Discussion of "Monetary Policy without Commitment"

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This paper

- Addresses two issues that are often ignored :
  - non-linearities
  - transition to steady state
- It shows No-commitment policies may imply large welfare losses

Would like to take this chance to point out

- transitions and non-linearities are important generically
- we should reconsider how we address time inconsistency

Greulich, Laczó and Marcet (2023) = Chamley with Heterogeneous agents

- A government funds expenditures using bonds, capital taxes and labor taxes (Chamley)
- Competitive equilibrium, flexible prices etc.
- We add: two agents, worker and capitalist, worker has much higher labor to capital income ratio
- Common result: capital taxes should be zero in the long run (Chamley, Straub and Werning)

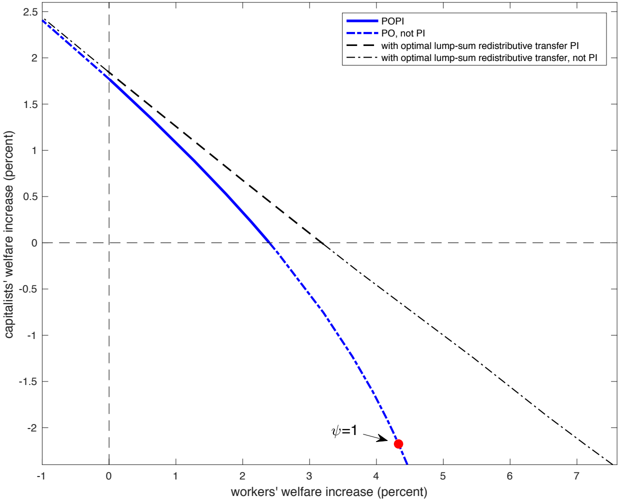
Greulich, Laczó and Marcet (2023) = Chamley with Heterogeneous agents

Features of Optimal Policy

- $\tau_t^k = 0$ for all t worker is worse off than status quo.
- Long transition: $\tau_t^k = \widetilde{\tau}$ between 16 to 24 years in pareto-improving range
- longer transition if we favor more the worker.
- Low initial τ_t^l promotes both redistribution and efficiency.
- Therefore, there is an equity/efficiency tradeoff even if $\tau_{ss}^k = 0$.

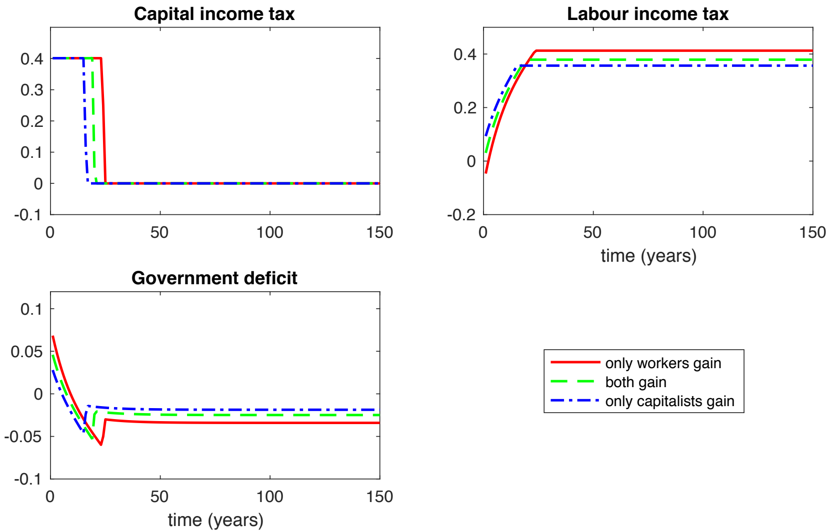
Baseline

Figure 2: The Ramsey Pareto frontier of Pareto-improving equilibria in the baseline m



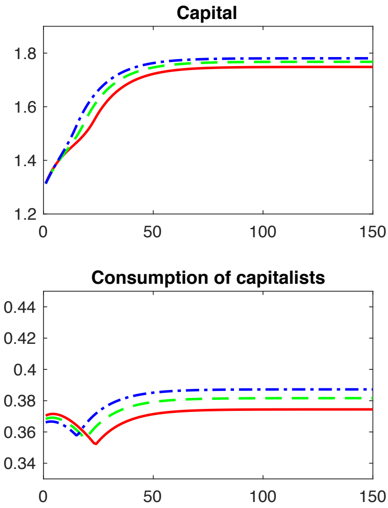
Baseline

Time Paths



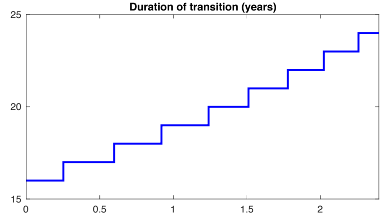
Baseline

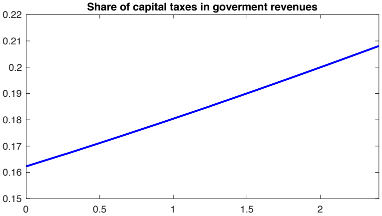
Time Paths



Baseline

Properties of Policy along PO Frontier





Worker' welfare increase % relative to Status Quo

Our new project: Reconsidering Time Inconsistency

Time Inconsistent full commitment policies are often seen as irrelevant.

There are probably two reasons for this view:

- "If the government reoptimizes it will want to change the continuation policy eventually"
- "Everybody will agree to change the policy eventually"

  $\Leftrightarrow$

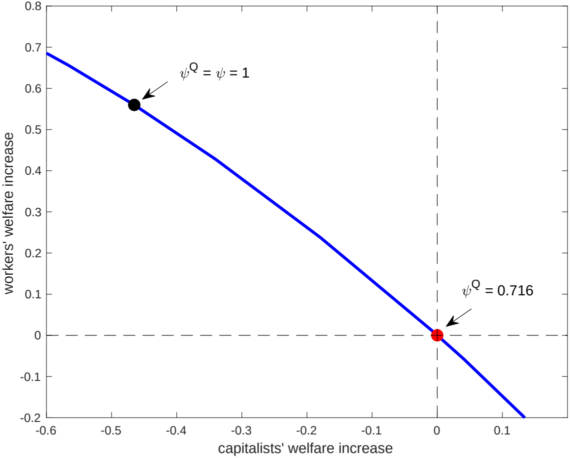
  The policy is not "renegotiation proof"

For example, in a Chamley model, when $\tau_t^k = 0$, the government will want to reset capital taxes to $\tau_t^k > 0$ so as to tax existing capital and reduce distorting taxes and all (homogeneous) agents would agree.

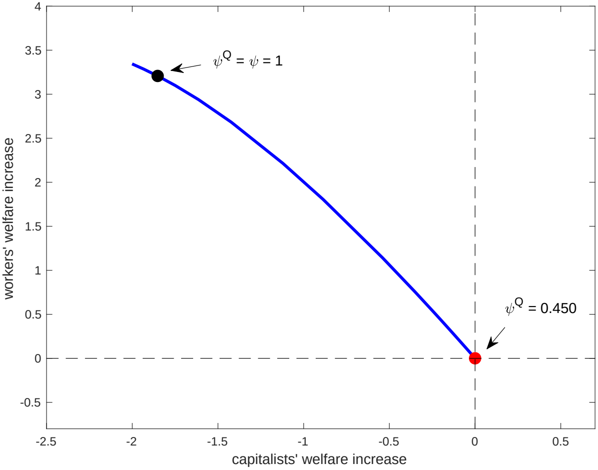
Reoptimization with heterogeneous agents

- What if the government "reoptimizes" at $t = Q$?
- Will everybody agree to change the continuation policy?

PO Frontier Baseline Heterogeneity, $Q = 5$



PO Frontier Baseline Heterogeneity, $Q = 30$



A Monetary model with heterogeneous agents

- Money in the utility function, two consumers $j = 1, 2$ with utility

$$\sum_{t=0}^{\infty} \beta^t \left[u(c_{j,t}) + v(l_{j,t}) + z(m_{j,t}) \right]$$

- no uncertainty, flexible prices
- Only assets: nominal bonds and money
- government funds expenditures with proportional taxes on labor τ_t^l, nominal bonds and money issuance

Well known results:

- Friedman rule does not hold, inflation "tax" lowers labor taxes.
- Policy time inconsistent: The government runs a huge inflation in initial period and promises to never do it again

Time Inconsistent Optimal Inflation

The consumer's budget constraint is given by

$$\sum_{t=0}^{\infty} \beta^t \frac{u'(c_t)}{u'(c_0)} \left(c_t + m_t \frac{i_{t+1}}{1 + i_{t+1}} - (1 - \tau_t^l) w_t l_t \right) = \frac{B_{-1}(1 + i_0) + M_{-1}}{P_0}$$

So with a representative agent high P_0 allows to lower i_t and τ_t^l.
Full commitment optimal policy: high P_0 and follow moderate inflation after that.
Time inconsistency: If the government can reoptimize at period Q it will send P_Q as high as possible

What changes under heterogeneous agents:
-most pareto optimal allocations do not imply P_0 as high as possible
-the continuation of the full commitment policy is likely to be Pareto Optimal at a future time Q.

A proof outline

Using primal approach, budget constraint is

$$\sum_{t=0}^{\infty} \beta^t \left(u'(c_t)c_t + z'(m_t)m_t - v'(l_t)l_t\right) = \frac{B_{-1}(1+i_0) + M_{-1}}{P_0} u'(c_0)$$

With homogeneous agents

Lagangean of full commitment at t=0

$$\sum_{t=0}^{\infty} \beta^t (u(c_t) + z(m_t) + v(l_t)l_t) + \tag{1}$$

$$\Delta \left[\sum_{t=0}^{\infty} \beta^t (u'(c_t)c_t + z'(m_t)m_t - v'(l_t)l_t) \right. \tag{2}$$

$$\left. - \frac{B_{-1}(1 + i_0) + M_{-1}}{P_0} u'(c_0) \right] \tag{3}$$

Continuation policy

Continuation of full commitment at $t = Q$ does optimise

$$\sum_{t=Q}^{\infty} \beta^t (u(c_t) + z(m_t) + v(l_t)l_t) + \tag{4}$$

$$\Delta \left[\sum_{t=0}^{\infty} \beta^t (u'(c_t)c_t + z'(m_t)m_t - v'(l_t)l_t) \right] \tag{5}$$

But the lagrangean for reoptimising at $t = Q$ has the "additional piece"

$$-\Delta \frac{B_{Q-1}(1 + i_Q) + M_{Q-1}}{P_Q} u'(c_Q)$$

so the solution would differ from the continuation

What changes under heterogeneous agents

There are two utilities, with welfare weight of the second agent ψ.
There are two budget constraints, hence two lagrange multipliers Δ^1, Δ^2.
One can always choose $\psi^Q, \Delta^{Q,1}, \Delta^{Q,2}$ so that the Lagrangeans of the continuation and the reoptimisation coincide.
In particular, the "additional piece" is now

$$-\sum_{j=1}^{2} \Delta^{Q,j} \frac{B^j_{Q-1}(1+i_Q)+M^j_{Q-1}}{P_Q} u'(c^j_Q)$$

and we can always choose $\Delta^{Q,j}$'s that make this equal to zero.
Hence the continuation of full commitment is pareto optimal.

Conclusion

Great paper.

We should reconsider the view that the full commitment policy is irrelevant.

Greulich, Laczó and Marcet (2023) = Chamley with Heterogeneous agents

The GLM model:

- No uncertainty, standard production and utility function, flexible labor supply, competitive markets.
- Government sets proportional labor and capital taxes τ_t^l, τ_t^k, funds fixed spending g and issues debt
- Optimal Policy under full commitment (Ramsey Equilibrium).
- Agents are heterogeneous (Two-agents) in their labor wage (different efficiency) and their initial wealth.
- Do not use welfare functions, study PO frontier, focus on Pareto-Improving policies.
- Standard choices for utility, production, etc. Yearly calibration.

The Model: Consumers

- Two consumers $j = 1, 2$ with utility

$$\sum_{t=0}^{\infty} \beta^t \left[u(c_{j,t}) + v(l_{j,t})\right]$$

- Heterogeneous in wage efficiency and initial wealth
- A policy is PO iff it maximizes

$$\sum_{t=0}^{\infty} \beta^t \Big[u(c_{1,t}) + v(l_{1,t}) + \psi[u(c_{2,t}) + v(l_{2,t})]\Big]$$

subject to equilibrium constraints
for some $\psi > 0$

The Model: Consumers

Budget constraint of consumer $j = 1, 2$

$$\begin{aligned} c_{j,t} + k_{j,t} - k_{j,t-1}(1-d) &= w_t \ \phi_j \ l_{j,t}(1-\tau_t^l) + \\ & k_{j,t-1} r_t - k_{j,t-1}(r_t - d)\tau_t^k \end{aligned}$$

$c_{j,t}$ consumption of agent j
$k_{j,t}$ capital agent j
w_t aggregate wages
ϕ_j efficiency of labor agent j
$l_{j,t}$ hours worked agent j
r_t rental price of capital

The Model: Firms

Notation: No j subscript means aggregate variable

Firms production function $F(k_{t-1}, e_t)$

Competitive

No uncertainty

The Model: Market clearing

$$e_t = \frac{1}{2}\sum_{j=1}^{2} \phi_j l_{j,t}$$

$$k_t = k_t^g + \frac{1}{2}\sum_{j=1}^{2} k_{j,t}$$

$$F(k_{t-1}, e_t) = \frac{1}{2}\sum_{j=1}^{2} c_{j,t} + g + k_t - (1-d)k_{t-1}$$

Characterization of Competitive Equilibrium

Consumer FOC

$$u'(c_{j,t}) = \beta \ u'(c_{j,t+1})R_{t+1}$$

where $R_{t+1} = 1 + (r_{t+1} - d)(1 - \tau^k_{t+1})$.

$$-\frac{v'(l_{j,t})}{u'(c_{j,t})} = w_t \ \phi_j \ (1 - \tau^l_t)$$

Firms' FOC

$$r_t = F_k(k_{t-1}, \ e_t) \text{ and } w_t = F_e(k_{t-1}, \ e_t).$$

Specific utility function

$$u(c) = \frac{c^{1-\sigma_c}}{1-\sigma_c} \qquad v(l) = -\omega \frac{l^{1+\sigma_l}}{1+\sigma_l}$$

consumer FOC imply

$$\begin{aligned} \frac{c_{2,t}}{c_{1,t}} &= \lambda \\ \frac{l_{2,t}}{l_{1,t}} &= \mathcal{K}(\lambda) \equiv \lambda^{-\frac{\sigma_c}{\sigma_l}} \left(\frac{\phi_1}{\phi_2}\right)^{\frac{1}{\sigma_l}} \end{aligned}$$

constant λ, for all t.

Implementability conditions

For $j = 1, 2$

$$\sum_{t=0}^{\infty} \beta^t \frac{u'(c_{1,t})}{u'(c_{1,0})} \left[c_{j,t} + \frac{v'(l_{j,t})}{u'(c_{j,t})} \; l_{j,t} \right] = k_{j,-1} R_0.$$

For j=1

$$\sum_{t=0}^{\infty} \beta^t \left[u'(c_{1,t}) c_{1,t} + v'(l_{1,t}) \; l_{1,t} \right] = u'(c_{1,0}) \; k_{1,-1} R_0$$

Implementability conditions

For j=1

$$\sum_{t=0}^{\infty} \beta^t \left[u'(c_{1,t})c_{1,t} + v'(l_{1,t})\ l_{1,t}\right] = u'(c_{1,0})\ k_{1,-1}R_0$$

For j=2, using $c_{2,t} = \lambda c_{1,t}$ and $l_{2,t} = \mathcal{K}(\lambda) l_{1,t}$ then

$$\sum_{t=0}^{\infty} \beta^t \left(u'(c_{1,t})\lambda c_{1,t} + \frac{\phi_2}{\phi_1} v'(l_{1,t})\ \mathcal{K}(\lambda) l_{1,t} \right) = u'(c_{1,0})\ k_{2,-1}R_0$$

These, plus feasibility are sufficient for a Competitive Equilibrium.

Optimize over $\tau_0^k, \lambda, \{c_t^1, k_t, l_t^1\}_{t=0}^{\infty}$

Constraints on Policy

1. No lump sum taxes
   - Focus on $T_w = T_c = 0$.
   - Sometimes use redistributive transfer $T_w = -T_c$.
   - Study robustness to a deductible $\mathcal{D} = -T_w = -T_c$.
2. Upper bound to capital taxes:

$$\tau_t^k \leq \widetilde{\tau} < 1$$

3. No immiseration:

$$c_t \geq \widetilde{c} \geq 0$$

Constrained PO problem

$$\max_{\tau_0^k, \lambda, \left\{c_t^1, k_t, l_t^1\right\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty} \beta^t \left[u(c_{1,t}) + v(l_{1,t})\right]$$

$$\text{s.t.} \quad \sum_{t=0}^{\infty} \beta^t \left[u(\lambda c_{1,t}) + v\left(\mathcal{K}(\lambda) l_{1,t}\right)\right] \geq \underline{U}^2$$

implementability, feasibility and policy constraints

We could trace the PO frontier by solving this for all feasible $\underline{U}^2$.

Lagrangean

$$
\begin{aligned}
\mathcal{L} \;=\; \sum_{t=0}^{\infty} \beta^t \Big[& \; u(c_{1,t}) + v(l_{1,t}) + \psi \left[u(\lambda c_{1,t}) + v(\mathcal{K}(\lambda) l_{1,t}) \;\right] + \\
& \xi_t (c_t - \widetilde{c}) + \\
& \Delta_1 [\; u'(c_{1,t}) c_{1,t} + v'(l_{1,t}) \; l_{1,t} \;] + \\
& \Delta_2 [\; u'(c_{1,t}) \lambda c_{1,t} + \frac{\phi_2}{\phi_1} v'(l_{1,t}) \mathcal{K}(\lambda) l_{1,t} \;] + \\
& \gamma_t [\; u'(c_{1,t}) - \beta \; u'(c_{1,t+1})(1 + (r_{t+1} - d)(1 - \tilde{\tau})) \;] - \\
& \mu_t \left(\frac{1+\lambda}{2} c_{1,t} + g + k_t - (1-d) k_{t-1} - F(k_{t-1}, e_t) \right) \; \Big] - \\
& \mathbf{W} - \psi \underline{U}^2
\end{aligned}
$$

Notice, generically $\lambda \neq \psi$

FOC for λ

To characterize optimum we need $\frac{\partial L}{\partial \lambda} = 0$:

$$\sum_{t=0}^{\infty} \beta^t \left\{ (\psi \lambda^{-\sigma_c} + \Delta_2) \left[u'(c_{1,t}) c_{1,t} + \frac{\phi_2}{\phi_1} \mathcal{K}'(\lambda) v'(l_{1,t}) l_{1,t} \right] - \frac{\Omega^l v'(l_{1,t})}{\phi_1 + \phi_2 \mathcal{K}(\lambda)} \phi_2 \mathcal{K}'(\lambda) l_{1,t} - \frac{\mu_t}{2} c_{1,t} \right\} - \mathbf{W_0}$$

Calibration

	Parameter	Value	Target status quo
Preference Parameters	β	0.96	yearly model
	σ_c	2	
	σ_l	3	
	ω	854	1/3 hours worked
Heterogeneity parameters	ϕ_w/ϕ_c	0.91	wage ratio
	$k_{c,-1}$	4.356	consumption ratio
	$k_{w,-1}$	-1.136	consumption ratio
Production parameters	α	0.394	capital income share
	d	0.074	
Government parameters	g	0.094	g/GDP ratio
	k^g_{-1}	-0.315	debt/GDP ratio
	$\widetilde{\tau}$	0.401	status quo tax

FOC from $t = 0$

to simplify consider Q high so that $\tau_Q^k = 0$:

- Ramsey FOC for c_t and l_t^1 give:

$$\Omega^l v'\left(l_{1,t}\right) = -F_e\left(k_{t-1}, e_t\right)\frac{\phi_1 + \phi_2\mathcal{K}(\lambda)}{1+\lambda}\Omega^c u'\left(c_{1,t}\right) \tag{6}$$

  where

$$\Omega^c \equiv 1 + \psi\lambda^{1-\sigma_c} + \left(\Delta_1 + \lambda\Delta_2\right)\left(1-\sigma_c\right),$$
$$\Omega^l \equiv 1 + \psi\mathcal{K}(\lambda)^{1+\sigma_l} + \left(\Delta_1 + \frac{\phi_2}{\phi_1}\mathcal{K}(\lambda)\Delta_2\right)\left(1+\sigma_l\right),$$

FOC from $t = 0$

Denote with * Continuation policy:

$$\Omega^{*,l} v'\left(l^*_{1,t}\right) = -F_e\left(k^*_{t-1}, e^*_t\right) \frac{\phi_1 + \phi_2 \mathcal{K}(\lambda^*)}{1+\lambda^*} \Omega^{*,c} u'\left(c^*_{1,t}\right) \qquad (7)$$

where

$$\Omega^{*,c} \equiv 1 + \psi^*(\lambda^*)^{1-\sigma_c} + \left(\Delta^*_1 + \lambda^* \Delta^*_2\right)\left(1 - \sigma_c\right),$$

$$\Omega^{*,l} \equiv 1 + \psi^* \mathcal{K}(\lambda^*)^{1+\sigma_l} + \left(\Delta^*_1 + \frac{\phi_2}{\phi_1}\mathcal{K}(\lambda^*)\Delta^*_2\right)\left(1 + \sigma_l\right),$$

CTC

There is CTC if and only if the continuation is PO $\Leftrightarrow$
There is a weight ψ^Q for which the continuation is optimal $\Rightarrow$

$$\frac{\Omega^{*,l}}{\Omega^{*,c}} = \frac{\Omega^{Q,l}}{\Omega^{Q,c}} \tag{8}$$

where

$$\Omega^{Q,c} \equiv 1 + \psi^Q(\lambda^*)^{1-\sigma_c} + \left(\Delta_1^Q + \lambda^*\Delta_2^Q\right)(1-\sigma_c)\,,$$
$$\Omega^{Q,l} \equiv 1 + \psi^Q\mathcal{K}(\lambda^*)^{1+\sigma_l} + \left(\Delta_1^Q + \frac{\phi_2}{\phi_1}\mathcal{K}(\lambda^*)\Delta_2^Q\right)(1+\sigma_l)\,,$$

CTC

Collecting all FOC's there is always a triplet $\psi^Q, \Delta_1^Q, \Delta_2^Q$ satisfying FOC's for continuation $*$ policy.

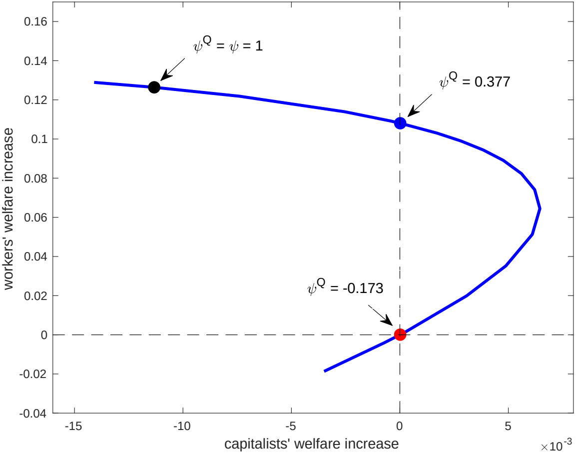
Question is : $\psi^Q > 0$?

Some analytic results:

Not if agents are almost equal $\lambda^* \approx \mathcal{K}(\lambda^*) \approx 1$.

Yes if λ^* large enough.

PO Frontier Low Heterogeneity, $Q = 5$



PO Frontier Low Heterogeneity, $Q = 30$