind in equilibrium.

marginal benefit of reserves in the form of reduced operating costs with the bank's opportunity cost of reserves. Looking at equation (16), the optimal choice of government bonds is when the bank's marginal costs of bond holdings are equal to the loss coming from the bank's lower return with a positive spread on government bonds.

Combining equations (13) and (14) yield

$$\chi^n_{t+1} - \Big[\Big(\nu_t(\zeta_{t+1}, \bar{\zeta}_{t+1}) - \theta_t\Big) + \chi^r_{t+1}\zeta_{t+1}\Big] = \frac{1}{\eta_{n,t+1}} \frac{R^n_{t+1}}{R^f_{t+1}}. \tag{13a}$$

This result implies that the bank's net benefit of issuing more deposits must equal the inframarginal cost of deposits. Combining equations (15) and (16) results in the following relation:

$$\chi^l_{t+1} - \chi^b_{t+1}\frac{R^l_{t+1}}{\theta_b}\left(1 + \frac{1}{\eta_{l,t+1}}\right) = \frac{1}{\eta_{l,t+1}} \frac{R^l_{t+1}}{R^f_{t+1}}. \tag{15a}$$

The marginal cost of taking on more central bank loans must equal the bank's net benefit of taking on more loans. This is given by the difference between the liquidity benefits given by the central bank loans and the marginal cost associated with the collateral constraint.

2.3 Firms

Neoclassical firms live for one period and rent capital, k_t, and labour, ℓ_t, to produce the output good to maximize the profit, Π^f_t. The representative firm takes wages, w_t; the rental rate of capital, $R^k_t + \delta - 1$; and the good price as given and solves

$$\begin{aligned} &\max_{k_t, \ell_t} \Pi^f_t \\ \text{s.t.} \qquad &\Pi^f_t = f(k_t, \ell_t) - k_t(R^k_t + \delta - 1) - w_t\ell_t, \end{aligned}$$

where f is the neoclassical production function. The first-order conditions read

$$k_t: \qquad f_k(k_t, \ell_t) = R^k_t + \delta - 1, \tag{17}$$

$$\ell_t: \qquad f_l(k_t, \ell_t) = w_t. \tag{18}$$

2.4 Consolidated government

The consolidated government collects taxes and subsidies deposits; lends to the bank against collateral, l_{t+1}; invests in capital, k^g_{t+1}; and issues CBDC and reserves. The government budget constraint reads

$$
\begin{aligned}
k_{t+1}^g + l_{t+1} - b_{t+1} - m_{t+1} - r_{t+1} =& k_t^g R_t^k + l_t R_t^l - b_t R_t^b - m_t R_t^m - r_t R_t^r \\
&+ \tau_t - n_{t+1}\theta_t - m_{t+1}\mu^m - r_{t+1}\rho, \qquad (19)
\end{aligned}
$$

where μ^m and ρ are the unit resource costs of managing CBDC and reserves payments, respectively.

2.5 Market clearing

Market clearing in the labour market requires that the firm's labour demand equals the household's labour supply:

$$\ell_t = 1 - x_t. \qquad (20)$$

Market clearing for capital requires that the firm's demand for capital equals capital holdings of the household, the bank, and the government:

$$k_t = k_t^h + (n_t + l_t - r_t - b_t) + k_t^g. \qquad (21)$$

Profits distributed to the household must equal the sum of the bank and firm profits:

$$\Pi_t = \Pi_{1,t}^b + \Pi_{2,t}^b + \Pi_t^f. \qquad (22)$$

By Walras' law, market clearing on labour and capital markets and the budget constraints of the household, bank, firm, and consolidated government imply market clearing on the goods market.

To derive the aggregate resource constraint for the economy, we plug equation (22) into the household's budget constraint, equation (1), and we impose market clearing conditions (20) and (21). Then, in combination with the government's budget constraint, equation (19), the resulting expression is the aggregate resource constraint:

$$k_{t+1} = f(k_t, 1 - x_t) + k_t(1 - \delta) - c_t\Omega_t^{rc}, \qquad (23)$$

where

$$\Omega_t^{rc} = 1 + \frac{m_{t+1}}{c_t}\mu^m + \frac{n_{t+1}}{c_t}(\nu_t(\zeta_{t+1}, \bar{\zeta}_{t+1}) + \zeta_{t+1}\rho).$$

3 Revisit the equivalence of payment systems

In this section, we revisit the result in the existing literature regarding the equivalence of payment systems by considering a collateral constraint for central bank lending to banks and different degrees of substitutability between CBDC and deposits. We assume a constant elasticity of substitution (CES) functional form for the household's real balances:

$$z_{t+1}(m_{t+1}, n_{t+1}) = \left(\lambda_t m_{t+1}^{1-\epsilon_t} + n_{t+1}^{1-\epsilon_t}\right)^{\frac{1}{1-\epsilon_t}}, \tag{24}$$

where $\lambda_t \geq 0$ represents the liquidity benefits of CBDC relative to deposits, and $\epsilon_t \geq 0$ is the inverse elasticity of substitution between payment instruments.

3.1 Perfect substitutability between payment instruments

We start by assuming perfect substitutability between CBDC and deposits. Let $\epsilon_t = 0$ for all t in expression (24):

$$z_{t+1} = \lambda_t m_{t+1} + n_{t+1}.$$

Consider a policy that implements an equilibrium with deposits, reserves, central bank loans, and government bonds. We analyze whether there exists another policy and equilibrium, indicated by circumflexes, with fewer deposits and reserves, more CBDC, central bank loans, and government bonds, a different ownership structure of capital, additional taxes on the household, and otherwise the same equilibrium allocation and price system.

Suppose that deposit holdings decrease by a magnitude of Δ from the initial equilibrium, i.e. $\hat{n}_{t+1} - n_{t+1} = -\Delta$. Suppose also that real balances, the aggregate capital stock and the reserves-to-deposits ratio remain unchanged in the new equilibrium, i.e.,

$$\hat{z}_{t+1} = z_{t+1}, \quad \hat{k}_{t+1} = k_{t+1}, \quad \hat{\zeta}_{t+1} = \zeta_{t+1}.$$

The above implies the following changes in the other equilibrium quantities:

$$\begin{aligned}
\hat{m}_{t+1} - m_{t+1} &= \frac{1}{\lambda_t}\Delta, & \hat{r}_{t+1} - r_{t+1} &= -\zeta_{t+1}\Delta, \\
\hat{l}_{t+1} - l_{t+1} &= (1 - \zeta_{t+1})\Delta, & \hat{b}_{t+1} - b_{t+1} &= \frac{\hat{l}_{t+1} R^l_{t+1}}{\theta_b}, \\
\hat{k}^h_{t+1} - k^h_{t+1} &= \left(1 - \frac{1}{\lambda_t}\right)\Delta, & \hat{k}^g_{t+1} - k^g_{t+1} &= -\left(1 - \frac{1}{\lambda_t}\right)\Delta + \hat{b}_{t+1},
\end{aligned}$$

where l_{t+1} and b_{t+1} will be normalized to zero in what follows.[8]

First, we show that the new policy is neutral to the economy, given an appropriate level of interest rate on central bank loans. Note that, before the implementation of the new policy, the cash flows generated in the first and second periods of the bank's operations are given by equation (10) and (11), respectively. Recalling that in equilibrium $\zeta_{t+1} = \bar{\zeta}_{t+1}$, the changes in bank profits at dates t and $t+1$ are, respectively:

$$\hat{\Pi}^b_{1,t} - \Pi^b_{1,t} = \Delta \left(\nu_t(\zeta_{t+1}, \zeta_{t+1}) - \theta_t \right), \tag{25}$$

$$\hat{\Pi}^b_{2,t+1} - \Pi^b_{2,t+1} = \Delta \left(R^n_{t+1} - \zeta_{t+1} R^r_{t+1} - (1 - \zeta_{t+1}) \left(1 + \frac{R^k_{t+1} - R^b_{t+1}}{\theta_b} \right) R^l_{t+1} \right). \tag{26}$$

Let $\hat{T}_{1,t}$ be a tax on the household at date t that compensates for the reduced bank losses:

$$\hat{T}_{1,t} = \hat{\Pi}^b_{1,t} - \Pi^b_{1,t} = \Delta \left(\nu_t(\zeta_{t+1}, \zeta_{t+1}) - \theta_t \right). \tag{27}$$

We denote $\hat{T}_{2,t+1}$ as a tax at date $t+1$ that compensates for the change in the household's portfolio return as well as for the change in bank profits that the household collects at date $t+1$:[9]

$$\begin{aligned} \hat{T}_{2,t+1} &= (\hat{k}^h_{t+1} - k^h_{t+1}) R^k_{t+1} + (\hat{n}_{t+1} - n_{t+1}) R^n_{t+1} + (\hat{m}_{t+1} - m_{t+1}) R^m_{t+1} + \hat{\Pi}^b_{2,t+1} - \Pi^b_{2,t+1} \\ &= \Delta \left[\left(1 - \frac{1}{\lambda_t} \right) R^k_{t+1} + \frac{R^m_{t+1}}{\lambda_t} - \zeta_{t+1} R^r_{t+1} - (1 - \zeta_{t+1}) \left(1 + \frac{R^k_{t+1} - R^b_{t+1}}{\theta_b} \right) R^l_{t+1} \right]. \end{aligned} \tag{28}$$

Let $\mathscr{T}_t = \hat{T}_{1,t} + \mathbb{E}_t \Lambda_{t+1} \hat{T}_{2,t+1}$ denote the market value of taxes at date t. Substituting the two expressions for taxes, equations (27) and (28), and using conditions from the household's optimization problem, we can rewrite $\mathscr{T}_t$ as

$$\mathscr{T}_t = \Delta \left[\left(\nu_t(\zeta_{t+1}, \zeta_{t+1}) - \theta_t \right) + \frac{R^n_{t+1} - \zeta_{t+1} R^r_{t+1} - (1 - \zeta_{t+1}) \left(1 + \frac{R^k_{t+1} - R^b_{t+1}}{\theta_b} \right) R^l_{t+1}}{R^f_{t+1}} \right].$$

[8]To guarantee the non-negativity of deposits, capital holdings, and reserves, Δ must not be too large. Specifically, we impose

$$\Delta \le n_{t+1}, \qquad \zeta_{t+1} \Delta \le r_{t+1}, \qquad \left(1 - \frac{1}{\lambda_t} \right) \Delta \le k^g_{t+1}, \qquad \left(1 - \frac{1}{\lambda_t} \right) \Delta \ge -k^h_{t+1}.$$

[9]Given the household's trades-off between CBDC and deposits, expression (9), it follows that: $\frac{\lambda_t}{R^m_{t+1}} = R^n_{t+1} - (1 - \frac{1}{\lambda_t}) R^f_{t+1}$.

In order for the new policy to be neutral to the economy, the market value of taxes must be zero. This is true if the central bank posts a loan rate equal to

$$R^l_{t+1} = \frac{R^n_{t+1} + (\nu_t(\zeta_{t+1}, \zeta_{t+1}) - \theta_t)\, R^f_{t+1} - \zeta_{t+1} R^r_{t+1}}{(1-\zeta_{t+1})\left(1 + \frac{R^k_{t+1} - R^b_{t+1}}{\theta_b}\right)}. \tag{29}$$

We denote the market value of the changes in bank profits at date t as $\mathscr{P}_t = (\hat{\Pi}^b_{1,t} - \Pi^b_{1,t}) + \mathbb{E}_t \Lambda_{t+1} (\hat{\Pi}^b_{2,t+1} - \Pi^b_{2,t+1})$. Plugging in the expressions for changes in bank profits, equations (25) and (26), and using the definition of the risk-free rate from the household's problem, $\mathscr{P}_t$ reads

$$\begin{aligned} \mathscr{P}_t &= \Delta\left(\nu_t(\zeta_{t+1}, \zeta_{t+1}) - \theta_t\right) \\ &+ \frac{1}{R^f_{t+1}} \Delta \left(R^n_{t+1} - \zeta_{t+1} R^r_{t+1} - (1-\zeta_{t+1}) \left(1 + \frac{R^k_{t+1} - R^b_{t+1}}{\theta_b}\right) R^l_{t+1} \right), \end{aligned}$$

which is equal to zero given equation (29). It follows that if the central bank offers an interest rate on central bank loans according to equation (29), the market values of the taxes and of the changes in bank profits are zero.

Next, we show that the government's dynamic and intertemporal budget constraints continue to be satisfied with the new policy. Before the implementation of the new policy, the government budget constraint at time t reads:

$$k^g_{t+1} - m_{t+1} - r_{t+1} = k^g_t R^k_t - m_t R^m_t - r_t R^r_t + \tau_t - n_{t+1}\theta_t - m_{t+1}\mu^m - r_{t+1}\rho. \tag{30}$$

The government budget constraint at time t with the new policy and changes is

$$\begin{aligned} \hat{k}^g_{t+1} + \hat{l}_{t+1} - \hat{m}_{t+1} - \hat{r}_{t+1} - \hat{b}_{t+1} = k^g_t R^k_t - m_t R^m_t - r_t R^r_t + \tau_t \\ - \hat{n}_{t+1}\theta_t - \hat{m}_{t+1}\mu^m - \hat{r}_{t+1}\rho + \hat{T}_{1,t}. \end{aligned}$$

Rearranging, simplifying, and collecting terms:

$$\begin{aligned} k^g_{t+1} - m_{t+1} - r_{t+1} + \Delta \left(\frac{\mu^m}{\lambda_t} - (\nu_t(\zeta_{t+1}, \zeta_{t+1}) + \rho\zeta_{t+1}) \right) = k^g_t R^k_t - m_t R^m_t - r_t R^r_t + \tau_t \\ - n_{t+1}\theta_t - m_{t+1}\mu^m - r_{t+1}\rho. \end{aligned} \tag{31}$$

The government budget constraints before and after the intervention at time t, equations (30) and (31), are identical as long as

$$\frac{\mu^m}{\lambda_t} = \nu_t(\zeta_{t+1}, \zeta_{t+1}) + \zeta_{t+1}\rho. \tag{32}$$

Condition (32) equalizes the resource cost the government supports to provide one unit of real balances through CBDC with the corresponding cost for deposits, which is the sum of the operating cost incurred by the bank, and the resource cost associated with reserves incurred by the government. In other words, when the public and private sectors are equally efficient in providing liquidity to the household, the government's budget constraint at date t is unaffected by the changes in allocation.[10]

Similarly, before the new policy, the government budget constraint at time $t+1$ reads

$$\begin{aligned} k^g_{t+2} - m_{t+2} - r_{t+2} = {} & k^g_{t+1}R^k_{t+1} - m_{t+1}R^m_{t+1} - r_{t+1}R^r_{t+1} \\ & + \tau_{t+1} - n_{t+2}\theta_{t+1} - m_{t+2}\mu^m - r_{t+2}\rho. \end{aligned} \tag{33}$$

With the new policy and changes, the government budget constraint at time $t+1$ becomes

$$\begin{aligned} k^g_{t+2} - m_{t+2} - r_{t+2} = {} & \hat{k}^g_{t+1}R^k_{t+1} + \hat{l}_{t+1}R^l_{t+1} - \hat{m}_{t+1}R^m_{t+1} - \hat{r}_{t+1}R^r_{t+1} - \hat{b}_{t+1}R^b_{t+1} \\ & + \tau_{t+1} - n_{t+2}\theta_{t+1} - m_{t+2}\mu^m_{t+1} - r_{t+2}\rho_{t+1} + \hat{T}_{2,t+1}. \end{aligned}$$

Rearranging, simplifying, and collecting terms:

$$\begin{aligned} k^g_{t+2} - m_{t+2} - r_{t+2} = {} & \hat{k}^g_{t+1}R^k_{t+1} + \hat{l}_{t+1}R^l_{t+1} - \hat{m}_{t+1}R^m_{t+1} - \hat{r}_{t+1}R^r_{t+1} - \hat{b}_{t+1}R^b_{t+1} \\ & + \tau_{t+1} - n_{t+2}\theta_{t+1} - \hat{r}_{t+1}R^r_{t+1} - m_{t+2}\mu^m_{t+1} - r_{t+2}\rho_{t+1} + \hat{T}_{2,t+1}. \end{aligned} \tag{34}$$

Using the expression for the central bank loan rate we derived, equation (29), it follows that the government budget constraints before and after the intervention at time $t+1$, equations (33) and (34), are the same. In other words, the central bank loan rate ensuring that the market values of taxes and changes in bank profits are zero, also ensures that the government budget constraint at time $t+1$ is unaffected by the changes in allocation.

We claimed initially that the proposed intervention does not change the price system. In this case, the firm's optimal production decisions and profits are unchanged. Lastly, we must show that the modified bank's portfolio is still optimal. Before the intervention, the bank's choice set is determined by the cost function, the subsidy rate, the household's stochastic

[10]This condition is plausible since one of the options central banks are considering for issuing a CBDC to the public involves using the existing commercial banks' deposit distributing systems.

discount factor, rates on returns on capital and reserves, and the deposit funding schedule. The new policy leaves unchanged the cost function, the subsidy rate, the stochastic discount factor, and the rates on returns on capital and reserves. After the intervention, as the household holds more CBDC, there is a modified deposit funding schedule, together with a central bank loan funding schedule. The central bank needs to post an appropriate loan funding schedule to induce the non-competitive bank to go along with the equivalent balance sheet positions as before the intervention. Subject to this schedule, the bank chooses loans that make up for the reduction in funding from the household, net of reserves, at the same effective price. The central bank loan rate that ensures this is true is the one we derived previously, equation (29):

$$R^l_{t+1} = \frac{R^n_{t+1} + \left(\nu_t(\zeta_{t+1}, \zeta_{t+1}) - \theta_t\right) R^f_{t+1} - \zeta_{t+1} R^r_{t+1}}{(1-\zeta_{t+1})\left(1 + \frac{R^k_{t+1} - R^b_{t+1}}{\theta_b}\right)}.$$

We can demonstrate that the term in parenthesis at the denominator on the RHS is positive. From the household's problem, we know that $R^k_{t+1} \leq R^f_{t+1}$, assuming that the rate of return on capital is not risky, we can approximate $R^k_{t+1} \simeq R^f_{t+1}$. We also know that due to the liquidity benefits the bank has from holding government bonds, $R^b_{t+1} < R^f_{t+1}$. Recalling that $\theta_b \in [0, 1]$, it follows that the additional term is positive.

The rate we derived is lower than the one derived in Niepelt (2022) precisely from the additional terms in parenthesis at the denominator on the RHS. It follows that, in case of perfect substitutability between CBDC and deposits, with a collateral constraint for central bank lending, the central bank must post a lower rate than in a no-collateral constraint scenario. The intuition is that when the bank is not collateral-constrained, it can borrow as much as it wants from the central bank. With a collateral requirement, the central bank needs to offer a lower lending rate to incentivize the bank to borrow the same quantity as in the absence of the constraint, such that its balance sheet remains unaffected.[11]

Figure 1 shows the equivalent loan rate we derived, R^l_{t+1}, as a function of the fraction of government bonds pledged as collateral, θ_b. The loan rate is at its highest when all bonds held by the bank can be pledged, i.e., $\theta_b = 1$. The maximum rate, the blue dashed line, is reported next to the orange line, representing the maximum value of the equivalent loan rate obtained in Niepelt (2022), which we call $\tilde{R}^l_{t+1}$.[12] As previously mentioned, we see that $R^l_{t+1} < \tilde{R}^l_{t+1}$. The equivalent loan rate is logarithmically increasing in the fraction of government bonds pledged as collateral and depends on how restrictive is the collateral constraint: the tighter

[11] Notice that we abstract from any social cost associated with central bank lending to banks.

[12] Niepelt (2022) does not account for the central bank's collateral requirement; therefore it is like considering $\theta_b \to +\infty$.

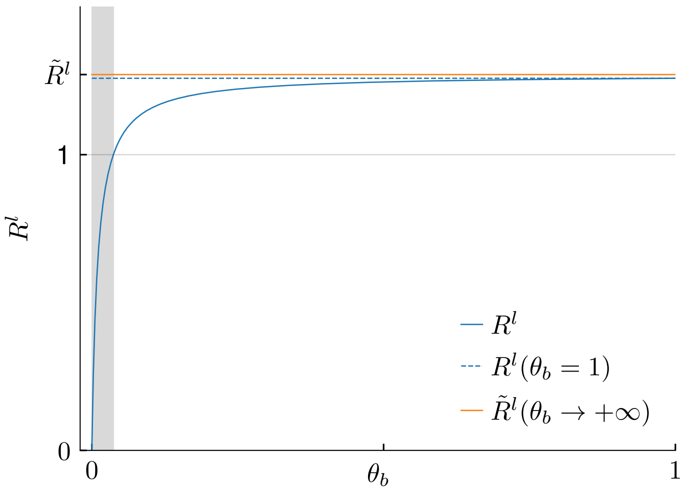

Figure 1: Equivalent loan rate, R^l_{t+1}, as function of the collateralized government bonds, θ_b

the constraint is (the lower θ_b), the lower the lending rate the central bank needs to offer (the lower R^l). The shaded-grey area corresponds to extremely low values of θ_b, indicating an exceptionally restrictive collateral constraint to the extent that the central bank may need to compensate the bank for accepting the loans. This scenario is unlikely as the haircut is too substantial to be observed. Empirically the value of θ_b is around 0.995 [see, e.g., Burlon et al. (2022)].

3.2 Imperfect substitutability between payment instruments

Imperfect substitutability between CBDC and bank deposits has been widely assumed in the existing literature.[13] However, there is little guidance on the degree of substitutability between the two payment methods. Barrdear and Kumhof (2022) offer a possible approach by calibrating the elasticity of substitution between CBDCs and bank deposits based on the elasticity of substitution across retail deposit accounts at different banks. They argue that the level of substitutability is low, given that even with high variability of interest rates offered on instant-access accounts by different banks, most households tend to remain with their current providers. They suggest that people tend to stick with what they know and are familiar with.

Although there is a lack of empirical evidence on the relationship between CBDC and bank deposits, CBDC would most likely not be a perfect substitute for bank deposits [see,

[13]See, e.g., Bacchetta and Perazzi (2022), Barrdear and Kumhof (2022), Burlon et al. (2022) and Kumhof and Noone (2021).

e.g., Bacchetta and Perazzi (2022)].[14] In this section, we study the equivalence of payment systems assuming that CBDC and deposits are imperfect substitutes, as in equation (24), with $\epsilon_t > 0$ for all t.

As in Section 3.1, consider a policy that implements an equilibrium with deposits, reserves, central bank loans, and government bonds. We analyze whether there exists another policy and equilibrium, indicated by circumflexes, with fewer deposits and reserves, more CBDC, central bank loans, and government bonds, a different ownership structure of capital, additional taxes on the household, and otherwise the same equilibrium allocation and price system.

Suppose again that deposit holdings decrease by a magnitude of Δ from the initial equilibrium and that real balances, the aggregate capital stock and the reserves-to-deposits ratio remain unchanged in the new equilibrium. This implies the same changes in equilibrium quantities as we have seen in Section 3.1, except for CBDC. Due to the imperfect substitutability between CBDC and deposits, in order for real balances to remain unchanged, the quantity of CBDC must change according to

$$\hat{m}_{t+1} - m_{t+1} = \left[\frac{1}{\lambda_t}\left(n_{t+1}^{1-\epsilon_t} - \hat{n}_{t+1}^{1-\epsilon_t}\right) + m_{t+1}^{1-\epsilon_t}\right]^{\frac{1}{1-\epsilon_t}} - m_{t+1}.$$

We define taxes at dates t and $t+1$, equations (27) and (28) respectively, as in Section 3.1, as well as the market values of taxes, $\mathscr{T}_t = \hat{T}_{1,t} + \mathbb{E}_t\Lambda_{t+1}\hat{T}_{2,t+1}$. The central bank loan rate ensuring that the market value of taxes is zero is:[15]

$$R_{t+1}^l = \frac{\mathscr{A}_t R_{t+1}^n - \zeta_{t+1}R_{t+1}^r + \left(\nu_t(\zeta_{t+1}, \zeta_{t+1}) - \theta_t + 1 - \mathscr{A}_t\right) R_{t+1}^f}{(1-\zeta_{t+1})\left(1 + \frac{R_{t+1}^k - R_{t+1}^b}{\theta_b}\right)}, \tag{35}$$

where

$$\mathscr{A}_t = \lambda_t \left(\frac{\hat{m}_{t+1} - m_{t+1}}{\Delta}\right)\left(\frac{n_{t+1}}{m_{t+1}}\right)^{\epsilon_t}.$$

Consider the changes in bank profits at dates t and $t+1$ as given from equations (25) and (26), respectively. We can check whether the market value of the changes in bank profits, $\mathscr{P}_t = (\hat{\Pi}_{1,t}^b - \Pi_{1,t}^b) + \mathbb{E}_t\Lambda_{t+1}(\hat{\Pi}_{2,t+1}^b - \Pi_{2,t+1}^b)$, also reduces to zero given the central bank

[14] Grodecka-Messi and Zhang (2023) use a historical experiment to empirically investigate the introduction of CBDC. They study the establishment of the Bank of Canada and the emergence of the central bank notes' monopoly, which introduced a new medium of exchange in competition with the privately issued currency by commercial banks -similar to the implications of CBDC introduction today.

[15] When deriving the central bank loan rate, we use expression (9) for the household's trade-off between CBDC and deposits. Given the CES functional form assumption, equation (24) it follows that: $\frac{m_{t+1}}{n_{t+1}} = \left(\lambda_t \frac{\chi_{t+1}^n}{\chi_{t+1}^m}\right)^{\frac{1}{\epsilon_t}}$.

loan rate in expression (35). It turns out this is not true. In particular, after making the appropriate substitutions, the market value of changes in bank profits reads:

$$\mathscr{P}_t = \mathbb{E}_t \frac{1}{R^f_{t+1}} \Delta \left(R^n_{t+1} - \mathscr{A}_t R^n_{t+1} - (1 - \mathscr{A}_t) R^f_{t+1} \right).$$

Note that if there were perfect substitutability between CBDC and deposits (i.e., $\epsilon_t = 0$), $\mathscr{A}_t$ equals 1, and the market value of the changes in bank profits reduces to zero. It follows that, in case of imperfect substitutability between CBDC and deposits, the central bank lending rate that renders the market value of taxes zero does not result in changes in bank profits being zero. In other words, the central bank cannot make the bank indifferent to the competition from CBDC. In fact, a change in the bank's profitability implies that the new policy does not guarantee the same allocation as before, implying that the introduction of CBDC has real effects on the economy. We will shed light on these effects in the subsequent dynamic analysis.

4 Dynamic effects of shifts in household's preferences

In Section 3, we revisited the result in the existing literature regarding the equivalence of payment systems. We showed that when CBDC and deposits are perfect substitutes, the central bank can offer loans to banks that render the introduction of CBDC neutral to the real economy. However, when CBDC and deposits are imperfect substitutes, the central bank cannot make the bank indifferent to the competition from CBDC, and it follows that the introduction of CBDC has real effects on the economy. Next, we investigate how the introduction of CBDC affects the real economy, more specifically, the potential threat to financial stability should CBDC crowd out deposits.

4.1 Functional forms and equilibrium conditions

The functional form for real balances is represented by equation (24). We assume that the household has utility function of the form

$$\mathscr{U}(c_t, x_t, z_{t+1}) = \frac{\left((1-\upsilon)c_t^{1-\psi} + \upsilon z_{t+1}^{1-\psi}\right)^{\frac{1-\sigma}{1-\psi}}}{1-\sigma} x_t^{v}, \tag{36}$$

where $\sigma > 0$ is the inverse intertemporal elasticity of substitution between bundles of consumption and real balances across times; $\psi > 0$ is the inverse intratemporal elasticity of substitution between consumption and real balances; and υ is the exponent of the power function for leisure. The bank's operating cost function has the following form:

$$\nu_t(\zeta_{t+1}, \bar{\zeta}_{t+1}) = \phi_1 \zeta_{t+1}^{1-\varphi} + \phi_2 \bar{\zeta}_{t+1}^{1-\varphi}, \tag{37}$$

where $\phi_1, \phi_2 \geq 0$ are the relative weights assigned to the bank's reserves-to-deposit ratio and to the other bank's ratio; and $\varphi > 1$. Lastly, the firm has the standard Cobb-Douglas production function:

$$f(k_t, \ell_t) = k_t^{\alpha} \ell_t^{1-\alpha}, \tag{38}$$

where k_t and l_t are the firm's demand for capital and labour, respectively, and α is the capital share of output.

Given the functional form assumptions, we characterize the general equilibrium. First, knowing the household's utility functional form, we can rewrite the stochastic discount factor, equation (3) as

$$\Lambda_{t+1} = \beta \frac{c_{t+1}^{-\sigma} x_{t+1}^{\upsilon} \Omega_{t+1}^{c}}{c_t^{-\sigma} x_t^{\upsilon} \Omega_t^{c}}.$$

The household's capital Euler equation (6) and leisure choice condition (5), and the aggregate resource constraint (23) become, respectively,

$$c_t^{-\sigma} x_t^{\upsilon} \Omega_t^c = \beta \mathbb{E}_t \left[c_{t+1}^{-\sigma} x_{t+1}^{\upsilon} \Omega_{t+1}^{c} R_{t+1}^{k} \right], \tag{39}$$

$$\frac{c_t^{1-\sigma}}{1-\sigma} \upsilon x_t^{\upsilon-1} \Omega_t^x = w_t c_t^{-\sigma} x_t^{\upsilon} \Omega_t^c, \tag{40}$$

$$k_{t+1} = k_t^{\alpha}(1-x_t)^{1-\alpha} + k_t(1-\delta) - c_t \Omega_t^{rc}, \tag{41}$$

where

$$\Omega_t^c = (1-\upsilon)^{\frac{1-\sigma}{1-\psi}} \left(1 + \left(\frac{\upsilon}{1-\upsilon} \right)^{\frac{1}{\psi}} \chi_{t+1}^{1-\frac{1}{\psi}} \right)^{\frac{\psi-\sigma}{1-\psi}},$$

$$\Omega_t^x = (1-\upsilon)^{\frac{1-\sigma}{1-\psi}} \left(1 + \left(\frac{\upsilon}{1-\upsilon} \right)^{\frac{1}{\psi}} \chi_{t+1}^{1-\frac{1}{\psi}} \right)^{\frac{1-\sigma}{1-\psi}},$$

$$\Omega_t^{rc} = 1 + \frac{m_{t+1}}{c_t} \mu^m + \frac{n_{t+1}}{c_t} \left((\phi_1 + \phi_2) \left(\frac{\chi_{t+1}^r}{\phi_1(\varphi-1)} \right)^{\frac{\varphi-1}{\varphi}} + \left(\frac{\chi_{t+1}^r}{\phi_1(\varphi-1)} \right)^{-\frac{1}{\varphi}} \rho \right).$$

Note that these equilibrium conditions closely parallel those of a standard real business cycle model, with the addition of the auxiliary variables Ω_t^c, Ω_t^x and Ω_t^{rc}, summarizing the impact of the household's preference for liquidity on consumption/savings choices, leisure choices and capital accumulation, respectively. We combine the household's first-order conditions for CBDC and deposits, equations (7) and (8), with the expression for real balances, equation (24) to derive the demand for real balances and their components:

$$z_{t+1} = c_t \left(\frac{\upsilon}{1-\upsilon} \frac{1}{\chi_{t+1}} \right)^{\frac{1}{\psi}}, \tag{42}$$

$$m_{t+1} = z_{t+1} \left(\lambda_t \frac{\chi_{t+1}}{\chi_{t+1}^m} \right)^{\frac{1}{\epsilon_t}}, \tag{43}$$

$$n_{t+1} = z_{t+1} \left(\frac{\chi_{t+1}}{\chi_{t+1}^n} \right)^{\frac{1}{\epsilon_t}}, \tag{44}$$

where

$$\chi_{t+1} = \frac{\chi_{t+1}^m \chi_{t+1}^n}{\left(\lambda_t^{\frac{1}{\epsilon_t}} \left(\chi_{t+1}^n\right)^{\frac{1-\epsilon_t}{\epsilon_t}} + \left(\chi_{t+1}^m\right)^{\frac{1-\epsilon_t}{\epsilon_t}} \right)^{\frac{\epsilon_t}{1-\epsilon_t}}} \tag{45}$$

is a weighted average of the spreads on CBDC and deposits, χ_{t+1}^m and χ_{t+1}^n, and it can be interpreted as the household's average cost of liquidity. Note that the spreads on CBDC and reserve are given as in equation (4).

From the firm's optimality conditions, equations (17) and (18), we derive the return on capital and the real wage:

$$R_t^k = 1 - \delta + \alpha \left(\frac{k_t}{1-x_t} \right)^{\alpha-1}, \tag{46}$$

$$w_t = (1-\alpha) \left(\frac{k_t}{1-x_t} \right)^{\alpha}. \tag{47}$$

Finally, from the bank's problem, we combine the first-order conditions for deposits and reserves, (13) and (14), to derive the expressions for the equilibrium deposit spread, χ_{t+1}^n:

$$\chi_{t+1}^n - \chi_{t+1}^n \left(\frac{1-s_t}{\psi} + \frac{s_t}{\epsilon_t} \right)^{-1} = (\phi_1 \varphi + \phi_2) \left(\frac{\chi_{t+1}^r}{\phi_1(\varphi - 1)} \right)^{\frac{\varphi-1}{\varphi}} - \theta_t, \tag{48}$$

where

$$s_t = \frac{\lambda_t^{\frac{1}{\epsilon_t}} \left(\chi_{t+1}^n\right)^{\frac{1-\epsilon_t}{\epsilon_t}}}{\lambda_t^{\frac{1}{\epsilon_t}} (\chi_{t+1}^n)^{\frac{1-\epsilon_t}{\epsilon_t}} + (\chi_{t+1}^m)^{\frac{1-\epsilon_t}{\epsilon_t}}}.$$

4.2 Shocks

After characterizing the general equilibrium, we aim to investigate the potential threat to financial stability should CBDC crowd out deposits. We address this concern by studying the economy's responses to changes in the household's relative preferences for CBDC over bank deposits.

Specifically, we first assess the effects of a positive shock to the liquidity benefits of CBDC over deposits, λ_t, and assume it follows a log AR(1) process of the form

$$\log(\lambda_t) = (1 - \rho^\lambda) \log(\lambda) + \rho^\lambda \log(\lambda_{t-1}) + e_t^\lambda,$$

where where ρ^λ is the persistence parameter, λ is the steady-state value, and e_t^λ is the exogenous one-time shock.

Secondly, we evaluate the impact of a positive shock to the inverse of the elasticity of substitution, ϵ_t, assuming it also follows a log AR(1) process of the form

$$\log(\epsilon_t) = (1 - \rho^\epsilon) \log(\epsilon) + \rho^\epsilon \log(\epsilon_{t-1}) + e_t^\epsilon,$$

where ρ^ϵ is the persistence parameter, ϵ is the steady-state value, and e_t^ϵ is the exogenous one-time shock. This shock corresponds to a negative shock to the substitutability between payment instruments, so the household is less willing to substitute between CBDC and deposits.

4.3 Parameters

The model is quarterly. We assume that in the steady state, the household perceives CBDC and deposits as equally useful for liquidity purposes, i.e., $\lambda = 1$. The household's discount factor, β, is set to the standard value of 0.99. Additionally, we assume that in the steady state, the inverse substitutability between the two liquid assets, ϵ, is 1/6. This corresponds to a medium degree of substitutability following Bacchetta and Perazzi (2022). We set the inverse intertemporal elasticity of substitution, σ, to 0.5. The leisure function coefficient, υ, is set to 0.85 to match a steady-state labour supply of approximately 1/3. We assume that consumption and liquidity services are complements. Therefore, the inverse intratemporal elasticity of substitution between the two, ψ, is set higher than σ and equal to 0.6. The capital share of output, α, and the rate of capital depreciation, δ, are set to the standard values of 1/3 and 0.025, respectively.

Throughout this Section, we assume that the government does not extend subsidies to the bank, i.e., $\theta_t = 0$. We follow Niepelt (2022) and set the government's marginal cost of providing reserves, ρ, to 0.0004. We follow the standard and set the persistence parameters in the log AR(1) processes, ρ^ϵ and ρ^λ, to 0.9.

Appendix C describes the model calibration of the remaining parameters. For simplicity, we assume that $\phi_1 = \phi_2 = \phi$. We calibrate the bank's cost function coefficients, ϕ and φ, as well as the utility weight of liquidity, $\boldsymbol{\upsilon}$, to match three steady-state quantities: CBDC-to-deposits ratio, m/n; reserve-to-deposits ratio, ζ; and consumption velocity, c/z. To minimize the compositional effect on the resource cost of liquidity provision, we set the unit resource costs of managing CBDC, μ^m, equal to the total resource cost of providing deposits.

Table 1 summarizes the parameter values.

Parameter	Value	Source
λ	1	Assumption
β	0.99	Standard
ϵ	1/6	Bacchetta and Perazzi (2022)
σ	0.5	Assumption
υ	0.85	Assumption (Match steady-state labour supply $\approx 1/3$)
ψ	0.6	Assumption (Ensure $\psi > \sigma$)
α	1/3	Standard
δ	0.025	Standard
θ_t	0	Assumption
ρ	0.0004	Niepelt (2022)
$\rho^\epsilon, \rho^\lambda$	0.9	Standard
ϕ	0.00061	Model
φ	2.00924	Model
$\boldsymbol{\upsilon}$	0.01200	Model
μ^m	0.00505	Model

Table 1: Model Parameters

4.4 Impulse responses

We assume that, in a steady state, the interest rate on CBDC is lower than on deposits, and we keep the rates of returns on CBDC, deposits and reserves constant. The impulse responses are reported as percentage point deviations. Figure 2 illustrates the impulse responses as deviations from the steady state to a 10% increase in the liquidity benefit of CBDC, λ_t.

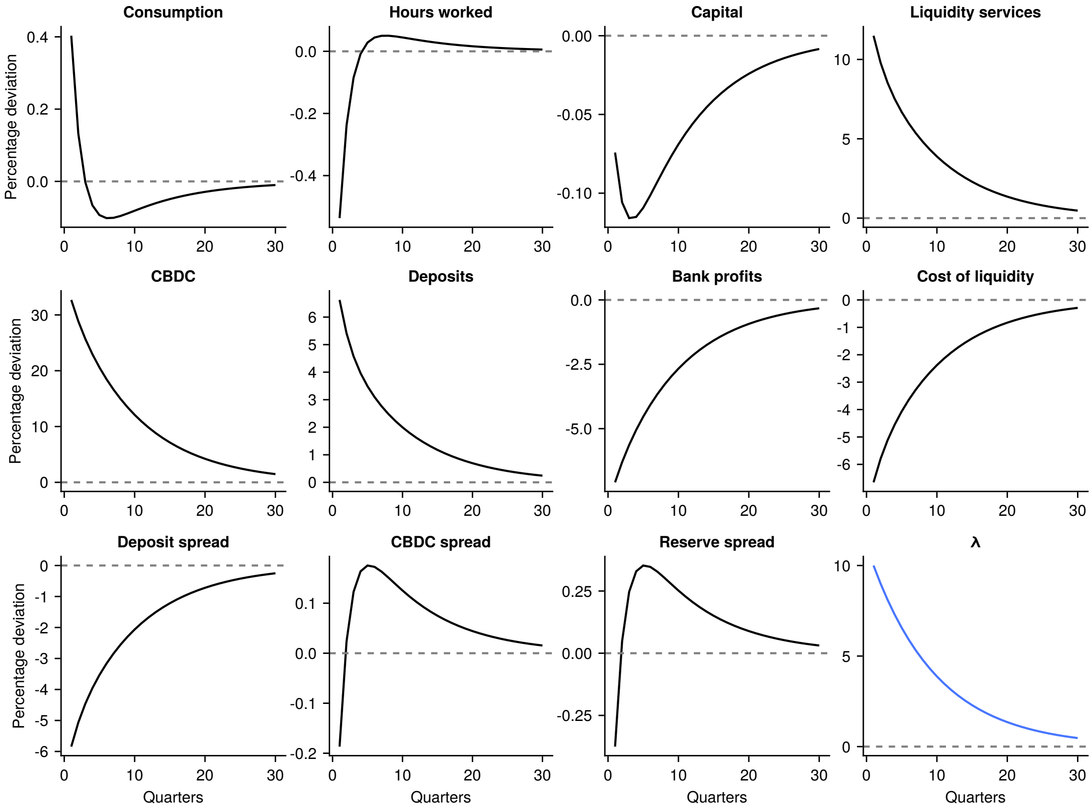

Figure 2: Impulse responses to 10% increase in the liquidity benefit of CBDC, λ_t

As the liquidity benefit of CBDC increases, the spreads on CBDC, deposits, and reserves decrease.[16] Since the CBDC rate is kept constant, the marginal decrease in the CBDC spread is due to a slight reduction in the risk-free rate.[17] The equilibrium deposit spread is pinned down by equation (48). It depends on the reserves spread, representing the marginal cost of issuing deposits, and the CBDC spread, which affects the markup the bank can charge on its prices over the marginal cost. Since both quantities decrease, the deposit spread drops by

[16]Recall that the spreads on each asset are essentially the differences between the risk-free rate and the CBDC, deposit, and reserve rates, respectively. Further, CBDC and reserve spreads have the same shape but different magnitudes as the initial steady-state levels differ.

[17]The risk-free rate response is not plotted in the figure, but it is analogous to that of the reserve spread.

large. An increase in λ_t means CBDC is becoming a more attractive source of liquidity for the household; thus, CBDC demand increases significantly. The bank, facing tougher competition from CBDC, decreased the price of deposits by a large margin to stem the potential deposit outflows. Intuitively, a more attractive CBDC diminishes the bank's market power. Thus, the markup on deposit spread (over the marginal cost of deposit issuance) that the bank imposes on the household is reduced, and bank profits drop.

With both CBDC and spreads decreasing, the household's average cost of liquidity, pinned down by equation (45), becomes lower, which induces the household to hold more liquidity services and increase consumption. This is because a lower cost of liquidity increases the household's current marginal utility of consumption. In other words, the opportunity cost of savings has increased, incentivizing the household to save less and consume more. At the same time, the household's marginal benefit of leisure is now higher than the marginal cost, inducing them to decrease labour supply. Lastly, with the increased liquidity holdings by the household, the societal resource costs associated with liquidity provision are higher and thus reduce capital accumulation.

Figure 3 illustrates the impulse responses as deviations from the steady state to a 10% increase in the inverse elasticity of substitution between CBDC and deposits, ϵ_t. The impulse responses are qualitatively the same as in Figure 2, except for deposit demand, which now decreases. The shock considered here reduces the substitutability between payment instruments. As a result, given the same liquidity benefit of the two instruments and their relative prices, CBDC demand increases while deposit demand drops mildly.[18] This result relates to the one in Bacchetta and Perazzi (2022), who study the effect of the elasticity of substitution between CBDC and deposits on the demand for CBDC, conditioning on the relative level of interest paid by CBDC with respect to the interest paid by deposits. They find that the effect of a change in the elasticity of substitution on CBDC and deposit demand depends on their relative interest rate. In the scenario where the interest rate on CBDC is below that of deposits, as we also assume, they find that the demand for CBDC decreases in the elasticity of substitution, as we find in Figure 3. The intuition is that when the two instruments are less substitutable, it takes more of one to replace the other. However, it is worth noting that the decrease in deposits is relatively small and should not raise significant concerns about CBDC crowding out deposits.

[18]Recall that, in the steady state, we assume equal liquidity properties for CBDC and deposits (i.e., $\lambda_t = 1$) and that the interest rate on CBDC is below that of deposits (i.e. $R^m < R^n$).

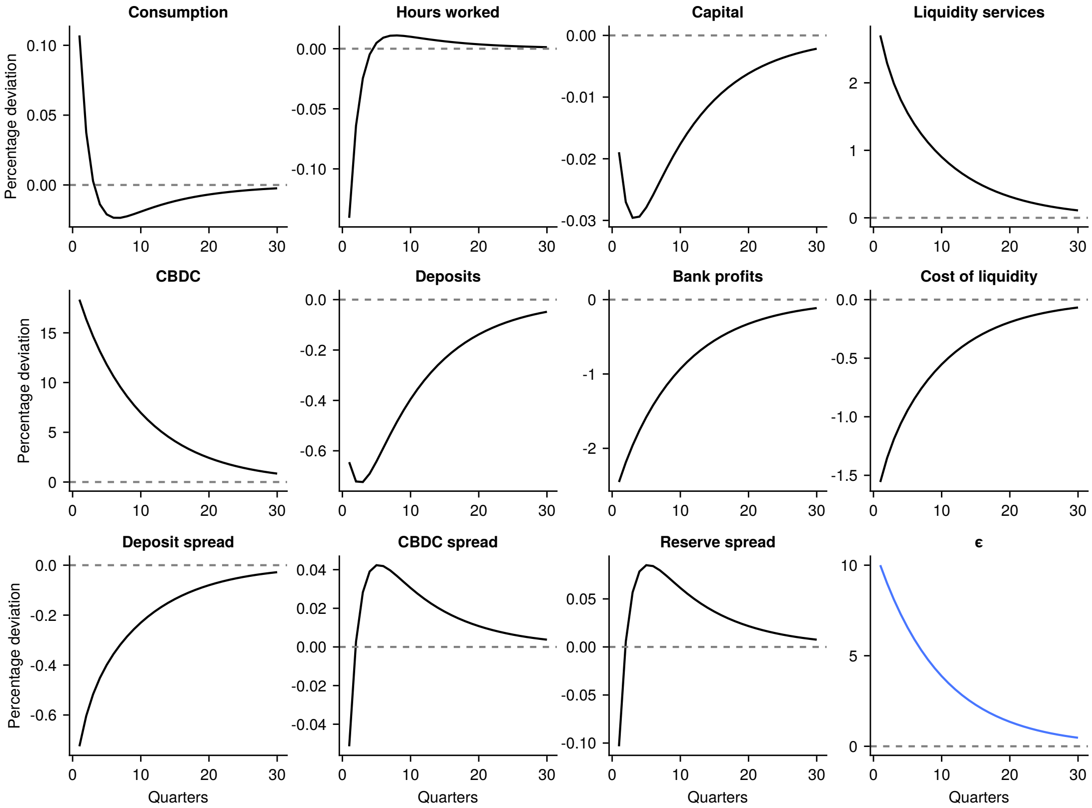

Figure 3: Impulse responses to 10% increase in inverse elasticity of substitution between CBDC and deposits, ϵ_t

4.5 Robustness checks

An aspect of uncertainty regarding any practical implementation of CBDC would be the household's perception of its usefulness relative to bank deposits. Therefore, we test the robustness of our results by changing the liquidity benefit of CBDC and the substitutability between CBDC and deposits in the steady state.

Firstly, we change the steady-state liquidity benefit of CBDC, λ, to 0.5 and 1.5. Figures 4 and 5 in the Appendix D show the impulse responses to a 10% increase in λ_t when its steady-state value is 0.5 and 1.5, respectively. Comparing these responses to the main specification in Figure 2, we see that the results remain qualitatively the same. The shapes of the impulse responses are identical, while there are some minor differences in magnitudes. Figures 8 and 9 show the impulse responses to a 10% increase in ϵ_t when steady-state λ is 0.5 and 1.5, respectively. Comparing these to Figure 3, we see that the main takeaways from Figure 8 stand. The only difference is that when the steady-state CBDC liquidity benefit is 0.5, the response of deposits is of the opposite sign. In other words, as the substitutability between payment instruments decreases, both CBDC and deposit demand increase. This is because now we assume that the liquidity benefit of holding deposits is higher than that of holding CBDC in the steady state. All other responses remain identical in shape and similar in magnitude.

Secondly, we change the elasticity of substitution between CBDC and deposits to 1/9 and 1/3. Figures 6 and 7 show the impulse responses to a 10% increase in λ_t when steady-state ϵ is 1/9 and 1/3, respectively. Figures 10 and 11 show the impulse responses to a 10% increase in ϵ_t when its steady-state value is 1/9 and 1/3, respectively. Comparing these impulse responses to their main counterparts in Figures 2 and 3, we see that they are all identical in shape and similar in magnitude.

5 Conclusion

We highlight the importance of the degree of substitutability between CBDC and bank deposits when evaluating the potential risk to financial stability following the introduction of a retail CBDC. We find that, when CBDC and deposits are perfect substitutes, as long as they have the same resource cost per unit of effective real balances, the central bank can offer loans to the bank such that it maintains the same balance sheet positions as before the introduction of CBDC, making it neutral to the real economy. Additionally, it is crucial to account for the collateral requirement that the bank must respect when borrowing from the central bank, as the central bank's lending rate depends on how restrictive the collateral constraint is. The tighter the constraint is (the lower the fraction of the bank's bond holdings that can be pledged as collateral), the lower the central bank's loan rate should be to keep the allocations unchanged when introducing a CBDC.

However, when CBDC and deposits are imperfect substitutes, issuing a CBDC changes the bank's profitability irrespective of the central bank's lending rate, meaning that the central bank cannot make the bank indifferent to the competition from CBDC. In fact, a change in the bank's profitability implies that the new policy does not guarantee the same allocations as before, implying that the introduction of CBDC has real effects on the economy. Nevertheless, based on our dynamic effects analysis, there seems not to be a significant risk of CBDC crowding out deposits when considering changes in the household's preferences for CBDC over deposits.

Overall, our findings can help policymakers and central bankers design and implement CBDCs to minimize the risk of financial instability. A possible extension in the analysis of the equivalence result is to investigate the transition from the steady state with no CBDC to the steady state after CBDC has been introduced and identify the driving forces governing the transition between the two.

A Condition under which the collateral constraint binds

Assuming interior solutions, the bank's optimality conditions for loans and bonds are, respectively:

$$\mathbb{E}_t\left[\Lambda_{t+1}(R^k_{t+1} - R^l_{t+1} - l_{t+1}\frac{\partial R^l_{t+1}}{\partial l_{t+1}})\right] = \mu_t\left(1 + \theta_b\frac{b_{t+1}}{R^{l\,2}_{t+1}}\frac{\partial R^l_{t+1}}{\partial l_{t+1}}\right),$$

$$\mathbb{E}_t\left[\Lambda_{t+1}(R^k_{t+1} - R^b_{t+1})\right] = \mu_t\frac{\theta_b}{R^l_{t+1}}.$$

Subtracting the condition for bonds from the one for loans:

$$\mathbb{E}_t\left[\Lambda_{t+1}(R^b_{t+1} - R^l_{t+1} - l_{t+1}\frac{\partial R^l_{t+1}}{\partial l_{t+1}})\right] = \mu_t\left(1 - \frac{\theta_b}{R^l_{t+1}} + \theta_b\frac{b_{t+1}}{R^{l\,2}_{t+1}}\frac{\partial R^l_{t+1}}{\partial l_{t+1}}\right). \tag{49}$$

To define the sign of the RHS, recall that $\theta_b \in [0,1]$, and since the rate of return on reserves is positive, and we assumed interior solutions, all the terms are positive.

We define the elasticity of central bank's loans with respect to their rate of returns as

$$\eta_{l,t+1} = \frac{\partial l_{t+1}}{\partial R^l_{t+1}}\frac{R^l_{t+1}}{l_{t+1}},$$

such that we can rewrite the last term on the LHS as

$$\frac{1}{\eta_{l,t+1}}R^l_{t+1}.$$

Expression (49) says that the collateral constraint is binding if:

$$R^b_{t+1} - R^l_{t+1} > \frac{1}{\eta_{l,t+1}}R^l_{t+1}.$$

Rearranging

$$R^b_{t+1} > R^l_{t+1} + \frac{1}{\eta_{l,t+1}}R^l_{t+1}.$$

We can conclude that the collateral constraint is binding if the sum of the cost of borrowing from the central bank and the bank's cost of taking on more loans is cheaper than the return the bank gets from holding government bonds:[19]

$$\mu_t > 0, \qquad l_{t+1} = \theta_b\frac{b_{t+1}}{R^l_{t+1}} \qquad \text{iff } R^b_{t+1} > R^l_{t+1} + \frac{1}{\eta_{l,t+1}}R^l_{t+1}.$$

[19] We replicated the same analysis in the setting by Burlon et al. (2022), and we got an analogous result.

B Steady State

Variables without time subscripts denote the steady-state values. The discount factor determines the return on capital and the risk-free rate:

$$R^k = R^f = \frac{1}{\beta}.$$

Conditional on the policy rates, R^m and R^r, we know CBDC and reserve spreads, χ^m and χ^r, respectively. We find the deposit spread, χ^n, using the equilibrium condition (48). Knowing the above-mentioned spreads, we derive χ^z, Ω^c, Ω^x. We find the capital-labour ratio by rearranging the return on capital (46):

$$\frac{k}{1-x} = \left(\frac{1}{\alpha}\left(R^k - 1 + \delta\right)\right)^{\frac{1}{\alpha-1}}.$$

Next, we divide the resource constraint (41) by the labour supply to get the consumption-labour ratio:

$$\frac{c}{1-x} = \left(\left(\frac{k}{1-x}\right)^{\alpha} - \delta\left(\frac{k}{1-x}\right)\right)\frac{1}{\Omega^{rc}}.$$

From the household's leisure choice, condition (40), we get the consumption-leisure ratio:

$$\frac{c}{x} = \frac{(1-\sigma)w}{\upsilon}\frac{\Omega_c}{\Omega_x},$$

where the wage rate is given by condition (47):

$$w = (1-\alpha)\left(\frac{k}{1-x}\right)^{\alpha}.$$

Lastly, we combine the consumption-leisure and consumption-labour ratios to derive the steady-state leisure:

$$x = \frac{c}{1-x}\left(\frac{c}{1-x} + \frac{c}{x}\right)^{-1}.$$

Having derived the above-mentioned steady-state values, it is straightforward to find all other quantities.

C Calibration

The household's first-order conditions for CBDC and deposits imply the following relation:

$$\chi^n = \frac{1}{\gamma}\left(\frac{m}{n}\right)^{\epsilon}\chi^m,$$

where m/n is the desired steady-state ratio of CBDC to deposits. To simulate a situation with limited CBDC adaptation, we set this value equal to 1/10.[20]

We use the last relation to derive χ^z and, consequently, the left-hand side of the bank's optimality condition (48), denoted by LHS. Having derived LHS, we again use expression (48) to find φ as

$$\varphi = \frac{LHS + \chi^r\zeta}{LHS - \chi^r\zeta},$$

where ζ is the desired steady-state reserves-to-deposits ratio. This ratio is set to 0.25 to align with U.S. data.[21]

We use the right-hand side of expression (48) to find ϕ:

$$\phi = \frac{\chi^r}{\zeta^{-\varphi}(\varphi - 1)}.$$

Rearranging the household's demand for effective real balances, expression (24), we derive $\boldsymbol{v}$ as

$$\boldsymbol{v} = \frac{\left(\frac{z}{c}\right)^{\psi}\chi^z}{1 + \left(\frac{z}{c}\right)^{\psi}\chi^z}.$$

where c/z is the consumption velocity, targeted at 1.147 to align with U.S. data.

Lastly, to minimize the compositional effect on the resource cost of liquidity provision, we set the unit resource costs of managing CBDC, μ^m, to be equal to the total resource cost of providing deposits:

$$\mu^m = 2\phi\zeta^{1-\varphi} + \zeta\rho.$$

[20] To find the steady-state level of the CBDC spread, χ^m, we set the steady-state CBDC rate, R^m, equal to 0.99, to simulate a negative net return in real terms. We make this conjecture since most of the central banks' research projects indicate that CBDCs may offer low or no nominal interests.

[21] To find the steady-state level of the reserve spread, χ^r, we set the steady-state reserve rate, R^r, equal to 1.0.

D Robustness checks

D.1 Impulse responses to 10% increase in λ_t with alternative specifications

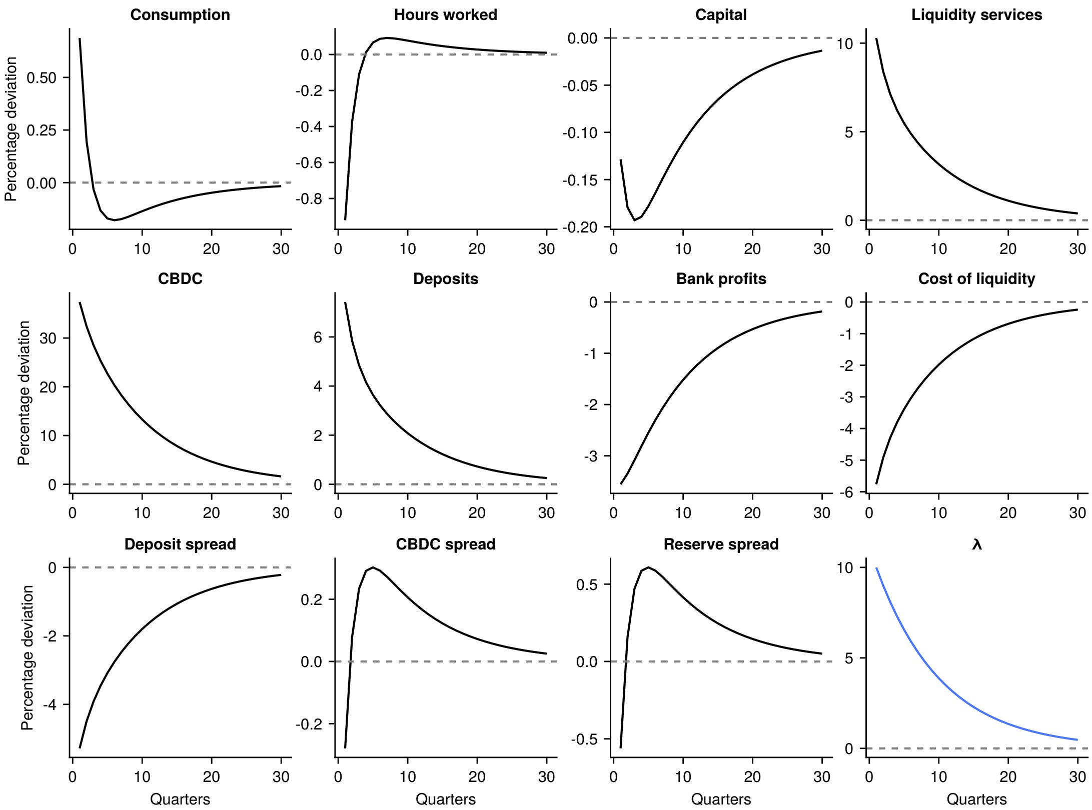

Figure 4: Lower steady-state $\lambda = 0.5$

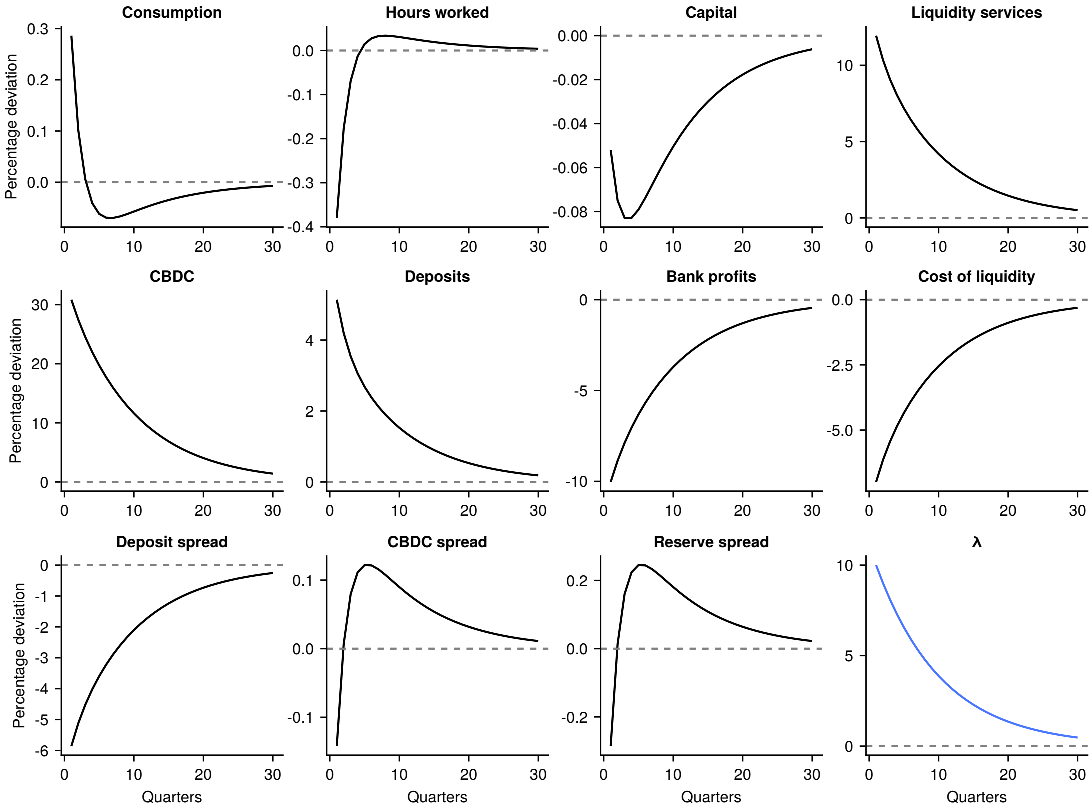

Figure 5: Higher steady-state $\lambda = 1.5$

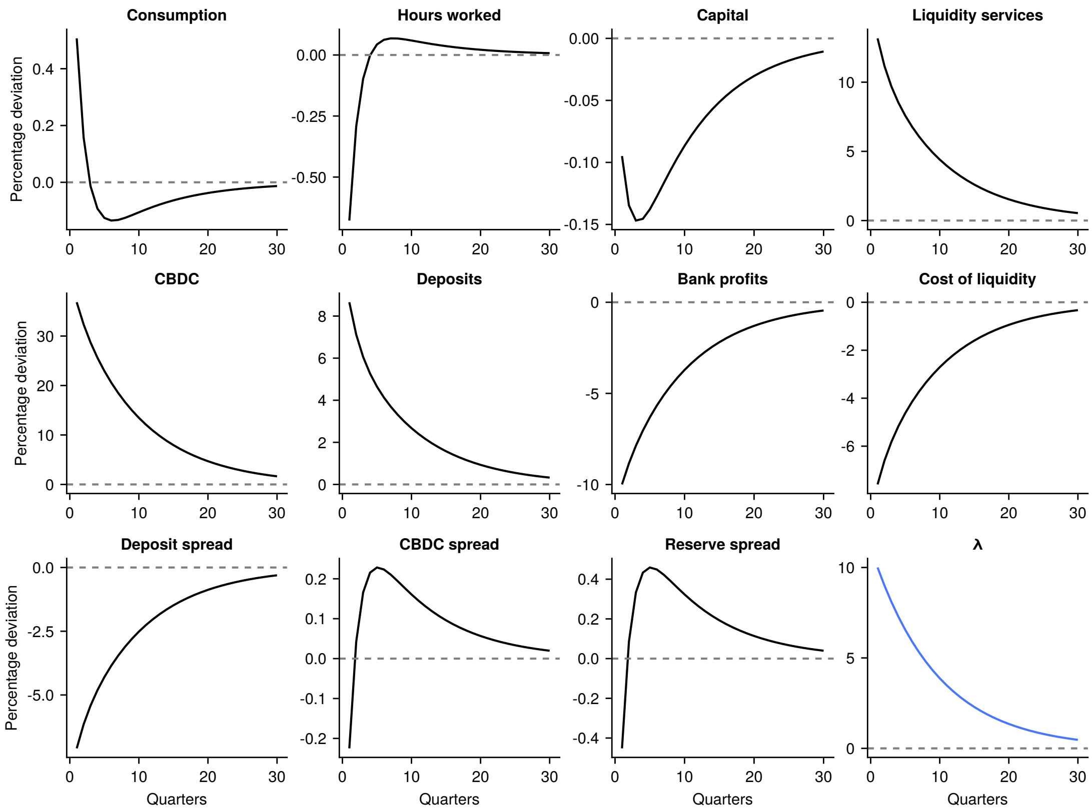

Figure 6: Lower steady-state $\epsilon = 1/9$

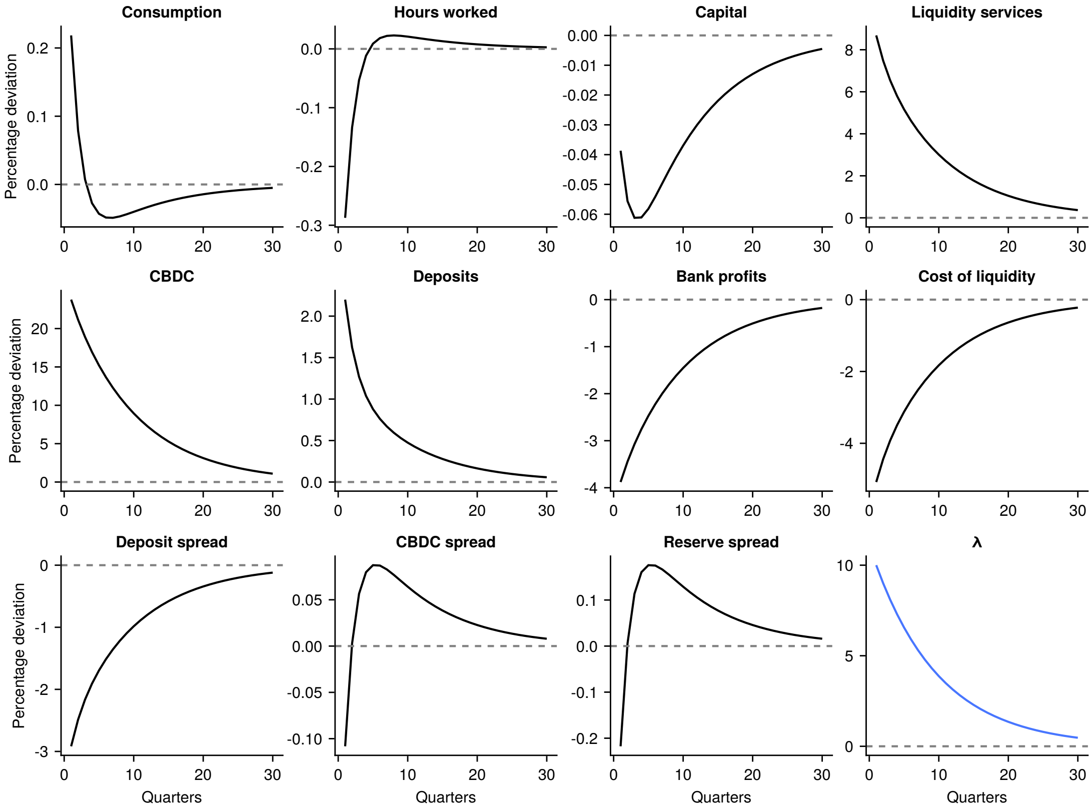

Figure 7: Higher steady-state $\epsilon = 1/3$

D.2 Impulse responses to 10% increase in ϵ_t with alternative specifications

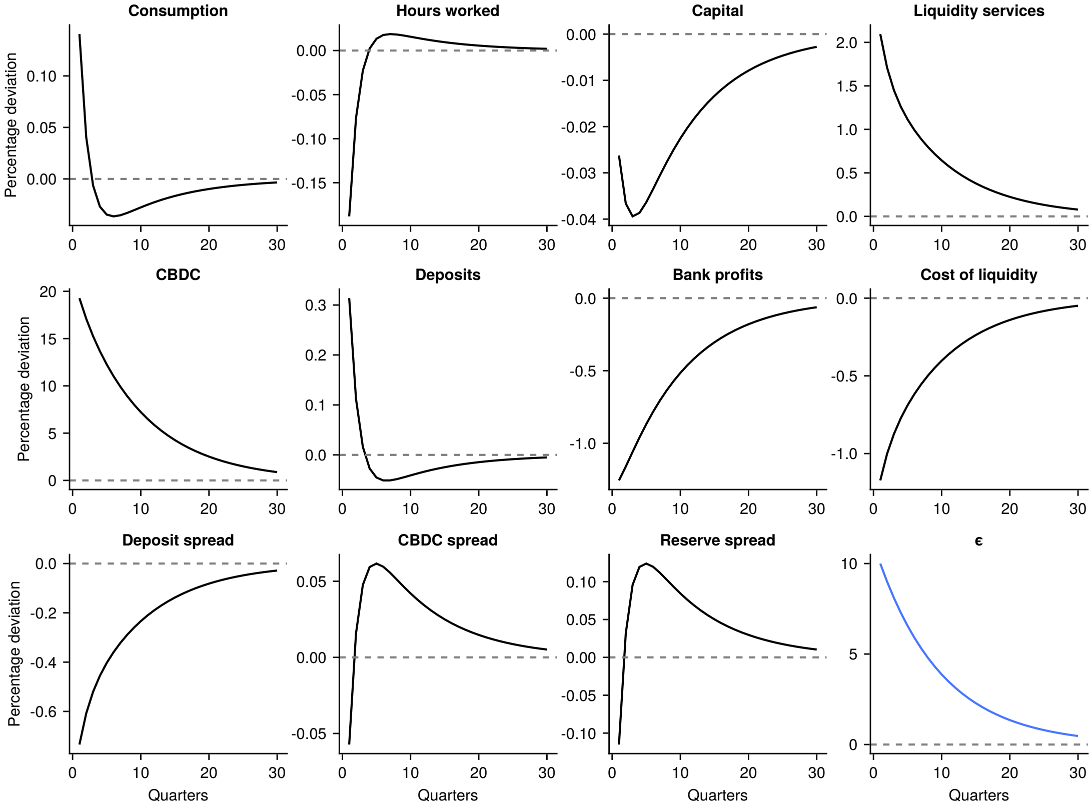

Figure 8: Lower steady-state $\lambda = 0.5$

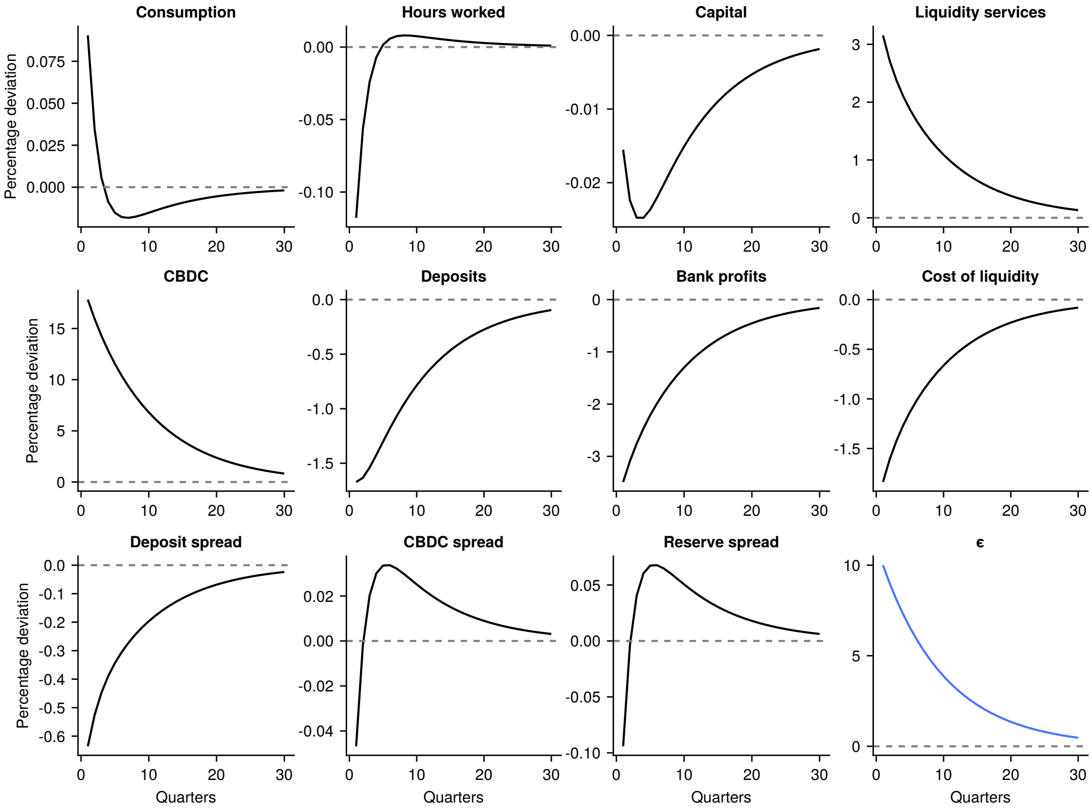

Figure 9: Higher steady-state $\lambda = 1.5$

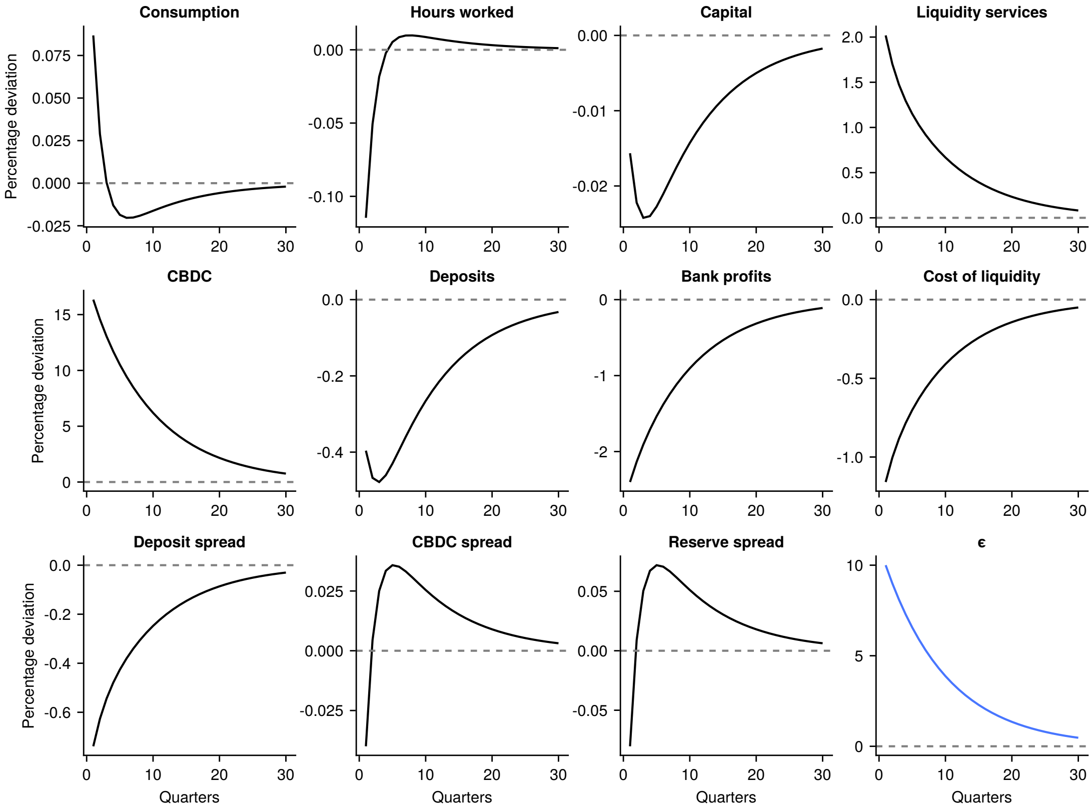

Figure 10: Lower steady-state $\epsilon = 1/9$

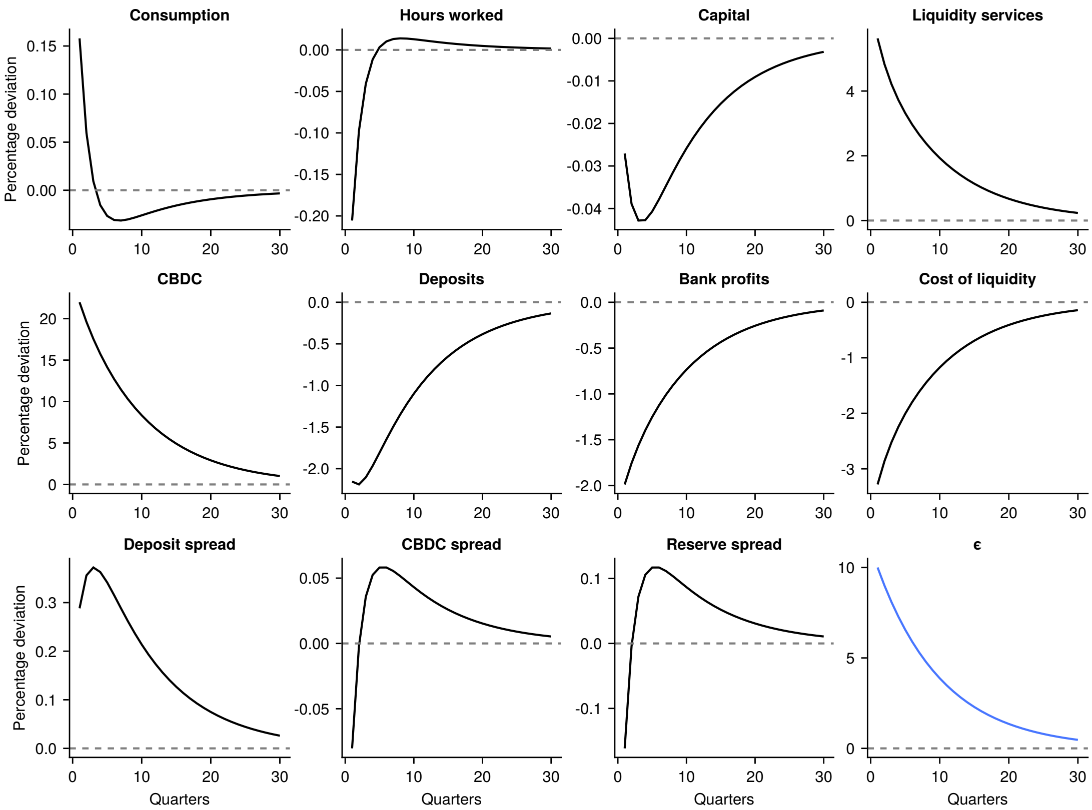

Figure 11: Higher steady-state $\epsilon = 1/3$

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