Sovereign default and the decline in interest rates*

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Abstract

Sovereign debt yields have undergone a historic decline over the last half-century. Standard explanations, including aging populations and increases in asset demand from abroad, encounter difficulties when confronted with the full range of evidence. We propose an explanation based on a decline in inflation and default risk. We show that a model with sovereign default captures the decline in interest rates, the stability of equity valuation ratios, and the reduction in investment and output growth. Calibrations of the model post-COVID suggest that sovereign default risk may have returned.

Keywords: Savings glut, Inflation expectations, Rare disasters, Secular stagnation
JEL codes: E31, E43, G12

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1 Introduction

The period from 1980 to 2021 marked a historic decline in sovereign yields. The prolonged combination of low interest rates and sluggish growth—especially following the Great Recession of 2009—has revived the idea of "secular stagnation," a persistent period of low investment and weak demand (Hansen, 1939). Summers (2015) and Gordon (2015) argue for the relevance of Hansen's concept from two angles: demand-side—an increase in desired savings arising from changing demographics (Auclert et al., 2021, Eggertsson et al., 2019) or growing inequality (Auclert and Rognlie, 2017, Mian et al., 2021)—and supply-side—stemming from a decline in the ideas and dynamism that fueled growth in the postwar era. A complementary idea is that of a global savings glut (Bernanke, 2005, Caballero et al., 2008): there is too great a supply of savings, mainly from patient investors outside the United States, relative to the demand for productive investment.

Yet the excess-savings explanation faces difficulties. If investors' desire to save had increased, both bonds and equities should have become more valuable, and investment should have surged. Instead, equity valuations rose only modestly and investment declined. A greater demand for savings, low interest rates, and low growth together thus present a puzzle: why did stock valuations and real investment not rise in line with bond prices? In response, Farhi and Gourio (2018) jointly consider growth, interest rates, and stock valuations in a neoclassical growth model with rare disasters (Barro, 2009, Gourio, 2012). They show that a substantial increase in the risk of rare disasters is needed to reconcile the levels of interest rates and stock prices.

Although Farhi and Gourio (2018) succeed in accounting for the joint behavior of stock prices and interest rates, an explanation based on heightened fears of disaster encounters its own problems. First, such fears should be reflected in option prices and in the VIX. Although the VIX varies over time, its average level in the first and second halves of our sample is nearly identical. Second, a rare-disaster explanation is fragile because it depends on the elasticity of intertemporal substitution (EIS). If the EIS exceeds one, rising disaster risk depresses valuations; if it is below one, it raises them—deepening the puzzle. Moreover, in a production economy, rising disaster risk and an EIS below one counterfactually imply rising investment and growth. One could instead posit a *decline* in disaster risk, but this only highlights the lack of evidence of an increase in disaster risk.

We therefore propose a different explanation—one based on a decline in the risk of sovereign default. Greater trust in the sovereign's ability and willingness to repay debts could have driven the historic fall in interest rates observed over recent centuries (Ferguson, 2018). In the modern era, this decline in perceived default risk likely manifests through reduced inflation expectations. Indeed, there is substantial evidence for a steady fall in inflation expectations from 1980–2020, coinciding with the decline in interest rates. Evidence from options markets indicates that inflation expectations became "anchored" in the twenty-first century, meaning investors no longer feared extreme inflation or deflation (Reis, 2020). This evidence makes it possible to jointly explain the decline in interest rates and the stability of equity valuation ratios. Because the true real rate has not fallen as much as the nominal rate, valuation ratios rise less, and there is no need to invoke a large increase in the probability of rare disasters.

If it is simply inflation expectations that have declined, why have *measured* real rates—nominal rates minus inflation expectations—also fallen? This apparent disconnect disappears once we account for inflation *risk*, the risk that the price level rises during recessions or depressions. With perfect foresight, a change in expected inflation would not affect ex post real rates. However, inflation (or the lack thereof) can surprise investors. A decline in inflation risk lowers the premium required to hold nominal securities. Interest rates will fall if this premium declines, even when measured in real terms ex post. Because inflation is a form of default, a decline in inflation risk is effectively a decline in the probability of default—one that may even influence yields on inflation-protected securities. Our first contribution is to show that a model combining rare disasters with a decline in sovereign default risk can explain both the fall in interest rates and the stability of valuations.

We find direct empirical support for this interpretation. First, we compare returns on nominal and inflation-indexed bonds. In the absence of an inflation risk premium, the average real returns on nominal and inflation-linked bonds should coincide. However, in the 1980s and 1990s, inflation-adjusted returns on U.K. nominal bonds were almost twice as high as those on inflation-linked bonds. This premium disappeared in the twenty-first century, as our model predicts. Second, we document the decline in inflation risk using survey expectations and the changing correlation between inflation and growth. This evidence aligns with recent findings that the once-positive correlations between inflation and output growth (Campbell et al., 2020) and between bond and stock returns (Campbell et al., 2017) turned negative in the 2000s. Finally, because sovereign risk depends on institutions that have evolved substantially over the centuries, our explanation can account for the striking fact that today's low rates are unprecedented not just in the last 40 years, but in the last 400 (Schmelzing, 2020).

We first account for the joint behavior of interest rates, inflation expectations, and stock prices in an endowment economy. However, the puzzle extends beyond an endowment setting. Just as the joint behavior of equities and interest rates poses a challenge, so too does that of investment and interest rates. A stronger desire to save should have produced an investment boom; instead, investment fell. When embedded in a production economy, our model also explains the joint decline in investment and growth, even as interest rates fell.

Our paper poses a puzzle: given the large decline in interest rates, equity valuations should have risen far more than they did. We offer a resolution: the drop in yields reflected lower sovereign-default risk, not a lower real rate. This claim naturally invites questions. One concern is that default risk appeared to fall even as debt-to-GDP ratios climbed across advanced economies. In our model—where government bonds are priced as claims on future surpluses—a smaller default premium can indeed coincide with higher debt, provided investors grow more confident in the government's capacity to generate those surpluses. The United States briefly ran surpluses in the 1990s, yet has not done so since, so it is natural to ask why confidence may have risen, especially in light of the magnitude of the debt. A second question arises from the surge in inflation and interest rates after 2021. In a quantitative exercise, we show that this episode can be interpreted as an increase in sovereign default risk. Because yields rose on both nominal Treasuries and TIPS, this increase cannot be attributed solely to inflation risk. The evidence is consistent instead with investors broadening their perception of default risk to include the possibility of losses on all government liabilities, whether nominal or inflation-protected. It is too early to tell whether this episode represents a historic shift, either in the risk of sovereign default, or in the form it might take.

The remainder of the paper is organized as follows. Section 2 summarizes the empirical evidence. Section 3 examines how an endowment economy can match this evidence, either through changes in the probability of disaster or changes in the probability of default. Section 4 extends the analysis to a production economy with inventory technology and derives implications for investment and growth. Section 5 concludes.

2 Summary of the data

Panel A of Figure 1 shows nominal government rates in a seven-century-long dataset collected by Schmelzing (2020). Interest rates are highly volatile, as Jordà et al. (2019) emphasize.[1] Periods of extreme spikes and also low rates occurred around the American Revolution, Napoleonic Wars, and World War II, reflecting a tension between an increase in risk of sovereign default and precautionary savings around disasters. The high rates in the 1970s and 1980s clearly stand out. Nonetheless, the figure shows a steady decline. Panel B shows the Bank of England lending rate starting from 1700, from the start of when the series was available. Only in the very most recent period did this rate reach a zero lower bound.

[1] Jordà et al. (2019) note that prior observations of a real rate of zero are not unusual. However, these are observations after subtracting ex post realized inflation, not ex ante inflation-adjusted yields. Although both returns are zero from an investor's perspective, one was a realization of zero because of high inflation, whereas the other is an expected value of zero.

Figure 1: Nominal government rates

Panel A shows a five-year moving average of long-term nominal sovereign yields in the United Kingdom, Holland, Germany, Italy, and the United States from 1311–2018. The solid black line represents an average of all of the plotted series. Yields are from Schmelzing (2020) and are in annual terms. Yields come from a variety of archival, primary, and secondary sources. Panel B shows the nominal lending rate for the Bank of England expressed in annual terms from 1694–2024.

A. Sovereign yields over the long run **B. Bank of England lending rate**

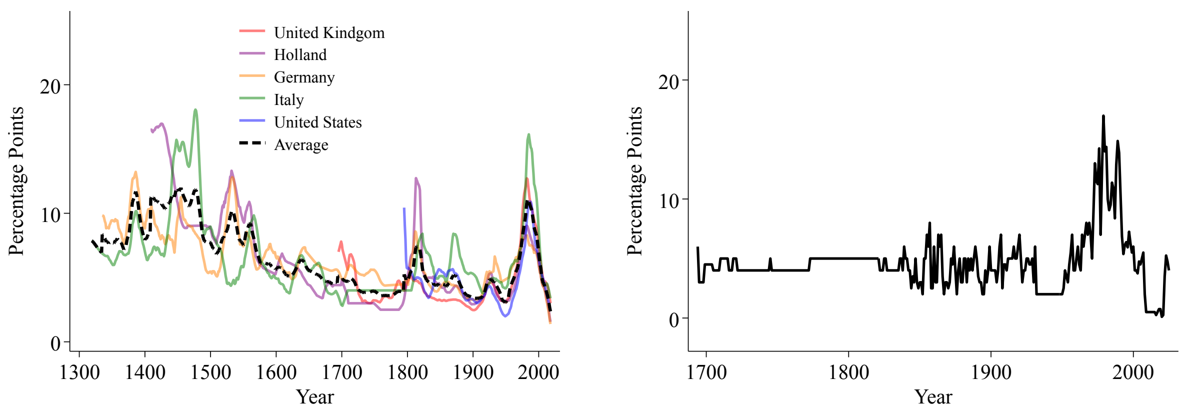
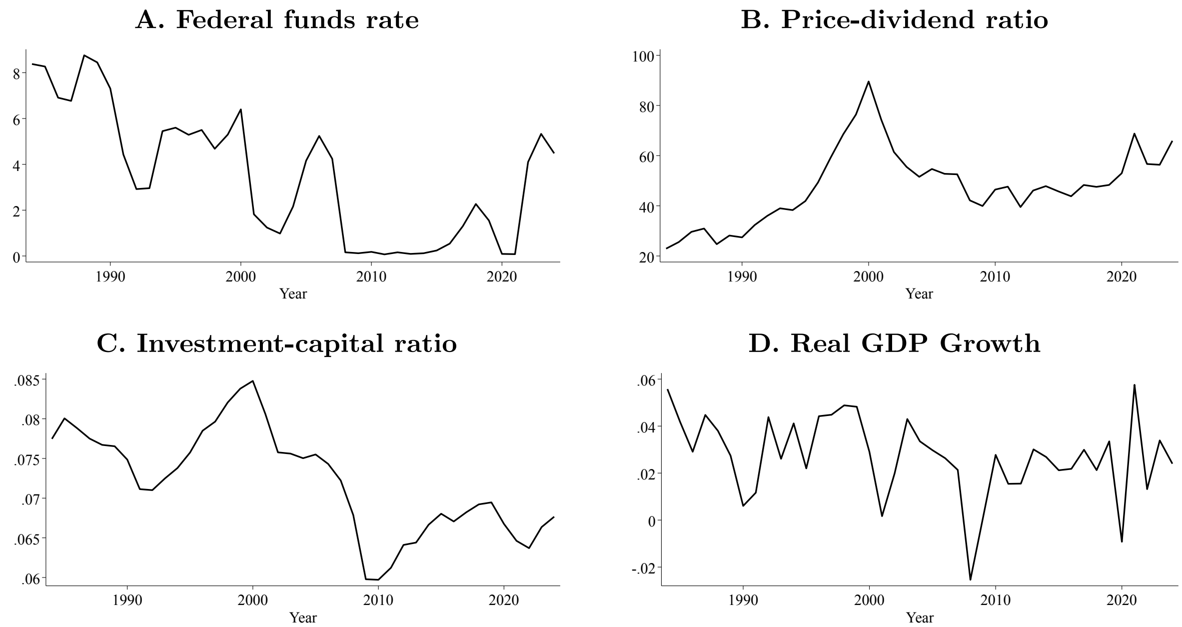


Figure 2 narrows down to the last 40 years, the focus of much of the literature. The federal funds rate in the U.S. declined sharply from 8.4% to 0.1% in 2021 before rising to 4.5% in 2024 (Panel A).[2] On the other hand, the price-dividend ratio went from 22.9 in 1984 to 68.8 in 2021 and rose further to 65.9 in 2024. This implies that a dividend yields went from 4.4% down to 1.5% in 2021—a smaller decline relative to interest rates (Panel B). The last row of Figure 2 shows the decline in the investment-capital ratio (Panel C) and the real GDP growth (Panel D) since the 1980s. Investment as a percentage of capital stock went from an average of 7.7% to 6.8% in 2024, while real GDP growth decreased from an average of 3.5% to 2.1%.

[2]In our quantitative model, we will focus on the one-year nominal yield. Notably, the literature studying monetary policy shocks and real interest rates finds that the secular decline in short-term rates has also shown up in long-term yields (Bianchi et al., 2022, Hillenbrand, 2024). For example, the 30-year yield fell from 11.5% to 1.9% in 2021 before rising to 4.8% in 2024, similar to the federal funds rate.

Figure 2: Data series, United States from 1984–2024

The figure shows the effective federal funds rate (shown in annual percentage points), the annual price-dividend ratio for the United States on the value-weighted CRSP index, the investment-capital ratio, and the annual real GDP growth rate for the United States.



3 Endowment economy

We first turn to a standard endowment economy with a representative agent. To interpret the secular decline in interest rates, we calibrate the model separately to two sample periods, 1984–2000 and 2001–2021. We identify the year of this structural break (2001) by conducting a break test on the time series of one-year inflation-adjusted Treasury bill yields. Details can be found in Appendix B. This is the same breakpoint used by Farhi and Gourio (2018), who perform a similar analysis; it is also consistent with evidence from Campbell et al. (2020), who find a structural break in the relationship between GDP growth and inflation in 2001. Although this approach of comparing sample averages means that certain features of the data (such as high-frequency volatility of prices and interest rates) remain outside the scope of the analysis, it allows us to consider the possibility of long-run unforeseen structural changes.[3] Farhi and Gourio assume a neoclassical growth model. We will return to such a model in the next section, but for the analysis at hand, the extra degree of complication is not necessary. As far as prices and interest rates are concerned, and in this i.i.d.-growth-rate setting, the production model and the endowment model yield the same predictions.

The aggregate endowment evolves according to

$$C_{t+1} = C_t e^{\mu}(1 - \chi_{t+1}), \tag{1}$$

where χ_{t+1} represents an occurrence of rare disaster:

$$\chi_{t+1} = \begin{cases} 0 & \text{with probability} \quad 1-p \\ \eta & \text{with probability} \quad p, \end{cases} \tag{2}$$

for $\eta \in (0,1)$. Note that p represents the probability of a disaster and η its magnitude. We assume that the representative agent has Epstein-Zin-Weil recursive preferences (Epstein and Zin, 1989, Weil, 1990) with risk aversion γ, elasticity of intertemporal substitution (EIS) ψ, and discount factor β. Let W_t denote the wealth of the representative agent, here assumed to be the cum-dividend value of the consumption claim. Let $R_{W,t+1} \equiv W_{t+1}/(W_t - C_t)$ denote the return on wealth from time t to time $t+1$. The stochastic discount factor (SDF) then equals

$$M_{t+1} \equiv \begin{cases} \beta^{\theta}(\frac{C_{t+1}}{C_t})^{-\frac{\theta}{\psi}} R_{W,t+1}^{\theta-1} & \text{if } \psi \neq 1, \\ \beta \mathbb{E}_t[(\frac{C_{t+1}}{C_t})^{1-\gamma}]^{-1}(\frac{C_{t+1}}{C_t})^{-\gamma} & \text{if } \psi = 1, \end{cases} \tag{3}$$

for $\theta \equiv (1-\gamma)/(1-1/\psi)$.

In this section, we assume that the aggregate stock market equals aggregate wealth (ex-dividend) and that the ex post real return on the Treasury bill equals the riskfree rate. We

[3] We will consider within-sample variation in Section 3.7.

relax these assumptions in the following sections. In equilibrium, $R_{W,t+1}$ must satisfy:

$$\mathbb{E}_t\left[M_{t+1}R_{W,t+1}\right] = 1. \tag{4}$$

Our assumptions on endowment and preferences imply a constant price-dividend ratio $(W_t - C_t)/C_t$, which we denote by κ. Standard arguments (see Appendix C) then imply that

$$\kappa = \frac{\beta e^{(1-\frac{1}{\psi})\mu}\left[1+p((1-\eta)^{1-\gamma}-1)\right]^{\frac{1}{\theta}}}{1-\beta e^{(1-\frac{1}{\psi})\mu}\left[1+p((1-\eta)^{1-\gamma}-1)\right]^{\frac{1}{\theta}}}. \tag{5}$$

Given the return on the wealth portfolio, the Euler equation provides the return on the one-period riskless bond:

$$\begin{aligned}\log R_f = -\log\beta + \frac{1}{\psi}\mu - \log(1+p((1-\eta)^{-\gamma}-1)) \\ + \left(\frac{\theta-1}{\theta}\right)\log(1+p((1-\eta)^{1-\gamma}-1)).\end{aligned} \tag{6}$$

Equations (5) and (6) constitute a system of two equations in two unknowns, p and β. Combining (5) and (6) gives the equity premium:

$$\begin{aligned}\log\mathbb{E}_t[R_{W,t+1}] - \log R_f &= \log(1-p\eta) + \log(1+p((1-\eta)^{-\gamma}-1)) \\ &\quad - \log(1+p((1-\eta)^{1-\gamma}-1)) \\ &\approx p\eta((1-\eta)^{-\gamma}-1)\end{aligned}$$

where the approximation is accurate for small p.

3.1 Increasing disaster probability

We calibrate this model using measured growth rates of real per capital consumption $\mu = 0.0253$ from 1984 to 2000 and $\mu = 0.0154$ from 2001 to 2021.[4] For comparability with Farhi and Gourio, we first show results for their calibration, corresponding to $\gamma = 12$, $\psi = 2$, and a disaster size $\eta = 0.15$. We find similar results in that we match the data using a discount factor (β) of 0.969 in the early period and 0.983 in the later period, and a disaster probability (p) of 2.10% in the early period and 4.82% in the later period. We thus arrive at our first result: matching the combined stability of valuations and the decrease in riskfree rates requires a large increase in the disaster probability, even after accounting for decreased growth.[5]

It may seem surprising that we require such a large increase in p. After all, the interest rate did fall because of decreased growth and increased patience. Moreover, a decrease in growth moderates the interest rate effect on stock prices, leading to a price-dividend ratio lower than it would have been, which is precisely the problem we are trying to solve. As it happens, the reason we require such a large increase is that, like the growth rate, the increase in disaster probability has two offsetting effects which cancel out when the elasticity of intertemporal substitution (EIS) equals unity. The EIS acts as a free parameter in this explanation. While there is no fundamental reason to believe that the EIS should differ greatly from the inverse of risk aversion (the former governs the desire of the agent to smooth across time, the latter to smooth across states), Farhi and Gourio (2018) follow others in the literature in assuming a high value of risk aversion and a high EIS in order to match the equity premium without other counterfactual implications. For an increase in disaster risk

[4] Throughout this paper, all figures, with the exception of Figure 1, Figure 4, Panel B (which uses data 5 years ahead), and Figure 13 (where one series was discontinued), report data up until the end of 2024. In this section, Section 3.3, and Section 4 the model is calibrated to two sample periods 1984–2000, and 2001–2021. Section 3.7 contains a detailed analysis of the 2022–2024 period.

[5] van Binsbergen (2020) states the puzzle as follows: given the decrease in interest rates and the duration of the stock market, one would have expected a much larger capital gain if the risk premium were to remain constant.

Table 1: Accounting for the data with a change in disaster probability

This table shows parameters necessary to match the data, assuming an endowment economy with rare disasters and no inflation risk. Unless otherwise noted, we take average consumption growth from the data, and calibrate the disaster probability p and the subjective discount factor β to fit average interest rates and the price-dividend ratios in each of two sample periods. Because there is no inflation premium, the Treasury bill yield minus forecasted inflation (ex ante inflation-adjusted Treasury yield) proxies for the riskfree rate, and the wealth-consumption ratio proxies for the price-dividend ratio on the aggregate market. The table shows how p and β change depending on assumptions regarding elasticity of intertemporal substitution (EIS) and on growth. Treasury yields in the data, and parameters in the model, are annual.

		Values	
	Parameter	1984–2000	2001–2021
Panel A: Moments in the data			
Price-dividend ratio	κ	42.35	50.86
Ex ante inflation-adjusted Treasury yield	$\bar{y}_b$	0.0287	-0.0053
Panel B: $\gamma = 12.0$, EIS $= 2.0$, $\eta = 0.15$			
Average consumption growth	μ	0.0253	0.0154
Discount factor	β	0.969	0.983
Probability of disaster	p	0.0210	0.0482
Panel C: $\gamma = 12.0$, EIS $= 0.5$, $\eta = 0.15$			
Average consumption growth	μ	0.0253	0.0154
Discount factor	β	0.993	0.977
Probability of disaster	p	0.0210	0.0482

to be the explanation, not only must the change be large enough to overcome the offsetting effects, but it is essential to assume that the EIS is above unity.

To illustrate this point, Panel C of Table 1 sets the EIS to 0.5 rather than 2, while keeping everything else the same. Lower growth and a rising disaster probability cause valuations to increase, not decrease. Matching (5) and (6) with p and β still requires an increase in p in the second period. However, β must now decline, implying that investors became less patient, not more, contradicting the demand-side intuition for the decline in interest rates (Summers, 2015).

3.2 Did the equity premium rise?

We now ask whether the equity premium did, in fact, increase. The literature studying long-term variation in the equity premium generally comes to the conclusion that the equity premium has declined during the postwar period, including from the first to the second periods that are our focus (Avdis and Wachter, 2017, van Binsbergen and Koijen, 2010, Fama and French, 2002, Lettau et al., 2008). This evidence contradicts an increase in the risk of disasters.

Options markets are another place to look for evidence of an increase in the equity premium (Barro and Liao, 2021).[6] Virtually any explanation for an increase in the ex ante equity premium involves an increase in risk or risk aversion. Although such a risk may not be realized in the sample, option prices incorporate the probability that market participants assign to this risk materializing. Figure 3 shows the VIX, reported by the Chicago Board Options Exchange (CBOE). The VIX is the risk-neutral expectation of quadratic volatility, which is tightly tied to the equity premium. Although the VIX is highly volatile at high frequencies, the average level of the VIX is remarkably stable between the two periods: equal to around 21 before 2001 and 20 after. It is hard to reconcile this stability with a secular increase in the equity premium.

3.3 Sovereign default risk

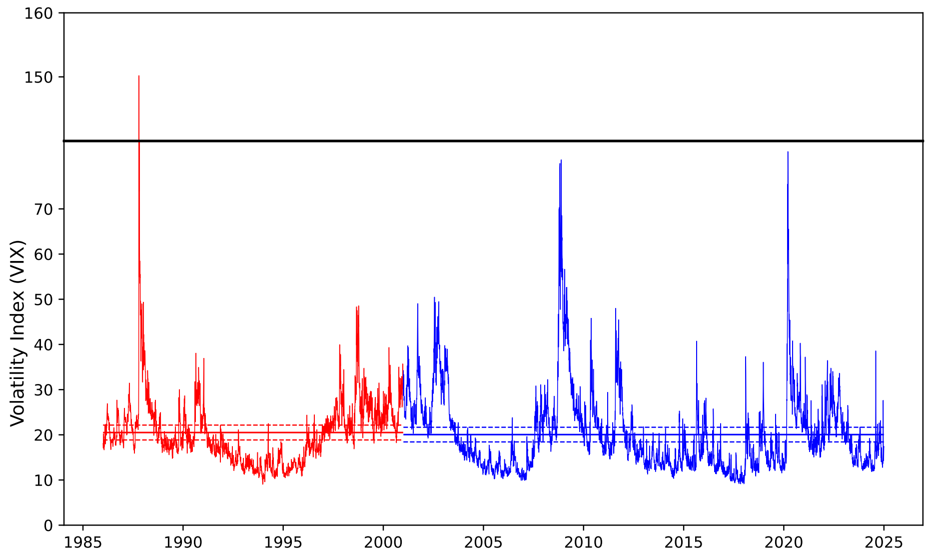
A standard proxy for the equilibrium riskfree rate is the real return on government debt; however, this return is not necessarily riskless, as the government can default either outright or through inflation. We now price the claim to defaultable debt.[7] A decline in default risk can explain the secular trends in riskfree rates and valuation ratios since 1980 without

[6] If interest rates contain a premium for default risk, as we argue in subsequent sections, then the equity premium as measured by the ex post sample average of returns on equities minus bond yields will be biased downwards by this premium. If equity returns rise relative to bond returns, this could seem like an increase in the equity risk premium, but in fact only be a decline in sovereign default risk.

[7] See Appendix C.3 for more detailed derivations.

Figure 3: Chicago Board Options Exchange Volatility Index (VIX)

The figure plots the VIX series from 1986 to 2024 from the Chicago Board Options Exchange (CBOE). The long dashed red line is the average VIX from the beginning of the series to the end of the year 2000. The long dashed blue line shows the average VIX since the beginning of 2001. Estimated averages in both samples are plotted with a two-standard-error confidence interval where standard errors are adjusted for heteroskedasticity and autocorrelation (Newey and West, 1987) with 2 lags on the monthly VIX.



appealing to rising disaster risk.

Suppose that, in a disaster, creditors lose a fraction $\lambda\eta$ relative to the face value of the bond. That is, a bond issued at time t pays $1 - L_{t+1}$ at time $t+1$, where loss $L_{t+1} = \lambda\chi_{t+1}$ represents a loss of zero if there is no disaster, and $\lambda\eta$ if a disaster should occur. If $\lambda = 1$, the loss to creditors is equal, in percentage terms, to the decrease in consumption η. If $\lambda = 0$, then the bond is riskfree. Let Q_t be the price of the defaultable bond. In equilibrium,

$$Q_t = \mathbb{E}_t \left[M_{t+1}(1 - L_{t+1}) \right]. \tag{7}$$

Let $R_{b,t+1} \equiv (1 - L_{t+1})/Q_t$ denote the return on the defaultable bond. The expected return is

$$\log \mathbb{E}[R_{b,t+1}] = \log R_f + \log(1 - p\lambda\eta) + \log\left(1 + p((1-\eta)^{-\gamma} - 1)\right) - \log\left(1 + p((1-\lambda\eta)(1-\eta)^{-\gamma} - 1)\right)$$
$$\approx \log R_f + p\lambda\eta((1-\eta)^{-\gamma} - 1). \quad (8)$$

The term $p\lambda\eta((1-\eta)^{-\gamma} - 1)$ is the default risk premium.

We have thus far been agnostic as to the means of default. Inflation offers one such means, and for simplicity, we now assume that it is the only means (we discuss this assumption further below). Let Π_t denote the price level, so that Π_{t+1}/Π_t is inflation. A capital loss of L_{t+1} through default is equivalent to an inflation of $1/(1 - L_{t+1})$. Our model will also allow for both expected and unexpected inflation outside of disasters. To do this, we introduce q_t to allow for non-disaster expected inflation and $\sigma_\pi \epsilon_{t+1}$ to be non-disaster unexpected inflation, where ϵ_{t+1} is iid normal, and assumed, for now, to be uncorrelated with consumption growth.[8] To summarize, the price level follows the process:

$$\Pi_{t+1} = \Pi_t e^{q_t + \sigma_\pi \epsilon_{t+1}} (1 - L_{t+1})^{-1}. \quad (9)$$

Let $Q_t^{\$}$ denote the nominal price of the nominal bond. In equilibrium,

$$Q_t^{\$} = \mathbb{E}_t\left[M_{t+1} \frac{\Pi_t}{\Pi_{t+1}}\right]. \quad (10)$$

Let $y_{b,t}^{\$}$ denote the continuously-compounded nominal yield on the nominal bond, namely $y_{b,t}^{\$} \equiv -\log Q_t^{\$}$. We will identify $y_{b,t}^{\$}$ with the yield on the one-year Treasury. Substituting

[8] We relax this assumption below.

(9) into (10) implies the nominal yield

$$y^{\$}_{b,t} = \log R_f + \log\left(1 + p((1-\eta)^{-\gamma} - 1)\right) \\ - \log\left(1 + p((1-\lambda\eta)(1-\eta)^{-\gamma} - 1)\right) + q_t - \frac{1}{2}\sigma_\pi^2. \quad (11)$$

It also implies the same expected real bond return (8) as in the case of outright default.

To map bond yields to the data in a way that allows us to identify the default parameter λ, we consider the nominal Treasury yield less expected inflation,

$$\bar{y}_b \equiv y^{\$}_{b,t} - \log \mathbb{E}_t\left[\frac{\Pi_{t+1}}{\Pi_t}\right]. \quad (12)$$

We refer to $\bar{y}_b$ as the *ex ante inflation-adjusted Treasury yield* (because it uses ex ante expected inflation rather than ex post realized inflation). We will use the log of one-year inflation forecasts from the data to measure expected inflation, equal to

$$\log \mathbb{E}_t\left[\frac{\Pi_{t+1}}{\Pi_t}\right] = q_t + \frac{1}{2}\sigma_\pi^2 + \log(1 + p\lambda\eta(1-\lambda\eta)^{-1}). \quad (13)$$

Subtracting (13) from (11), we obtain

$$\bar{y}_b = \log R_f + \log\left(1 + p((1-\eta)^{-\gamma} - 1)\right) \\ - \log\left(1 + p((1-\lambda\eta)(1-\eta)^{-\gamma} - 1)\right) - \sigma_\pi^2 - \log(1 + p\lambda\eta(1-\lambda\eta)^{-1}), \quad (14)$$

The difference between $\bar{y}_b$ and the log riskfree rate includes the inflation risk premium (the same as in (8)) and an adjustment for Jensen's inequality.[9] Absent data on inflation forecasts, one could instead use the *ex post* inflation-adjusted yield, defined as the nominal yield (11)

[9] Specifically, the Jensen's term is $\bar{y}_b - \log \mathbb{E}_t[R_{b,t+1}] = -\log\left(\mathbb{E}_t[\Pi_{t+1}/\Pi_t]^{-1}\mathbb{E}_t[\Pi_t/\Pi_{t+1}]\right)$, which is strictly negative for $\lambda\eta \neq 0$ or $\sigma_\pi \neq 0$.

less average log inflation in the data sample:

$$\hat{y}_b \equiv y^{\$}_{b,t} - \mathbb{E}_t\left[\log\frac{\Pi_{t+1}}{\Pi_t}\middle| L_{t+1}=0\right] = y^{\$}_{b,t} - q_t, \tag{15}$$

where we have imposed the fact that there were no disasters in our data sample. The ex post and ex ante measures differ only in the Jensen's inequality terms.

We now calibrate the model, keeping p constant at 2.10% (the calibrated value from 1984–2000), and allowing λ to vary. The form of Table 2 is the same as Table 1. We first show the price-dividend ratio and the ex ante inflation-adjusted Treasury yield for the two sample periods. For the ex ante inflation-adjusted, we use the yield on one-year Treasuries and one-year SPF inflation forecasts. Note that the model with inflation implies a different and more precise interpretation of the ex ante inflation-adjusted Treasury yield. The model counterpart is (14), whereas in the previous model it was simply the riskfree rate.[10]

Similarly to the previous exercise, we fix all parameters other than consumption growth (which is taken from the data), patience β, and the inflationary default parameter λ. A higher λ corresponds to a greater exposure, and hence a higher inflation premium. In line with the disaster literature, we consider a lower value of risk aversion γ (equal to 5) and a correspondingly larger disaster (a consumption decline of 30%). This calibration forms our benchmark; however, our points are qualitatively similar with higher γ and smaller disasters. We first consider the case of an EIS equal to 2. We first note that the model is capable of matching the data, assuming a λ such that 16.5% of the bond value is lost in disasters in the first sample whereas in the second sample the bond rises in value by 6.0%. Crucially, it does so with a smaller increase in the discount rate β. Rather than 1.4 percentage points, β increases by 0.9 percentage points. This is because the model has a fundamentally different explanation for the decline in the interest rate, namely the reduced

[10]Under the inflation process (9) and imposing the fact that there were no disasters in the data sample ($L_t = 0$), we can recover the variance of non-disaster inflation shocks from the difference between inflation realizations and forecasts: $\text{var}(\log(\Pi_{t+1}/\Pi_t) - \log \mathbb{E}_t[\Pi_{t+1}/\Pi_t]|L_{t+1}=0) = \sigma_\pi^2$.

inflation premium. Indeed, the inflation premium (which we can calculate using (8)), is 1.7 percentage points in the first half of the sample, falling to negative 62 basis points in the second half, accounting for the majority of the decline in the observed interest rate.[11] The remaining decrease in $\bar{y}_b$ comes from a decrease in the real rate, driven by a combination of lower growth and greater patience.

Panel C assumes an EIS equal to 1. Note that, unlike the previous explanation, the model does not depend on an EIS greater than 1. Parameters in Panel C are similar to those in Panel B, with an even smaller increase in β. When the EIS is equal to one, the decline in growth now does not affect the price-dividend ratio (although it still affects the riskfree rate). Thus, β now must increase by a mere 0.4 percentage points, because there is no need to counteract the effect of lower growth on the price-dividend ratio and because growth also has a larger effect on the interest rate. In summary, changes in λ help explain the decline in observed interest rates, making the assumption of a decrease in disaster premia unnecessary. This explanation is more robust, in that it does not require a knife-edge combination of hard-to-observe risk, patience, and willingness to substitute over time.

3.4 Evidence for declining inflation risk

In contrast to the explanation based on disaster risk, independent evidence points to a decrease in the risk of sovereign default. We focus on the case that all sovereign default takes place through inflation. We first examine relative yields on nominal and inflation-protected

[11] Not surprisingly, this is a larger decline than that estimated using models assuming stationarity (see Favero et al. (2024) for a discussion of the role of the stationarity assumption in interest rate modeling). For instance, Haubrich et al. (2012) assume stationarity and Gaussian shocks, implying that the inflation risk premium depends directly on the variance of inflation. They find an average premium of 0.4%. Greenwald et al. (2022, 2024) find similar results. Ang et al. (2008)—more similar to our results—that the 5-year inflation risk premium has declined over time from a peak of approximately 2% in the early 1980s to around 0.15% after the 2001 recession. Similarly, Bekaert and Wang (2010) find recent estimates of the inflation to be close to zero. Our model differs from these both in that we allow for a structural break and because we allow for rare inflation events, implying not only that true volatility is difficult to capture in-sample, but also that the premium does not depend solely on this volatility. That said, our estimation implies that, if averaged across the samples, the premium is 0.5%, almost exactly what is implied by these prior estimates.

Table 2: Accounting for the data with inflationary default risk

This table shows parameters necessary to match the data, assuming an endowment economy with rare disasters and inflationary default. We take average consumption growth from the data in each sample. We calibrate the discount factor β and the decline in bond value $\lambda\eta$ to match the average price-dividend ratio and Treasury bill yield minus forecasted inflation (ex ante inflation-adjusted Treasury yield). We assume the disaster probability equals 2.10%, its benchmark value in Table 1. Parameters and yields are in annual terms.

		Values	
	Parameter	1984–2000	2001–2021
Panel A: Moments in the data			
Price-dividend ratio	κ	42.35	50.86
Ex ante inflation-adjusted Treasury yield	$\bar{y}_b$	0.0287	-0.0053
Panel B: $\gamma = 5.0$, EIS $= 2.0$, $\eta = 0.3$			
Average consumption growth	μ	0.0253	0.0154
Discount factor	β	0.972	0.981
Fraction of bond value lost	$\lambda\eta$	0.165	-0.060
Model-implied riskfree rate	r_f	0.0140	0.0003
Panel C: $\gamma = 5.0$, EIS $= 1.0$, $\eta = 0.3$			
Average consumption growth	μ	0.0253	0.0154
Discount factor	β	0.977	0.981
Fraction of bond value lost	$\lambda\eta$	0.165	-0.060
Model-implied riskfree rate	r_f	0.0140	0.0003

bonds. Given that Treasury Inflation-Protected Securities (TIPS) were first introduced in 1997 and did not trade with sufficient liquidity for prices to be reliably measured until around 2004 (Fleming and Krishnan, 2004), we cannot use this information to distinguish inflation premia before and after the structural break separating our two samples.[12] Index-linked Gilts from the U.K., on the other hand, have traded since the early 1980s and provide the ideal asset to examine this difference over the last four decades. We examine the ex-post

[12]We discuss the TIPS series further below and in Section 3.7

inflation-adjusted yield on nominal bonds relative to the yield on index-linked bonds, namely (15) in the model. Our explanation predicts that the difference is positive in the first sample period, declining to zero in the second.

Panel A of Figure 4 shows the difference between the ex post inflation-adjusted yield on the one-year U.K. government nominal bond and the yield on five-year index-linked Gilts. The figure shows that in the first sample, this difference was significantly positive and large—over 1.5 percentage points. In the second sample, this excess return is on average zero, consistent with our estimate of no default risk. This suggests an economically large decline in the inflation risk premium.

A possible concern with this measure is that it compares bonds of different maturities. It may, for example, be that differences in these bond returns arise from changes in the term structure. Panel B addresses this concern by considering five-year nominal bond yields minus (annualized) realized inflation over five years. The result is essentially identical: 5-year nominal bonds earned a premium of more than two percentage points in the 1980s and 1990s and no premium in the 2000s and 2010s. The parameter values in Table 2 imply a strikingly similar decline of 1.8 percentage points, validating our model calibration.

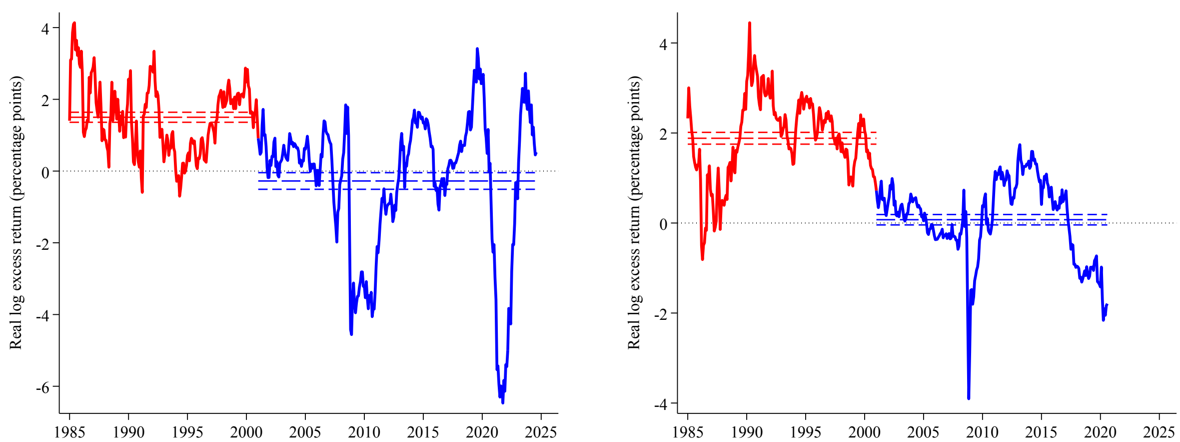
As described above, there are also reasons that yield-based break-even estimates of the sovereign default premium may be too low. For example, the above calculation assumed that all defaults occur through inflation. Non-zero prices on credit-default swaps suggest otherwise (Chernov et al., 2020).[13] Even if one were to relax this assumption (and allow an additional term for outright default), there remain two reasons why the estimate may be too low: (1) inflation indexation is inexact (or, more precisely, errors may be correlated with factors investors care about, such as inflation itself), and (2) recovery rates may be lower on inflation-indexed bonds. Indeed, TIPS explicitly do not index for deflation. Anderson and Sleath (1999) discuss errors in inflation indexation for Gilts. The government of Canada has

[13]Some of the premium for default may reflect the probability of a temporary halt in payment (technical default), whereas the premium (8) assumes missed payments are not made at a later date (Bomfim, 2022).

Figure 4: Excess real returns on nominal bonds over inflation-indexed bonds

The figure shows the ex post inflation-adjusted yields on nominal U.K. government bonds in excess of the five-year gilt yield. In Panel A, the solid line is the one-year inflation-adjusted nominal bond yield (the yield less realized inflation over the next year) minus the contemporaneous five-year gilt yield. Panel B is the same difference for the five-year nominal bond yield less the next five years of inflation (annualized). For this reason, Panel B only extends into 2020. Dashed lines represent sample means with 95% confidence intervals.

A. One-year nominal bond minus gilt **B. Five-year nominal bond minus gilt**

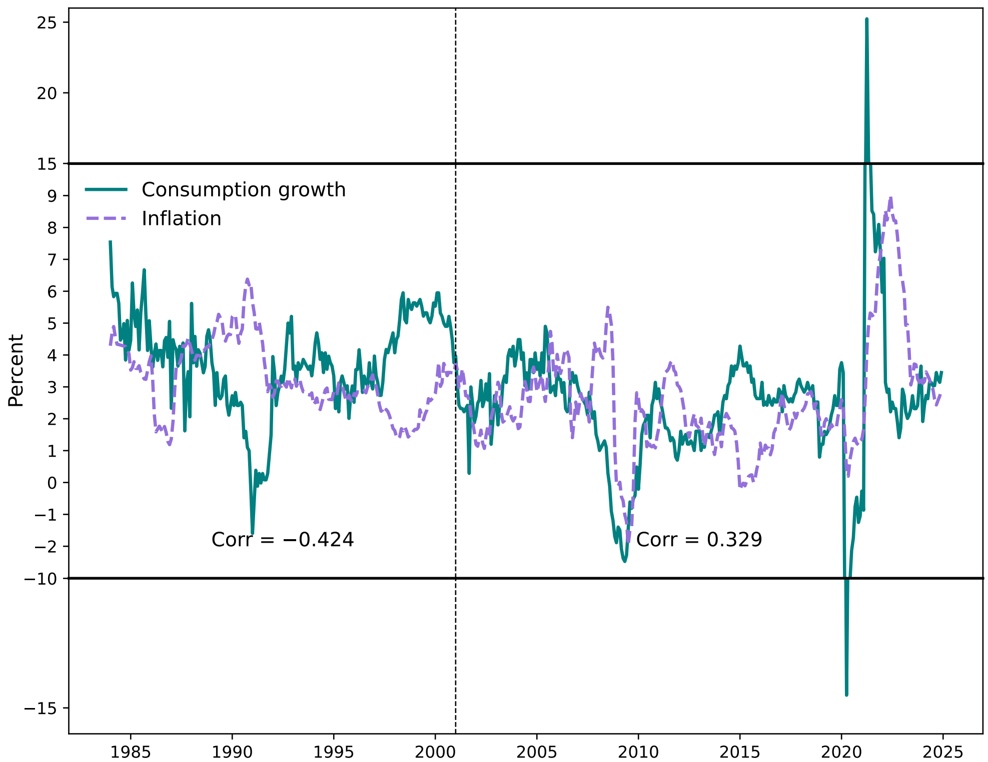


discussed stopping the issuance of inflation-linked securities due to concerns about the size of potential payouts (Czitron, 2022). Dittmar et al. (2024) argue that the prices of TIPS reflect a greater risk of outright default than the prices of nominal Treasury bonds. This is not surprising as investors have no experience with whether TIPS would pay off should there be a shock to inflation.

Another place to look for evidence of a declining inflation risk premium is in sample correlations between inflation and consumption growth. Ideally, to capture changes in λ, one would use these disaster observations to construct a correlation that is disaster specific. That is the literal interpretation of the model in the previous section. However, both expected and realized behavior during disasters are hard to observe; disaster expectations are inherently unobservable, and disasters are too few for accurate measurement regarding price behavior. Absent these ideal data moments, we look directly at realized inflation and consumption

Figure 5: Consumption growth and inflation

This figure shows one-year realized consumption growth and the one-year realized inflation rate from 1984 to 2024. The vertical line separates the sample periods at the beginning of 2001.



growth with the view that the line between a downturn and disaster is ultimately arbitrary. Figure 5 shows the result. The recession in the early 1990s was clearly accompanied by inflation, whereas the boom of the late 1990s was accompanied by deflation. However, the financial crisis was deflationary. The COVID recession saw deflation during the 2020 contraction, followed by high inflation.

Survey data also provide evidence for the declining risk of sovereign default. Using data from the University of Michigan and the Survey of Professional Forecasters, Figure 6 shows a decline in inflation forecasts over four decades, leveling off in more recent years. Reis (2020) finds an anchoring of inflation expectations using survey data. Figure 6 shows that break-even inflation measured by TIPS prices relative to comparable nominal bonds is consistent with low expected inflation in the second half. Tables 1 and 2 adjust for the expected inflation component of yields, and show that there is still a marked decline in ex ante inflation-adjusted rates. Though the evidence in Figure 6 does not directly address the puzzle in 1, it is relevant because it is reasonable to assume that higher inflation expectations would be accompanied by higher risk, and indeed our hypothesized process for inflation (9) explicitly links the two.[14]

The link between expected inflation and the likelihood of disasters provides a further test of our model. Indeed, our assumed process for inflation makes predictions regarding expectational errors, assuming disasters do not occur in-sample: forecasters should over-predict inflation during the sample in which inflationary disasters occur and underpredict it (or accurately predict it) otherwise.[15] Data on one-year-ahead forecasts from the Survey of Professional Forecasters in fact show that in the first sample, forecasters consistently over-estimate inflation, whereas in the second sample, their estimates are on average correct (Figure 7). Researchers have interpreted these expectation errors as evidence of slow learning due to highly persistent underlying processes (Farmer et al., 2024) or the strong pull of past experience (Goetzmann et al., 2022), both of which are also in the spirit of our model. Regardless of the interpretation, in the first sample, investors predicted inflation that did not occur, whereas in the second, they ceased to forecast inflation. This is consistent with a structural break such that, in the first sample, inflation exhibits positive skewness ($\lambda > 0$), which vanishes in the second ($\lambda \approx 0$). A comparison of the estimates in Table 2 together with the estimates of Figure 7 indicates that about half of expectational "errors" are due to the absence of disasters in the sample.

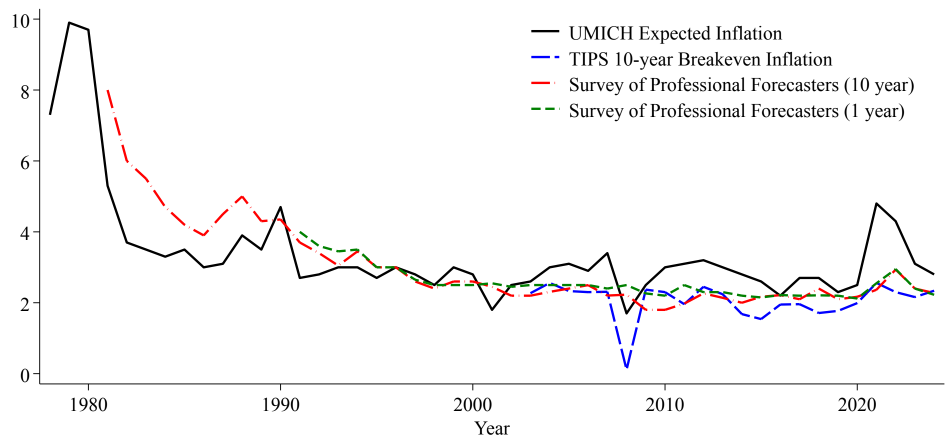
Finally, any amount of variability in the disaster probability within each period will induce

[14]This is also why direct observations of yields over the long term in Figure 1 is relevant.

[15]In a sample in which disasters do not occur, the difference between forecasted and the measured ex post average inflation will be $\log \mathbb{E}_t[\Pi_{t+1}/\Pi_t] - \log \mathbb{E}_t[\Pi_{t+1}/\Pi_t | L_{t+1} = 0] = -\log(1 - p\lambda\eta(1-\lambda\eta)^{-1}) \approx p\lambda\eta(1-\lambda\eta)^{-1}$, which, according to our theory, is positive in the first sample but close to zero in the second.

Figure 6: Expected inflation in the United States

The solid black line shows expected inflation from the Surveys of Consumers of University of Michigan. The dashed blue line shows the 10-year breakeven inflation rate computed from Treasury Inflation-Indexed Constant Maturity Securities. The dashed-dotted red line shows 10-year expected inflation from the Survey of Professional Forecasters.



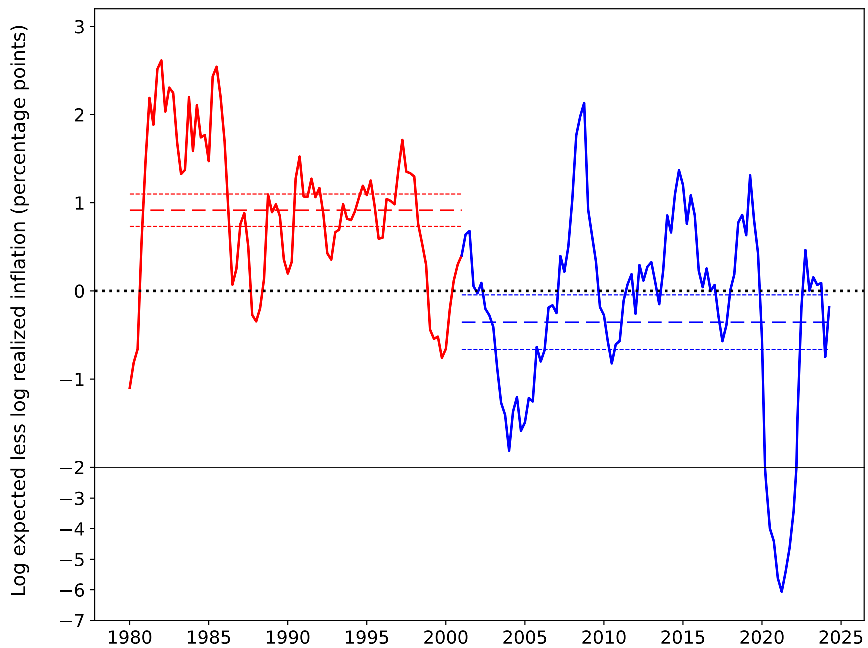
a positive correlation between bond and stock returns in the first period and a negative correlation in the second. In the model of Wachter (2013), an increase in the risk of rare disasters causes stock prices to fall. If bond payoffs are also negatively exposed to disaster risk ($\lambda > 0$), then bond prices fall with when disaster risk increases, leading to a positive correlation. If, on the other hand, bonds experience deflation during a disaster ($\lambda < 0$), then the correlation flips sign. Indeed, Campbell et al. (2017) and Campbell et al. (2020) observe a substantial shift in the bond-stock beta, going from positive to negative between the first and second sample.[16] Thus the changing correlation in bond and stock returns also provides evidence in support of declining risk of sovereign default.

To summarize, our model and calibration imply that the *true* riskfree rate and equity risk premium have remained relatively stable over time, a conclusion that is consistent with evidence from valuation ratios, which have remained relatively flat; and from the VIX.

[16] Relatedly, Cieslak and Vissing-Jorgensen (2021) show that the so-called "Fed Put"—the tendency of the Fed to reduce rates (increasing bond prices) when the stock market falls—began only in the late 1990s.

Figure 7: Expected versus realized one-year inflation

The figure plots the difference between expected and realized one-year inflation, where expectations are taken from the Survey of Professional Forecasters. The horizontal dashed lines show the average difference in each of our respective samples along with 95% confidence intervals.



This is *not* to say that riskfree rates have not declined at all. In the calibrations that we present above, the riskfree rate decreases from the first sample to the second. This is mirrored in the data: the yield on index-linked Gilts has also fallen. Our point is that this decline is substantially smaller than what is reflected in declines in inflation-adjusted nominal yields, and accounting for this explains the joint evolution of valuation ratios and bond yields.

3.5 The government's ability to repay

The previous discussion describes the puzzle posed by declining interest rates, unaccompanied by a rise in valuation. It also proposes a solution: that the decline in interest rates arises primarily from a decreased risk of sovereign default, rather than a decline in the riskfree rate.

In recent work, Jiang et al. (2024b) highlight a tension between the growing debt burden in the developed world, the tendency to add to this burden by running deficits in good times and bad times, and the low interest rates that developed nations still enjoy (and which are the focus of this paper). Rather than pricing government bonds, Jiang et al., following Cochrane (2023), price the totality of government debt as a claim to future surpluses (more precisely, their framework expresses the real value of government debt, divided by GDP, as an infinite sum of real future surpluses as a fraction of GDP). Our approach does not require a government budget constraint, and so the path of future surpluses remains in the background. However, the two frameworks are tightly connected.

Appendix F shows how the inflation process (9) arises from as a process for real government surpluses as a fraction of GDP. Then it relates the default parameter λ to this surplus process. The greater the risk of default, the less the sovereign pays back after a disaster and inflation. Appendix F establishes several results. The first is that a one-time decrease in λ leads to an increase in the debt-to-GDP ratio. Thus, a decrease in the risk of sovereign default will imply that government borrowing as a fraction of the real economy will rise, which is in fact what occurred. The second point is that, if recent past behavior is a guide to future behavior when it comes to government surpluses, there is an increase, not a decrease, in the risk of sovereign default, and there have been economically significant pricing errors in government bonds. That is, we in effect replicate the main point of Jiang et al. (2024b). However, our approach is also consistent with a growing debt-to-GDP ratio, as long as investors are willing to accept future promises of surpluses instead of current surpluses (the recent past may not be a good guide to future behavior). This is in fact the solution claimed for the puzzle by Cochrane (2023). Although outside the scope of this paper, our view is that it is not unrealistic for an investor to assume that the US government would pay back its debts. Outright default in the developed world is a rare event. Since the 1980s, the Federal Reserve has shown a commitment to keeping inflation low. Moreover, in the 1990s, the U.S. proved that it is possible to run surpluses.

Our point in this paper is not to comment on the wisdom of running large deficits or on any element of fiscal policy. There is no bubble in our model, as in Kocherlakota (2023), and the analysis in Section 3.6 suggests that convenience yields did not increase substantially over the 40-year period from 1980 to 2020, implying that convenience alone could not be the reason why interest rates fell despite the increase in the debt burden. Thus, our model simply implies that *investors believed*, during the 2000–2021 period, that the current fiscal behavior of developed countries in running large deficits will end. This belief was highly beneficial to U.S. taxpayers, at least in the short run, as it implied that interest payments were low. As Section 3.7 shows, our leading interpretation of events after 2021 marked the end of that belief.

3.6 The role of convenience yields

The above sections argue that a decline in sovereign default risk is necessary to explain the joint behavior of interest rates and equity valuation ratios. However, an alternative to sovereign default risk is a decline in the convenience yield. The convenience yield hypothesis states that certain assets can act as a substitute for cash and therefore have a utility that goes beyond their intrinsic value. A substantial literature examines the convenience yield (e.g., van Binsbergen et al., 2022, Fleckenstein and Longstaff, 2024, Krishnamurthy and Vissing-Jorgensen, 2012). Like a decline in sovereign default risk, a decline in the convenience yield affects the Treasury bill rate (which presumably has convenience value), while leaving equities unaffected. Specifically, suppose that the observed Treasury yield is equal to a sum of the traditional financial return and a "convenience" yield, representing the role of Treasury securities as cash. The latter is negative. An increase in the magnitude of the convenience yield could thus explain a decline in the observed Treasury yield.

The literature on the convenience yield uses several proxies. Ideally, one would find a proxy in which convenience could be disentangled from risk. For our purposes, which involve finding a plausible upper bound on the effect of convenience, it suffices to examine the spread between highly rated corporate bonds and Treasury bonds (Cieslak et al., 2024). This spread widened by an average of 37 basis points between the first and second sample, indicating that an increase in the convenience yield plays a minor role in explaining the 3–4 percentage-point decline in interest rates. Replacing the Treasury bill rate series with a highly rated corporate debt series thus does not change our results very much: rather than a decline in fraction of bond value lost of 22.5 percentage points, we see a decline of 20.0 percentage points. Thus, while acknowledging that a rise in convenience may be part of the reason for the decline in Treasury rates, it is secondary to other factors driving government bond yield declines.

3.7 Valuation ratio and interest rate dynamics within each sample

Earlier in this section, we argued that reduced sovereign default risk is a primary driver of the decline in interest rates over the past 40 years and perhaps the past 400 years. There are many possible explanations for the decline in interest rates if taken in isolation. Jointly considering the price-dividend ratio, the interest rate, and the growth rate considerably narrows down the possible explanations. For example, if an increased propensity to save had driven down interest rates, the price-dividend ratio would have risen to a much greater degree than what we observe. We have argued that the answer lies in interpreting the decline in interest rates as a decline in the risk of sovereign default, and not entirely as a decline in the real rate.

Although the decline in the risk of sovereign default explains the average levels across samples, we do not claim that it explains the variations within samples. For a full answer as to what drives rates and valuation ratios, we generalize the model to allow for Gaussian shocks to consumption, to expected inflation, and to unexpected inflation. We examine how much of the decline in interest rates can be accounted for by survey measures of expected inflation alone, and we incorporate the shift in the consumption-inflation correlation. Finally, we allow for time variation in dividend growth expectations that occur apart from consumption growth expectations. By definition, an increase in expected inflation represents an increase in the degree of default since all inflation is a form of default on nominal bonds. Moreover, the change in sign in the consumption-inflation correlation represents a decrease in the risk of sovereign default. However, incorporating these changes into the model will allow us to isolate the effect of rare disasters.

Define $z_t \equiv \log(C_t/D_t)$, the consumption-dividend ratio, and let ε_{t+1} be a four-dimensional vector of independent standard normal shocks. Assume

$$
\begin{aligned}
C_{t+1} &= C_t e^{\mu + \sigma_C^\top \varepsilon_{t+1}} (1 - \chi_{t+1}), && (16)\\
z_{t+1} &= (1 - \rho_z)\bar{z} + \rho_z z_t + \sigma_z^\top \varepsilon_{t+1} - \xi \log(1 - \chi_{t+1}), && (17)\\
\Pi_{t+1} &= \Pi_t e^{q_t + \sigma_\pi^\top \varepsilon_{t+1}} (1 - \lambda \chi_{t+1})^{-1}, && (18)\\
q_{t+1} &= (1 - \rho_q)\bar{q} + \rho_q q_t + \sigma_q^\top \varepsilon_{t+1}. && (19)
\end{aligned}
$$

Following Longstaff and Piazzesi (2004) and Wachter (2005), we assume that the consumption-dividend ratio is stationary and use σ_z and ξ to represent financial leverage that can cause dividends to fall by more than consumption after negative shocks. Appendix D derives closed-form expressions for interest rates and valuation ratios in this economy.[17]

We use survey expectations measures to proxy for expected dividend growth (determined by z_t) and expected inflation (determined by q_t). Following De la O and Myers (2021) and Bordalo et al. (2024), we base dividend-growth expectations for the aggregate market on I/B/E/S earnings forecasts for individual firms. We use earnings rather than dividend forecasts because I/B/E/S only has dividend expectations for enough firms to reasonably construct a series starting in 2003. Moreover, earnings expectations, and in particular long-term earnings expectations, may better capture dividend expectations in the model in that

[17] For simplicity, these equations do not reflect the unforeseen structural breaks, for example, in μ and λ.

decisions to pay a dividend or not over a given time horizon may be driven by dynamics that the simple model (17) does not capture. De la O and Myers and Bordalo et al. develop methodologies for aggregating these individual-firm forecasts into a forecast for the aggregate market. Whereas De la O and Myers use one-year ahead growth expectations, Bordalo et al. focus on the long-term growth variable (LTG). We follow the Bordalo et al. approach, extending their data, which ends in 2020. The mean of LTG is consistently above realized earnings growth.[18] Explaining the disconnect between stated expectations and reality on the part of I/B/E/S forecasters is beyond the scope of this paper, nor would accounting for this effect change our results as it appears equally in both samples. Due to this disconnect, we de-mean LTG before backing out implied values for z_t. Appendix A contains details on data construction; Appendix D contains further details on our calibration. As in Figure 7, we use the one-year-ahead inflation expectations from the Survey of Professional Forecasters. The log of this measure is equal to (13).

We match the cyclically-adjusted price-earnings ratio (CAPE) from Shiller (2000), converting the price-dividend ratio in the model to a price-earnings ratio using the payout ratio.[19] As in our previous analysis, we allow for the change in average consumption growth between the two samples. We also allow the consumption-inflation correlation to change, as well as the discount factor β and the default parameter λ.[20] Other parameters remain constant between the samples. Table 3 gives the parameters that change between the samples, with further details in Appendix D. As Table 3 shows, this more complex model has nearly the same implications for the change in default risk as the simpler model presented in Table 2. The fraction of the value of the bond lost in a disaster goes from 13.2% to −3.2%.

Figure 8 shows the time series of interest rates (Panel A) and valuation ratios (Panel B)

[18] LTG fluctuates around 13%, far higher than average consumption growth, and indeed higher than average realized log real earnings growth—around 4% in annual data from Shiller (2000).

[19] This number is 53% in the first sample and 52% in the second. Given that these numbers are so close, setting them as a single value does not noticeably change our results.

[20] Note that the correlations in the table are between shocks to consumption and inflation (i.e., after filtering out expectations), whereas the correlations in Figure 5 were between the raw time series.

in the model and the data. The model succeeds in capturing the interest rate decline around 2000. It also matches some of the fluctuations in rates both before and after the structural break, but these fluctuations are more muted in the model compared to the data. As we explain below, the model has the potential to match the more recent increase in rates. Turning to the valuation ratios in Panel B, the model again can match the levels both before and after the structural breaks, and has some success in matching fluctuations, although there are some fluctuations in I/B/E/S that do not appear in prices.

We also perform the following counterfactual exercise: suppose that there was no change in the risk of default, leaving the decrease in growth and the increase in β to account for the decline in interest rates. What would have been the consequence for equity valuations? Figure 8 shows that they would be impossible to ignore, rising to many multiples of the true price-earnings ratio. This is because β needs to rise to 0.997 to match the decline in interest rates observed in the data.[21] It is hard to imagine what force could have led to a mispricing of this magnitude. On the other hand, if the inflation premium is what changed, then one finds a much smaller real rate decline and a smaller increase in the price-dividend ratio.

Our focus has so far been on the period spanning the 1980s to December 2021. Interest rates increased substantially in 2022 and remained elevated, relative to recent years, at the time of this writing. Equations (6) and (11) show that three mechanisms that could play a significant role in an increase in interest rates are (1) a decline in β, (2) an increase in expected economic growth μ (or a decrease in the disaster probability p), (3) an increase in expected inflation q_t, or (4) an increase in the risk of default during disasters, λ.[22] A decrease in patience β implies a reversal of the long-term economic forces described above, and thus seems unlikely.[23] Moreover, a decrease in β would cause the stock market to crash,

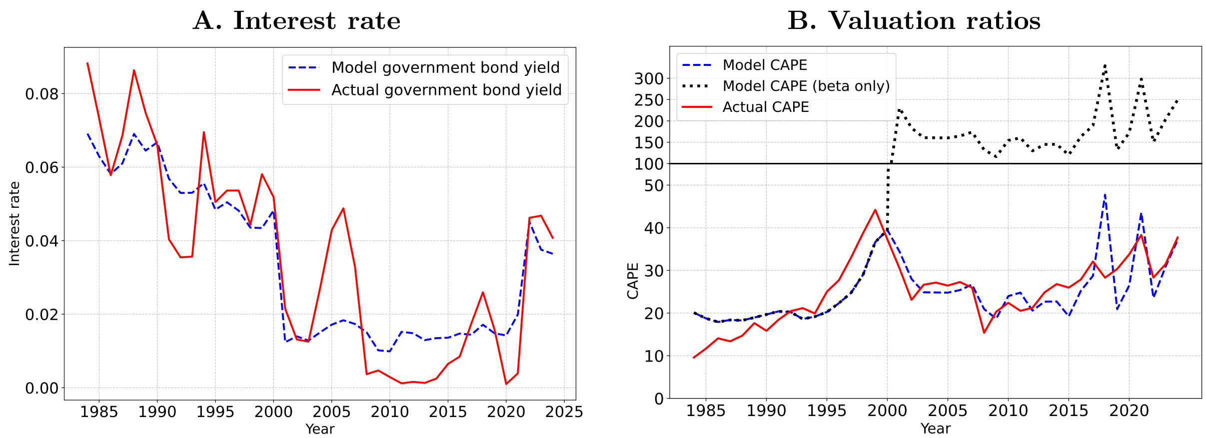
[21] A β of 0.997 is consistent with an interest rate of -0.003, whereas we estimate the Treasury yield less expected inflation over 2001–2021 to be -0.005. Given our other parameters, that value is inconsistent with a finite price-dividend ratio.

[22] A reversal of the correlation of consumption and inflation outside of disasters does not play a sufficiently large role economically, given realistically low risk aversion, to be a consideration.

[23] A caveat is that, to the extent β reflects the mix of investors accessing U.S. markets, it is possible

Figure 8: Model and data implied interest rate and equity valuation ratios

The figure shows the time series of interest rates and valuation ratios in the generalized endowment model. Panel A shows the one-year Treasury yield in the data and the corresponding short-term nominal bond yield in the model. Panel B shows the ratio of price divided by current earnings from the S&P 500 in the data, and the price-dividend ratio in the model.



which did not occur. This is the mirror image of the previous analysis that shows a boom in the stock market should the entire decrease in interest rates arise from this mechanism. Likewise, there is no evidence for an increase in expected economic growth in 2022, with the Survey of Professional Forecasters anticipating one-year-ahead GDP growth at 1.5% in 2022, nor is there evidence for a decline in rare-event risk. The evidence for an increase in expected inflation is mixed, with very little increase in the Survey of Professional Forecasters (0.6 percentage points), though a more sizable increase in the University of Michigan survey. However, even this falls far short of the increase in interest rates (3.9 percentage points) between 2021 and 2022. The remaining possibility is an increase in the risk of sovereign default.

Figure 8 and Table 3 show the result of recalibrating the model to the 2022–2024 period. In accordance with the previous discussion, we keep the discount rate β and the growth

that this mix could fluctuate if, say, sovereign wealth of foreign nations such as China, for political reasons, partially withdraws from U.S. markets.

Table 3: Generalized endowment model with inflationary default risk

This table shows results for the generalized endowment model which includes rare disasters and inflationary default, and allows for time-variation in non-disaster inflation expectations and in expected dividend growth. We estimate processes (16–19) using data from 1984–2024, allowing expected consumption growth μ and the correlation between shocks to consumption growth and inflation (here denoted $\text{Corr}(C, \pi)$) to change between periods. We calibrate the discount factor β and the decline in bond value $\lambda\eta$ to match the average level of the price-dividend ratio and the ex ante inflation-adjusted Treasury yield in each sample. In both samples, $\gamma = 5$, $\eta = 0.30$, the disaster probability equals 2.10%, and the EIS equals 1. Appendix D contains additional details. Parameters are in annual terms.

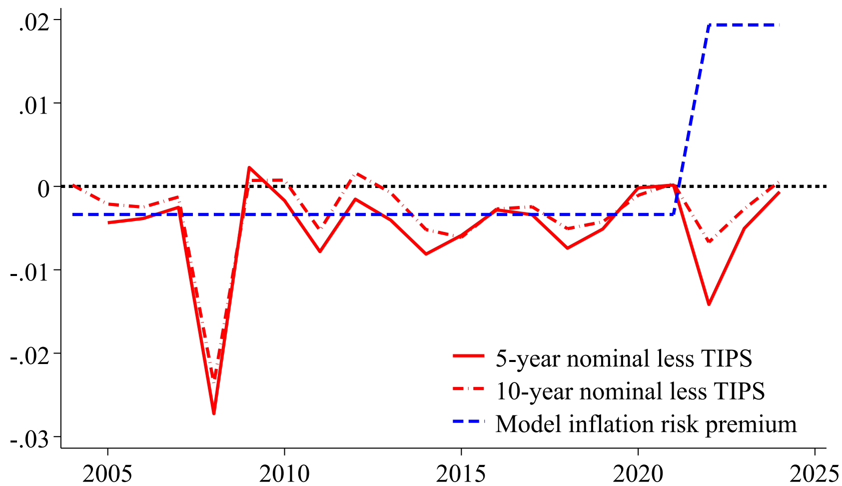
		Values		
	Parameter	1984–2000	2001–2021	2022–2024
Panel A: Moments in the data				
Price-dividend ratio	κ	42.35	50.86	59.65
Ex ante inflation-adjusted Treasury yield	$\bar{y}_b$	0.0287	-0.0053	0.0197
Panel B: $\gamma = 5$, EIS $= 1$, $\eta = 0.3$				
Average consumption growth	μ	0.0253	0.0154	0.0154
Consumption-realized inflation correlation	$\text{Corr}(C, \pi)$	-0.205	0.736	0.736
Discount factor	β	0.976	0.981	0.981
Fraction of bond value lost	$\lambda\eta$	0.132	-0.032	0.186

rate μ at their levels from the second sample 2000–2021. We continue to use data from the Survey of Professional Forecasters to proxy for q_t and the LTG measure to proxy for z_t. We find that, to match the level of the interest rate in 2022, we require an increase in λ such that nearly 18.6% of the value of a bond lost in a disaster, significantly more than the 13.2% in the first sample period.[24] One question is why we need such a sizable increase, given that interest rates still remain lower than in the first sample (though high when compared with recent years). The reason is that we are operating with a higher value of β, which increases asset prices overall, and a lower growth, which decreases interest rates. Altering these assumptions would at best lead us to a slightly reduced λ that would still represent

[24]The model matches the slight decline in interest rates from 2022-2024 nearly perfectly based on the decrease in inflation expectations.

Figure 9: Premium for sovereign default in model and data, assuming TIPS are default-free

This figure plots the 5 and 10 year expected inflation-adjusted nominal yield less the TIPS yield of the same maturity. Expected inflation is from the Survey of Professional Forecasters who provide both 5 year and 10 year ahead inflation expectations.



a substantial increase relative to the 2001–2021 value. Further evidence in support of this mechanism comes from the reversal in sign between the correlation of bond and stock returns (Antolin-diaz, 2025).[25] Thus the 2022–2024 episode leads to the conclusion that reduction in sovereign wealth risk achieved between the first and second sample period has now been reversed.

However, evidence from inflation-protected securities suggests an important addendum to this conclusion. Figure 9 shows the implied inflation premium from TIPS, together with the premium from sovereign default, from the start of the TIPS sample period. This figure shows that, other than in 2008 when nominal bonds experienced a high premium for default, this premium has fluctuated little and, more importantly, remained negative throughout

[25] As Section 3.4 explains, any amount of variability in the disaster probability within each sample would cause stock and bond returns to be positively correlated if $\lambda > 0$ and negatively correlated with $\lambda < 0$. The change in signs of the bond-stock correlation matches the change in our λ parameter. Data suggest that the sign of this correlation, which switched from positive to negative between the first and second period, has subsequently become positive in the 2022–2024 period.

the entire sample period.[26] The data are thus consistent with our model prior to 2022. After 2022, the model and data diverge: the model suggests a sharp increase in the inflation risk premium, and while the premium did rise, the level remained at or near zero. The data appear at odds with our assumptions that (a) all default takes place through inflation, (b) TIPS are a perfect hedge, and (c) the risk of default is higher than it was. As we discussed in Section 3.4, (a) and (b) are simplifying assumptions, and not essential for us. Eliminating these assumptions only affects the interpretation of our model for inflation; the main implications, in Table 3 and Figure 8 for example, remain unchanged.

4 Production economy

A central part of the puzzle regarding valuation ratios and interest rates lies in the fact that economic growth and capital investment remained low despite historically high asset prices. In this section, we show how our conclusions under an endowment economy translate to a production economy, explaining investment-capital ratios. We introduce a model with a riskless inventory asset that can account for the zero lower bound on interest rates.

4.1 No-inventory case

We consider a standard production model in which capital quality can decline suddenly and unpredictably.[27] Let K_t denote the quantity of productive capital at time t. Given K_t and constant productivity A, output equals

$$Y_t = AK_t. \tag{20}$$

[26]The 2008 episode can be thought of as a temporary increase in the risk of a disaster, which would have led to declining equity valuations, plummeting riskfree rates, and a higher yield on inflation-protected securities given our estimate of λ.

[27]See Barro (2009), Gabaix (2011), and Gourio (2012).

Let δ denote depreciation and X_t investment. Capital evolves according to:

$$\tilde{K}_{t+1} \equiv X_t + (1-\delta)K_t \tag{21}$$

$$K_{t+1} \equiv \tilde{K}_{t+1}(1-\chi_{t+1}), \tag{22}$$

where χ_{t+1}, defined in (2), represents destruction of capital. We assume $A > 1-\delta$, consistent with a growing economy. Following Gomes et al. (2019), we refer to $\tilde{K}$ as planned capital, the quantity of capital available if the disaster does not occur.

We can restate the agent's problem as a consumption-portfolio choice decision in which the agent allocates savings to capital and the riskfree bond. Let B_t denote the time-t dollar allocation to the riskfree asset. Define the agent's wealth at time t as

$$W_t \equiv C_t + B_t + \tilde{K}_{t+1}. \tag{23}$$

If investment in capital grows at the stochastic rate $R_{K,t+1}$, wealth at time $t+1$ must equal

$$W_{t+1} = B_t R_{f,t+1} + \tilde{K}_{t+1} R_{K,t+1}. \tag{24}$$

What is $R_{K,t+1}$? Equations (20–22) indicate that, should a disaster not occur, a single unit of capital creates A units of output. A fraction δ is lost prior to the next period. Should a disaster occur, then a fraction χ_{t+1} is lost. Given the remaining capital, A units of output are created and an additional fraction δ is lost. Therefore, the return on capital is

$$R_{K,t+1} = (1-\delta+A)(1-\chi_{t+1}). \tag{25}$$

We can rewrite the budget constraint in terms of flow variables. Equating (23) with (24) at time t and substituting in for $R_{K,t}$ implies

$$C_t + B_t + \tilde{K}_{t+1} = B_{t-1}R_{f,t} + \tilde{K}_t(1 - \delta + A)(1 - \chi_t).$$

Using (21) and (22), then subtracting $(1-\delta)K_t$ from both sides implies

$$C_t + B_t + X_t = Y_t + B_{t-1}R_{f,t}. \tag{26}$$

That is, output from the capital stock plus wealth in bonds can be used toward consumption, bond purchases at time t, or investment in the productive asset.

We can also rewrite the budget constraint in terms of the evolution of wealth. Define the share of savings invested in capital as

$$\alpha_t \equiv \frac{\tilde{K}_{t+1}}{W_t - C_t}.$$

Substituting in for B_t in (24) from (23) implies that

$$W_{t+1} = (W_t - C_t)(R_{f,t+1} + \alpha_t(R_{K,t+1} - R_{f,t+1})) \tag{27}$$

is an equivalent expression for the budget constraint.

Under Epstein and Zin (1989) and Weil (1990) preferences with unit EIS. the agent chooses consumption C_t and the capital portfolio share α_t to solve

$$\max_{C_t,\alpha_t} \left(C_t^{1-\beta} \left(\mathbb{E}_t \left[V(W_{t+1})^{1-\gamma} \right] \right)^{\frac{\beta}{1-\gamma}} \right), \tag{28}$$

subject to (27). Conjecturing that $V(W_t)$ equals a constant multiplied by W_t, and applying the first-order condition for optimal consumption implies the standard unit EIS result $C_t/W_t = 1 - \beta$.

In equilibrium, the bond is in zero net supply ($\alpha_t = 1$), and (26) reduces to

$$C_t + X_t = Y_t = AK_t. \tag{29}$$

Furthermore, the conditions $\alpha = 1$ and $C_t = (1-\beta)W_t$ imply that consumption is a fixed percentage of planned capital:

$$C_t = \frac{1-\beta}{\beta}\tilde{K}_{t+1} = \frac{1-\beta}{\beta}(X_t + (1-\delta)K_t), \tag{30}$$

where the second equality follows from the capital accumulation equation (21).

In Appendix G.1, we derive asset returns in this economy and show that there is an isomorphism between this economy and the endowment economy in Section 3. Indeed, equilibrium prices in the two models are identical if the parameters are such that the equilibrium consumption growth processes are the same.[28] What is new in this production model is that investment and economic growth are endogenous. Substituting in for C_t in (29) gives us the equilibrium investment-capital ratio:

$$\frac{X_t}{K_t} = \beta(1-\delta+A) - (1-\delta). \tag{31}$$

Our previous discussion implies that the riskfree rate has declined slightly, thus implying a slight increase in β. In order to match the declining investment-capital ratio (Figure 2, Panel C), one needs either productivity A to fall or depreciation δ to rise.

4.2 Inventory case

Suppose that, in addition to capital and a riskfree bond, the agent can put funds into inventory, namely a riskfree storage technology with a zero net return. If we impose the

[28] This occurs when $\beta^{-1}e^{\mu} = (1-\delta+A)$. One can verify this by comparing (6) and (G.6).

condition that riskfree storage be in zero supply, then the economy reduces to that in the previous section. The innovation in this section is that the inventory asset can be in net positive supply.

Why would there be a positive-supply riskfree asset? As mentioned in the Introduction, any store of value from one period to another could count as inventory, provided that it is, in fact, riskfree and can be frictionlessly interchanged between consumption and investment. Physical goods do not exactly fit this description because they cannot easily be changed into something other than what they are. Money fits this description if inflation is equal to zero. For the period 2000–2021, as discussed above, there is evidence that investors viewed money as indeed riskless in this sense. That said, our model is agnostic about whether the inventory is money or not; we simply assume that an inventory asset exists.

Consider the agent's problem in Section 4.1, except here the agent can invest in a storage technology with quantity I_t. The agent maximizes unit-EIS recursive utility by choosing consumption and B_t, I_t, and $\tilde{K}_{t+1}$. That is, the agent solves

$$\max_{C_t, B_t, I_t, \tilde{K}_{t+1}} \left(C_t^{1-\beta} \left(\mathbb{E}_t \left[V(W_{t+1})^{1-\gamma} \right] \right)^{\frac{\beta}{1-\gamma}} \right), \tag{32}$$

subject to

$$\begin{aligned} W_t &= C_t + B_t + I_t + \tilde{K}_{t+1} \\ W_{t+1} &= B_t R_{f,t+1} + I_t + \tilde{K}_{t+1} R_{K,t+1} \\ I_t &\geq 0. \end{aligned} \tag{33}$$

The solution is still characterized by $V(W_t) \propto W_t$, implying the result $C_t/W_t = 1 - \beta$.

We now characterize the equilibrium. Let R_f^* denote the equilibrium riskfree rate in the no-inventory economy Then:

1. If $R_f^* > 1$, then in equilibrium $I_t = 0$, and the equilibrium is the same as in Section 4.1.

2. If $R_f^* < 1$, then $I_t > 0$. Investment in inventory crowds out investment in productive capital.

The argument is as follows (Appendix G gives an alternative, more formal proof). First, consider the case of $R_f^* > 1$, and conjecture that $R_{f,t+1} = R_f^*$ constitutes an equilibrium in (32–33). This investor never chooses $I_t > 0$ because bonds offer superior returns; on the other hand, (33) implies that the agent cannot short-sell inventory. Therefore $I_t = 0$, namely, the inventory asset is irrelevant, and thus $\alpha = 1$ is still the market-clearing condition. Equilibrium quantities and returns are the same as in Section 4.1.

Now assume that $R_f^* < 1$. The only possible equilibrium value for R_f is unity. This is because $R_f < 1$ implies an arbitrage opportunity: the investor would borrow at R_f and invest the proceeds in the inventory asset. If instead $R_f > 1$, the reasoning in the above paragraph implies the agent holds no inventory. That means $R_f^* = R_f > 1$, contradicting the assumption. Intuitively, we can find an equilibrium with inventory for the following reason: if the agent does not hold inventory ($\alpha = 1$) and the riskfree rate equals $R_{f,t+1}^* < 1$, then the agent will wish to hold more inventory, as it is a marginally better asset. Doing so, however, reduces the volatility of the return on the wealth portfolio and stochastic discount factor and thus increases the equilibrium riskfree rate. The agent will increase holdings of inventory until the equilibrium rate is equal to the return on inventory.

Assume parameters are such that $R_f^* < 1$; as the above argument shows, this is where inventory matters. We show it is also empirically relevant in that it prevails in the second sample period. Bonds are redundant, so we can assume $B_t = 0$. The requirement $R_f = 1$ replaces $\alpha = 1$ as the market-clearing condition. Given that the equilibrium takes this form, the optimization problem reduces again to (28), but this time subject to

$$W_{t+1} = (W_t - C_t)(1 + \alpha_t r_{K,t+1}), \tag{34}$$

where $r_{K,t+1} = R_{K,t+1} - 1$ is the net return on capital and α is the share of capital in savings, as it was in Section 4.1. Note that technology and depreciation pins down $r_{K,t+1}$, see (25).

Appendix G shows that $C_t/W_t = 1 - \beta$ and the stochastic discount factor $M_{t+1} = \mathbb{E}_t[R_{W,t+1}^{1-\gamma}]^{-1} R_{W,t+1}^{-\gamma}$. The first-order condition with respect to α shows that

$$E_t\left[\frac{1}{(1+\alpha r_{K,t+1})^{\gamma}} r_{K,t+1}\right] = 0, \tag{35}$$

which is special case of $E_t[M_{t+1}R_{W,t+1} - R_f] = 0$, with $R_f = 1$, and $R_{W,t+1} = 1 + \alpha_t r_{K,t+1}$, from (34).

Let $r_{K,0} \equiv (1-\delta+A) - 1$ and $r_{K,\eta} \equiv (1-\delta+A)(1-\eta) - 1$ denote the net returns on capital in the non-disaster and disaster states, respectively. Then (35) implies

$$\frac{p r_{K,\eta}}{(1+\alpha r_{K,\eta})^{\gamma}} + \frac{(1-p) r_{K,0}}{(1+\alpha r_{K,0})^{\gamma}} = 0, \tag{36}$$

Solving for α implies:

$$\alpha = \min\left\{1, -\frac{((1-p)r_{K,0})^{1/\gamma} - (-p r_{K,\eta})^{1/\gamma}}{((1-p)r_{K,0})^{1/\gamma} r_{K,\eta} - (-p r_{K,\eta})^{1/\gamma} r_{K,0}}\right\}, \tag{37}$$

The investor holds inventory when expected risk-adjusted capital returns are sufficiently low.

It follows from $C_t/W_t = 1 - \beta$ and (34) that

$$\frac{C_{t+1}}{C_t} = \beta R_{W,t+1} = \beta(1+\alpha r_{K,t+1}) = \beta\left(\alpha(1-\delta+A)(1-\chi_{t+1}) + 1 - \alpha\right). \tag{38}$$

Relative to the model in Section 4.1, consumption growth is less volatile because, in aggregate, agents use inventory to smooth aggregate fluctuations. It is also, on average, lower, because less is invested in the productive asset. Output growth, however, is *more* volatile. Consumption growth is no longer tethered to output as in Section 4.1. Still, the relation between growth in the capital stock and growth in wealth remains the same:

$$\frac{K_{t+1}}{K_t} = \frac{\tilde{K}_{t+1}}{\tilde{K}_t}\frac{1-\chi_{t+1}}{1-\chi_t} = \frac{W_t}{W_{t-1}}\frac{1-\chi_{t+1}}{1-\chi_t}.$$

Substituting in from (38) then implies

$$\frac{Y_{t+1}}{Y_t} = \frac{K_{t+1}}{K_t} = \beta\left(\alpha(1-\delta+A)(1-\chi_{t+1}) + (1-\alpha)\left(\frac{1-\chi_{t+1}}{1-\chi_t}\right)\right). \tag{39}$$

Output growth is more volatile than consumption growth because it bears the full brunt of disasters: note that $1-\chi_{t+1}$ multiplies both the term with α (representing investment in the risky technology) and $1-\alpha$. By definition, the disaster applies to the entire existing capital stock. While this effect makes output growth more volatile than consumption in the present model, it does not, by itself, raise the volatility relative to the model in Section 4.1. There is, however, a second effect, represented by $1-\chi_t$ in the denominator. Coming out of a severe recession featuring capital destruction $\chi_t > 0$, output growth is higher because agents invest more to get back to the optimal allocation. This raises the volatility of output growth relative to the model in Section 4.1.

What are the properties of investment? Rewriting the capital accumulation equation (21) so that X_t is on the left-hand side, and dividing by K_t, implies

$$\frac{X_t}{K_t} = \frac{\tilde{K}_{t+1}}{\tilde{K}_t}\frac{\tilde{K}_t}{K_t} - (1-\delta) \tag{40}$$

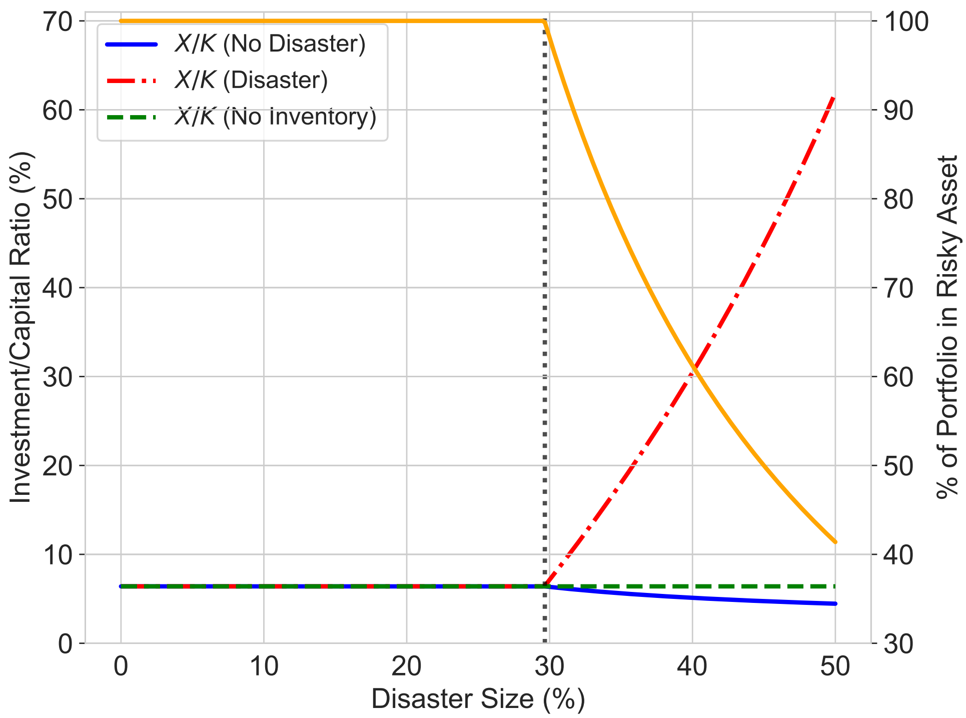
$$= \beta R_{W,t}(1-\chi_t)^{-1} - (1-\delta) \tag{41}$$

$$= \beta(\alpha(1-\delta+A) + (1-\alpha)(1-\chi_t)^{-1}) - (1-\delta), \tag{42}$$

where we have used the fact that $\tilde{K}_{t+1}/\tilde{K}_t = \beta R_{W,t}$. After capital disasters, the agent invests at a higher rate to replenish the capital stock. Consequently, the investment-capital ratio is time-varying in this economy, despite i.i.d. shocks and a balanced growth path.

Figure 10: Investment capital ratio in the model

The figure shows how capital investment varies with the size of the consumption decline in a disaster for the production models with and without inventory. The figure plots the investment-capital ratio X/K in the model with inventory when there is and is not a disaster, and in the model without inventory. It also plots α, the share of savings invested in capital. Risk aversion $\gamma = 5$, the EIS $\psi = 1$, the patience parameter $\beta = 0.981$, depreciation $\delta = 0.05$, the probability of disaster $p = 2.10\%$, and the marginal product of capital $A = 0.0839$. The dotted black line represents the point at which the riskfree rate is equal to 0 in the model without inventory.



A disaster in the prior period increases investment in productive capital. This is because the disaster affects capital disproportionately, and the agent must re-invest to return capital back to its pre-crisis level. For an illustration, see Figure 10, which shows the investment-capital ratio for $\chi_t = 0$ (no disaster) and $\chi_t = \eta$ (disaster) for various values of the disaster size.[29] The figure also shows the optimal planned capital to wealth ratio α. For compari-

[29] A higher disaster size has the same effect in the model as a higher disaster probability. What matters for these mechanism is total disaster risk.

son, the figure also shows quantities in the case of no inventory. Fixing other parameters, for disaster sizes of less than 25%, the gross riskfree rate is above one, implying that the economies with and without inventory are the same. As the size of the disaster increases, the equilibrium riskfree rate in the no-inventory economy falls sharply (we illustrate this in Figure 12). It becomes optimal to hold inventory and investment in productive assets falls. At that point, investment depends on the occurence of a disaster in the prior period. The greater the size of the disaster, the greater the increase in investment. In contrast, with no inventory, the investment-capital ratio is always the same.

We define the stock market as the claim to output Y_t in all future periods. As (39) shows, the growth rate of capital is no longer i.i.d. but depends on χ_t (note that χ_{t+1} is i.i.d. given time-t information). Therefore, the price-dividend ratio on the output claim is a function of χ_t and solves

$$\kappa^Y(\chi_t) = \mathbb{E}_t\left[M_{t+1}\left(1+\kappa^Y(\chi_{t+1})\right)\frac{Y_{t+1}}{Y_t}\right],$$

Under our distributional assumptions:

$$\kappa^Y(0) = \frac{\beta}{1-\beta}\left(\nu + (1-\nu)\left(\frac{1+\alpha r_{K,0}}{1+\alpha r_{K,\eta}}\right)(1-\eta)\right), \tag{43}$$

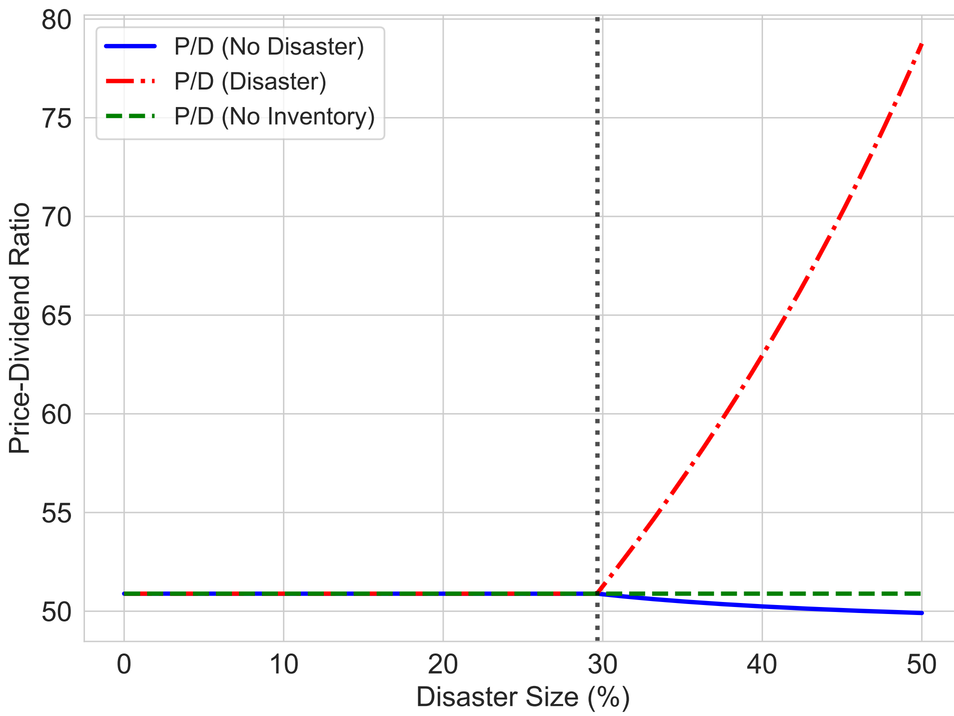
$$\kappa^Y(\eta) = \frac{\beta}{1-\beta}\left((1-\nu) + \nu\left(\frac{1+\alpha r_{K,\eta}}{1+\alpha r_{K,0}}\right)(1-\eta)^{-1}\right), \tag{44}$$

where $\nu \equiv ((1-p)(1+\alpha r_{K,0})^{1-\gamma})/((1-p)(1+\alpha r_{K,0})^{1-\gamma} + p(1+\alpha r_{K,\eta})^{1-\gamma})$. See Appendix G for details. In the case where $\alpha = 1$, the price-dividend ratio is the constant $\kappa^Y = \beta/(1-\beta)$.

Figure 11 shows the price-dividend ratio for various levels of the disaster size, both in the economy with inventory and in the economy without. The economy without inventory has a constant price-dividend ratio solely determined by β. When there is inventory, the price-dividend ratio rises in disasters because dividends are temporarily depressed (they are also low because of the disaster). This increase is due to the endogenous investment response, whereby inventory is liquidated after a disaster to rebuild the capital that is destroyed.

Figure 11: Price-dividend ratio in the model

The figure shows how the price-dividend ratio varies with the size of the consumption decline in a disaster for the production models with and without inventory. The figure plots the price-dividend ratio in the model with inventory when there is and is not a disaster, and in the model without inventory. Risk aversion $\gamma = 5$, the EIS $\psi = 1$, the patience parameter $\beta = 0.981$, depreciation $\delta = 0.05$, the probability of disaster $p = 2.10\%$, and the marginal product of capital $A = 0.0839$. The dotted black line represents the point at which the riskfree rate is equal to 0 in the model without inventory.



In contrast with standard production models, the price-dividend ratio in the no-disaster case declines (in a comparative statics sense) as a function of the disaster size (see Figure 11). In the case without inventory, the price-dividend ratio is independent of disaster risk. Models with production that seek to match business-cycle fluctuations in investment and valuation ratios require the EIS to be greater than 1. Endowment models achieve the same effect by imposing exogenous leverage (dividends more sensitive to shocks than consumption). In this model, leverage is endogenous, and qualitatively correct price-dividend ratio dynamics could in principle occur, even with an EIS of one. The magnitude of the decline in Figure 11 suggests that the effect is small under our calibration.

Figure 12 shows that the equity risk premium in this economy loses its usual dependence on disaster risk. The equity premium equals $rp \equiv \log \mathbb{E}_t[R_{Y,t+1}] - \log R_f$, where the return on the output claim is

$$R_{Y,t+1} = \left(\frac{\kappa^Y(\chi_{t+1}) + 1}{\kappa^Y(\chi_t)}\right)\left(\frac{Y_{t+1}}{Y_t}\right).$$

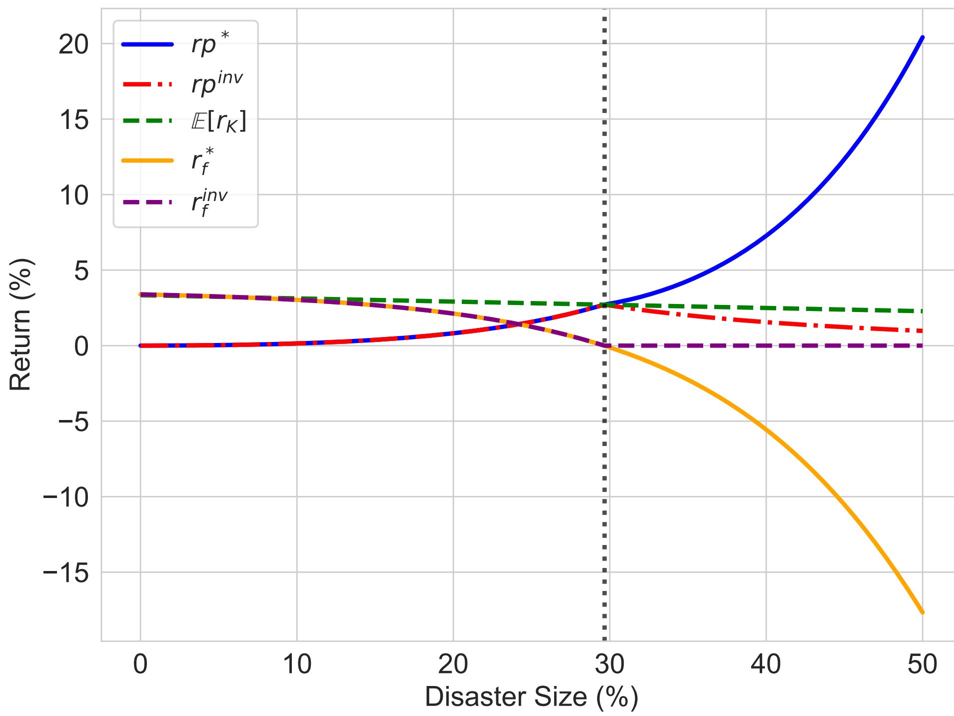
The blue line in the figure shows the equity premium in the model without inventory: it is highly dependent on the disaster size, as is the riskfree rate. However, the return on capital—which, in the economy with no inventory, is the equity return—is only very slightly decreasing. This is a standard result in disaster-risk economies: the full discount rate on the equity claim decreases slightly with the probability of a disaster.

While this might seem counterintuitive, it arises from the fact that, while the equity premium increases, the riskfree rate declines and more than offsets the effect. Also recall that the continuously compounded return in a standard i.i.d. economy can be expressed as the log dividend yield plus the log growth in cash flows. When the EIS equals one, the dividend yield does not depend on disaster risk, and so the only effect is the small effect of expected cash flows. In the economy with inventory, the return on capital is the same as in the economy without (this is defined by the production opportunities), and thus is slightly decreasing. The riskfree rate is constant, implying that the premium on capital is also slightly decreasing. The equity premium decreases slightly more in the disaster size as compared to $\log \mathbb{E}[R_K] - \log R_f$. This is because the increase in the price-dividend ratio counteracts the decline in output due to the disaster.

We calibrate the model to match the ex-ante inflation-adjusted Treasury yield, price-dividend ratio, and GDP growth in the U.S., as in the sections above. Calibrating to match these data requires solving a system of three equations in three unknowns, where the unknowns are the parameters β, λ, and A and the three equation are the price-dividend ratio

Figure 12: Risk premia and riskfree rate in the model

The figure shows how the riskfree rate and risk premium vary with the size of the consumption decline in a disaster for the production models with and without inventory. The moments plotted are: the equity premium in the models with and without inventory, rp^{inv} and rp^{*}; the riskfree rates in the models with and without inventory, r_f^{inv} and r_f^{*}; and the expected return on capital, $\mathbb{E}[r_K]$. The equity premium is defined as the log expected return on the output claim minus the log riskfree rate. Risk aversion $\gamma = 5$, the EIS $\psi = 1$, the patience parameter $\beta = 0.981$, depreciation $\delta = 0.05$, the probability of disaster $p = 2.10\%$, and the marginal product of capital $A = 0.0839$. The dotted black line represents the point at which the net riskfree rate is equal to 0 in the model without inventory.



(43), the ex ante inflation-adjusted bond yield (G.26), and

$$\frac{Y_{t+1}(0)}{Y_t(0)} = \beta\left(\alpha(1-\delta+A) + (1-\alpha)\right) \tag{45}$$

which is GDP growth when the disaster does not occur. Indeed, each of the moments to which we calibrate parameters is the value in the no-disaster state ($\chi_t = 0$), consistent with the fact that we do not observe any disasters in our sample. We then solve for the values of the parameters that equate the data moments with their corresponding model moments.

Table 4 displays the results from calibrating the production model. The model explains the data moments with a quantitatively reasonable calibration of β, λ, and A. The slight increase in β matches the modest rise in the price-dividend ratio; lower capital productivity A matches the lower growth in the second sample.[30] Inflationary default risk $\lambda\eta$ falls, in line with the estimates in Section 3.[31] A striking result of Table 4 is the model's ability to match real investment. It turns out that matching the growth rate and asset prices is sufficient, in this model, to capture both the levels and the decline in the investment-capital ratio. Specifically, while the average in the data is 7.7% from 1984 to 2000, it is 7.4% in the model. This falls to an average of 6.9% in the data from 2001–2021. The model matches this decline, producing an average of 6.3%. As in Section 3.7, a β sufficiently high to explain interest rates on its own implies an investment boom (see equation 42, or the without-inventory version, equation 31). In fact the opposite occurred, offering further evidence that a force other than a savings glut must have led to the decline in interest rates.

Table 4 also calibrates a production economy without inventory, allowing us to discern the impact of allowing this additional investment opportunity. In the model with inventory, reduced GDP growth occurs in part because of investment in unproductive capital. To match the growth rates, capital productivity must fall by less than in the model without. Moreover, given the same discount factor β, the unconstrained riskfree rate also falls by less because the economy is less risky. This implies a greater change in the fraction lost in default that is needed to explain the observed decline in interest rates. Overall, while inventory acts as both an insurance policy and as a drag on growth, its effects, which stem from the riskfree rate hitting the zero lower bound, are relatively small.

If inventory is indeed money, then this should show up in the data as an increase in

[30] We could, equivalently, keep A constant and estimate an increase in δ; they are isomorphic for explaining the growth decline.

[31] The estimates of λ are slightly different than in Table 2 because we calibrate to average GDP growth instead of consumption growth.

Table 4: Inventory and inflationary default in a model with production

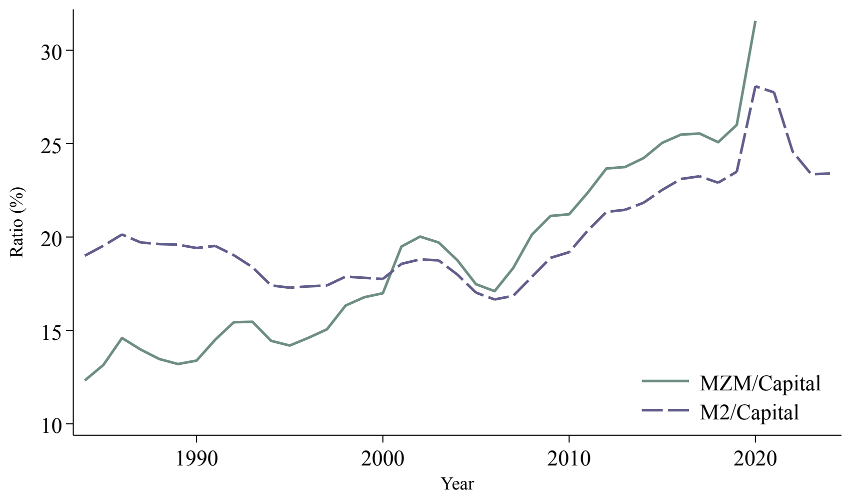
This table shows parameters necessary to match the data, assuming a production economy with rare disasters and inflationary default. We take average GDP growth from the data in each sample. We take average consumption growth from the data in each sample. We calibrate the discount factor β and the decline in bond value $\lambda\eta$ to match the average price-dividend ratio and Treasury bill yield minus forecasted inflation (ex ante inflation-adjusted Treasury yield). The model is solved with risk aversion $\gamma = 5$ and EIS = 1. Consumption declines 30% in a disaster ($\eta = 0.30$), the probability of disaster $p = 2.10\%$, and depreciation $\delta = 0.05$. Panel A assumes the existence of inventory, a riskfree production opportunity, whereas Panel B is a standard production model.

		Values	
	Parameter	1984–2000	2001–2021
Panel A: Moments in the data			
Price-dividend ratio	κ	42.35	50.86
Ex ante inflation-adjusted Treasury yield	$\bar{y}_b$	0.0287	-0.0053
US GDP growth	$\frac{\Delta Y}{Y}$	0.0241	0.0133
Investment-capital ratio	$\frac{X}{K}$	0.0773	0.0689
Panel B: Calibration, $\gamma = 5$, EIS = 1, $\eta = 0.30$			
Discount factor	β	0.977	0.981
Fraction of bond value lost	$\lambda\eta$	0.178	-0.059
Capital productivity	A	0.0986	0.0839
Risky capital share	α	1.000	0.981
Investment-capital ratio	$\frac{X}{K}$	0.0744	0.0634
Unconstrained riskfree rate	r_f^*	0.0130	-0.0012
Panel C: Without inventory			
Discount factor	β	0.977	0.981
Fraction of bond value lost	$\lambda\eta$	0.178	-0.037
Capital productivity	A	0.0986	0.0833
Risky capital share	α	1.000	1.000
Investment-capital ratio	$\frac{X}{K}$	0.0744	0.0634
Unconstrained riskfree rate	r_f^*	0.0130	-0.0018

the money supply relative to the capital stock. To assess this prediction of the inventory model, we plot this ratio in Figure 13. We report two definitions of the money supply, both

Figure 13: Money supply relative to the capital stock

The figure shows the ratio of the money supply to the capital stock in the U.S. economy. We report two measures of the money supply. The first is M2, which adds to M1 savings accounts, small time deposits, and retail money market mutual funds. The second is MZM (zero-maturity money), which is constructed by the Federal Reserve Bank of St. Louis and includes M2 less small-denomination time deposits plus institutional money market funds. The line for MZM stops in 2020 because the series was discontinued.



of which have risen relative to the capital stock in the twenty-first century. The first is the common M2 measure. The second, which we argue is a better measure of inventory, is the more inclusive “zero-maturity money” measure (MZM) from the St. Louis Fed, which takes M2, removes small illiquid time deposits, and adds institutional money market mutual funds (whereas M2 only includes retail money market funds). The rise in MZM relative to capital over the past two decades has been sizable, in line with our model’s prediction of an increasing share of wealth in money-like inventory assets.

5 Conclusion

Declining interest rates is not merely a feature of the 1980–2021 period, they are a feature of the last several centuries. They also pose a joint puzzle: why have persistently low interest rates not been accompanied by higher valuation ratios and investment rates?

This article offers an explanation rooted in the decline of perceived sovereign-default risk. We show that when investors require a smaller premium for default, yields can fall substantially without an equivalent decline in the true real rate. This mechanism accounts for declining interest rates, and stable valuation ratios. It is supported by reduced inflation risk premia, and a reversal in the correlations among inflation, output growth, and asset returns.

Our analysis, however, abstracts from the deeper sources of changing default risk. The model is silent on why investors' confidence in sovereign repayment—and, equivalently, their expectations of inflationary default—have improved. These expectations appear to have evolved along both short- and long-term dimensions: institutionally, through credible monetary and fiscal policy over recent decades, and historically, through the slow consolidation of sovereign legitimacy. Understanding these forces lies beyond the scope of this paper but is essential to a full account of the historic decline in yields.

Our results suggest several areas for future work. First, our model is based on investors' lower expectations of default, as well as lower default during disasters. Qualitatively, these can account for higher debt-to-GDP ratios. However, the increase in debt-to-GDP ratios across advanced economies has been so pronounced that it raises questions about whether default risk truly has declined relative to the 1980s or 1990s. Second, we match intra-period dynamics using expectations data. These appear to be insufficient: a model incorporating time-varying disaster risk could explain additional facts, including the changing bond–stock correlations and the sharp decline in rates during 2008. Further work could also investigate why inflation-protected and nominal yields sometimes move together—most notably after 2021—when an inflation-based channel of default risk predicts divergence between them. These questions point toward a richer understanding of how perceptions of sovereign and macroeconomic risk jointly shape asset prices over time.

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Internet Appendix

A Data description

We use various series to illustrate the secular decline in interest rates in the short- and long-run. To obtain interest rates from 1311–2018, we rely on data from Schmelzing (2020). The dataset contains nominal interest rate and inflation time series for several developed economies over the last eight centuries. Specifically, the data include long-term sovereign borrowing rates with an average maturity that hovers around 10 years; however, this varies over time and across countries. From these data, we plot the nominal sovereign borrowing yields for the United Kingdom, Holland, Germany, Italy, and the United States in Panel A of Figure 1. The data are collected from a variety of sources, outlined in detail in the paper and online appendix. The U.K. borrowing rates come from the Calendar of State Papers and the Bank of England. Data before 1694 for the U.K. (before the founding of the Bank of England) are not used, since the data are incomplete. Data for the Netherlands come from Dormans (1991), Weeveringh (1852), the European Central Bank, and various sources from Leiden, Haarlem, Utrecht, Schiedam, and Amsterdam. German data come from various sources from several German principalities. U.S. data come from Durand and Winn (1947), Homer and Sylla (2005), the NBER Macrohistory database, and Federal Reserve Economic Data (FRED) from the Federal Reserve Bank of St. Louis.

We also report the Bank of England (BoE) short-term lending rate (series IDGBRD) from Global Financial Data. From 1694 to 1971, the "bank rate" is used; from 1972 to 1981, the minimum lending rate is used; from 1981 to 1997, the BoE base rate is used; and from 1997 to the present, the BoE Operational interest rate is used. For more information see the Bank of England research datasets webpage.

For the U.S., our primary interest rate series—in modern times—is the effective Federal Funds Rate (series FEDFUNDS), the rate corresponding to the median volume of overnight unsecured loans between depository institutions. These data come from FRED. This series is plotted in Panel A of Figures 2 and 8. This is also the interest rate series used in all of our calibration exercises. Figure 9 presents data on the 5-year and 10-year nominal and inflation-protected Treasury bonds (FRED series DGS5, DGS10, DFII5, and DFII10). Data for interest rates in Appendix E come from Jordà et al. (2019).

U.K. inflation-linked and nominal Gilts yields are taken from Global Financial Data, which sources the yields from the Bank of England. Inflation in the U.K. at one- and five-year horizons is calculated from the monthly CPI on all items, as reported by the OECD (see Global Financial Data series IGGBR1D, IGGBR5D, and CPGBRCM).

Data on U.S. inflation expectations for Figure 6 come from the Survey of Professional Forecasters and FRED. From the Survey of Professional Forecasters, we use the 1-year, 5-year, and 10-year ahead inflation expectations in different parts of the paper. For 1-year inflation expectations, we use the median forecast of the price index for GDP (series: PGDP) as used in Angeletos et al. (2021). This is subtracted from the Fed Funds Rate to form our inflation-adjusted interest rate measure in all calibration exercises. This is also what we use to calibrate q in the model presented in Section 8. These data are also used to construct the deviation of expected inflation from realized inflation shown in Figure 7. In Figure 9, we use 5-year and 10-year expected CPI inflation to construct the difference between U.S. nominal bonds and TIPS. We do this because long-term forecasts for the price index of GDP are not available in the SPF. From FRED, we use the inflation expectations from the Surveys of Consumers of University of Michigan (series MICH), which covers short-term inflation expectations, and the expected 10-year-ahead inflation implied from Treasury Inflation-Indexed Constant Maturity Securities (series T10YIE).

Data on realized inflation in the U.S. is used in Section 3.7 where we use the annual realized change in the price level of GDP. This is also used in Figure 6 meaning we use the same forecast errors as in Angeletos et al. (2021). In Figure 5 we use the realized change in the CPI for all urban consumers.

Data on long-term earnings growth expectations come from two sources: Bordalo et al. (2024) and I/B/E/S. In particular, we use LTG which represents cumulative, annualized earnings growth over the next 3-5 years (Bordalo et al., 2024). Since Bordalo et al. (2024) present these data until the end of 2020, we fill in the remaining years with the estimate of LTG from I/B/E/SGlobal Aggregates for the S&P 500 Index. These data are used to calibrate the consumption-dividend ratio z in the generalized endowment model. Also used are the Personal Consumption Expenditures series (FRED: PCE) from FRED economic data and earnings from Robert Shiller's webpage (Shiller, 2000).

Growth data come from different sources. In Tables 1–2, the U.S. growth parameter μ is set to match per capita consumption growth, series A794RX0Q048SBEA from FRED Economic Data hosted by the St. Louis Federal Reserve. In Figure 2 and Table 4, we use real per capita GDP growth rates from FRED (series A939RX0Q048SBEA) as the growth rate for the U.S. Average annual growth rates are used, which are computed using December-to-December values. When calibrating to the international evidence in Appendix E, we use the real GDP growth series from Jordà et al. (2019).

Data on investment and capital stock come from the Bureau of Economic Analysis (BEA) Fixed Assets Accounts Tables. Investment data come from Table 1.5, Line 2 and capital stock data come from Table 1.1, Line 2. In these data, investment as a fraction of capital averaged 7.7% from 1984–2000 and 6.9% from 2001–2021.

Price-dividend ratio data for the U.S. from 1984 to 2021 are from the Center for Research in Security Prices (CRSP). Specifically, we use cum-dividend returns (series VWRETD) and ex-dividend returns (series VWRETX). To calculate the price-dividend ratio, we back out prices and dividends from cum- and ex-dividend returns. This series is plotted in Panel B of Figure 2. We use this procedure to calculate our price-dividend ratio moments for all of the calibration exercises.

We combine price dividend ratio data with data on the cyclically-adjusted price-earnings ratio (CAPE)—the price divided by the average inflation-adjusted earnings from the previous

10 years—to calibrate the model in Section 3.7. See Shiller (2000) and online data description. In particular, we use the difference between CAPE and the price dividend ratio to determine how earnings are paid out as dividends. Valuation ratio data for the international evidence in Appendix E come from Jordà et al. (2019).

Finally, we obtain the Volatility Index (VIX) series from the Chicago Board Options Exchange (CBOE). The CBOE calculates the risk-neutral expected 30-day quadratic variation using option prices. There are small differences in the calculation methodology over the years; see CBOE white paper.

B Structural break test

Throughout the main text, we calibrate the model to data from two subsamples. We determine the most likely date for a structural break in the average one-year Treasury yield. Specifically, for each potential break year t_{break} since 1985, we estimate the regression[32]

$$y^{\$}_{b,t} - \Delta\pi_t = \beta_0 + \beta_1 \mathbb{1}\{t > t_{\text{break}}\} + \epsilon_t, \tag{B.1}$$

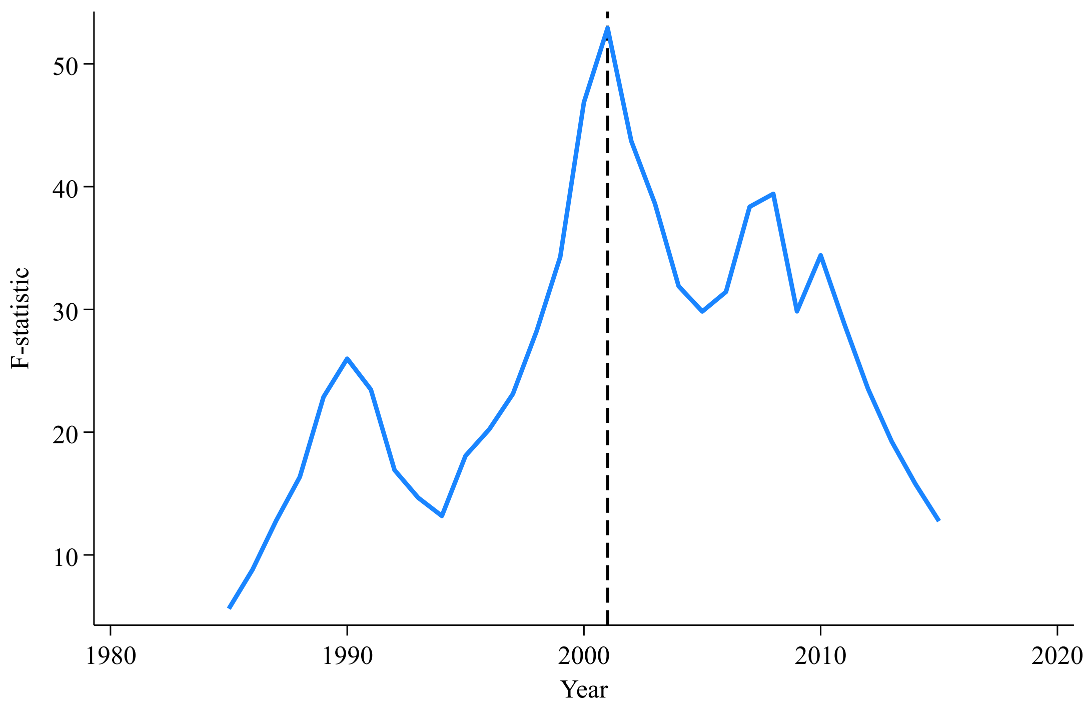
where $\Delta\pi_t \equiv \log(\Pi_{t+1}/\Pi_t)$ is log realized inflation.[33] Figure B.1 plots the F-statistic from this regression as a function of the break point. Evidently, 2001 stands out as the best fit for a structural break in inflation-adjusted yields.

As we mention in the main text, our choice of 2001 as a break date is also consistent with prior work studying secular changes in macroeconomic time series since the 1980s. First, Farhi and Gourio (2018) calibrate their model to two separate data samples around this date. Second, using a regression-based break test very similar to ours, Campbell et al. (2020) identify 2001 as the most likely year for a structural break in the relation between

[32]Recall that $y^{\$}_{b,t}$ is the yield on the one-year Treasury bill and $\Delta\pi_t$ is one-year log realized inflation, so this difference represents the ex post return on the one-year nominal bond, (15).

[33]Results are similar using forecasts instead of realizations.

Figure B.1: Structural break test on interest rates



Notes: The figure presents F-statistics for the linear regression (B.1), estimated using OLS for all potential break dates from 1985–2015 using data on one-year nominal Treasury bill yields less inflation.

GDP growth and inflation. They find that the two series were negatively correlated prior to 2001 and became positively correlated thereafter.

C Endowment model with general EIS and constant within-period growth rates

C.1 Price-consumption ratio

Given the SDF (3), the Euler equation with respect to the consumption claim is

$$1 = \mathbb{E}_t\left[\beta^\theta \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}} R_{W,t+1}^\theta\right]. \tag{C.1}$$

Conjecture a constant price-consumption ratio

$$\kappa \equiv (W_t - C_t)/C_t. \tag{C.2}$$

Substituting (C.2) into (C.1) and using $R_{W,t+1} = W_{t+1}/(W_t - C_t)$ implies

$$1 = \beta^\theta \mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{\theta(1-\frac{1}{\psi})} \left(\frac{\kappa+1}{\kappa}\right)^\theta\right]. \tag{C.3}$$

Given (1–2),

$$\frac{\kappa}{\kappa+1} = \beta e^{(1-\frac{1}{\psi})\mu}\left[1 + p((1-\eta)^{1-\gamma} - 1)\right]^{\frac{1}{\theta}}. \tag{C.4}$$

A solution exists provided that the right-hand side of (C.4) is less than one. We restrict attention to parameter combinations satisfying this restriction. Finally,

$$\kappa = \frac{\beta e^{(1-\frac{1}{\psi})\mu}\left[1 + p((1-\eta)^{1-\gamma} - 1)\right]^{\frac{1}{\theta}}}{1 - \beta e^{(1-\frac{1}{\psi})\mu}\left[1 + p((1-\eta)^{1-\gamma} - 1)\right]^{\frac{1}{\theta}}}, \tag{C.5}$$

verifying the conjecture.

C.2 Riskfree rate

The riskfree rate is given by the Euler equation for the riskfree asset

$$R_f = \mathbb{E}_t \left[\beta^\theta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{\theta}{\psi}} R_{W,t+1}^{\theta-1} \right]^{-1}. \tag{C.6}$$

This simplifies to

$$R_f = \mathbb{E}_t \left[\beta^\theta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \left(\frac{\kappa}{\kappa+1} \right)^{1-\theta} \right]^{-1}. \tag{C.7}$$

where $\kappa/(\kappa+1)$ is given by (C.4). Solving this yields the expression for the gross riskfree rate

$$R_f = \beta^{-1} e^{\frac{1}{\psi}\mu} \Big[1 + p((1-\eta)^{-\gamma} - 1) \Big]^{-1} \Big[1 + p((1-\eta)^{1-\gamma} - 1) \Big]^{\frac{\theta-1}{\theta}} \tag{C.8}$$

which implies that the log riskfree rate is given by

$$\log R_f = -\log \beta + \frac{1}{\psi}\mu - \log(1 + p((1-\eta)^{-\gamma} - 1)) + \left(\frac{\theta - 1}{\theta} \right) \log(1 + p((1-\eta)^{1-\gamma} - 1)). \tag{C.9}$$

C.3 Yield and expected return with sovereign default risk

Consider the defaultable short-term government bond paying $(1 - L_{t+1})$ dollars—that is, 1 dollar in the case of no default and $1 - \lambda\eta$ dollars in the case of default. The price of this claim is obtained by solving the Euler equation

$$Q_t = \mathbb{E}_t \left[\beta^\theta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{\theta}{\psi}} R_{W,t+1}^{\theta-1} (1 - L_{t+1}) \right], \tag{C.10}$$

which simplifies to

$$Q_t = \mathbb{E}_t\left[\beta^\theta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \left(\frac{\kappa}{\kappa+1}\right)^{1-\theta} (1 - L_{t+1})\right], \tag{C.11}$$

where $\kappa/(\kappa+1)$ is given by (C.4). This gives the price of the defaultable claim as

$$Q_t = \beta e^{-\frac{1}{\psi}\mu} \left[1 + p((1-\eta)^{1-\gamma} - 1)\right]^{\frac{1-\theta}{\theta}} \left[1 + p((1-\lambda\eta)(1-\eta)^{-\gamma} - 1)\right]. \tag{C.12}$$

The yield on the defaultable claim is defined as $y_{b,t} \equiv -\log Q_t$, and is thus equal to the constant

$$y_b = \log R_f + \log\left(1 + p((1-\eta)^{-\gamma} - 1)\right) - \log\left(1 + p((1-\lambda\eta)(1-\eta)^{-\gamma} - 1)\right), \tag{C.13}$$

where $\log R_f$ is given by (C.9). The expected excess return on the bond is the expected payoff divided by the price, less the log riskfree rate, and therefore equals

$$\begin{aligned} \log \mathbb{E}_t\left[R_{b,t+1}\right] - \log R_f &= \log\left(1 + p((1-\lambda\eta) - 1)\right) \\ &\quad + \log\left(1 + p((1-\eta)^{-\gamma} - 1)\right) - \log\left(1 + p((1-\lambda\eta)(1-\eta)^{-\gamma} - 1)\right). \end{aligned} \tag{C.14}$$

Suppose instead of being subject to outright default, the bond is a nominally riskfree asset and so the government partially defaults through inflation. Assume inflation is given by the process (9). The price of this defaultable claim is obtained by solving the Euler equation

$$Q_t^{\$} = \mathbb{E}_t\left[\beta^\theta \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}} R_{W,t+1}^{\theta-1} \frac{\Pi_t}{\Pi_{t+1}}\right], \tag{C.15}$$

which simplifies to $Q_t^{\$} = Q_t e^{-q_t + \sigma_\pi^2/2}$ for the price Q_t given by (C.12). Subsequent results in the main text then follow straightforwardly.

D Generalized endowment model with unit EIS

For ease of exposition in what follows, we use the following notation. Standard deviations σ_x should be understood to equal $\sqrt{\sigma_x^\top \sigma_x}$ for variables x. Covariances σ_{xy} should be understood to equal $\sigma_x^\top \sigma_y$.

D.1 Discount factor and riskfree rate

With a unit EIS, the consumption-wealth ratio equals $C_t/W_t = 1-\beta$, and thus the return on wealth equals[34]

$$R_{W,t+1} = \frac{W_{t+1}}{W_t - C_t} = \beta^{-1}\frac{C_{t+1}}{C_t}. \tag{D.1}$$

The SDF is given by the latter case in (3), namely

$$M_{t+1} = \beta \mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{1-\gamma}\right]^{-1}\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} = \mathbb{E}_t[R_{W,t+1}^{1-\gamma}]^{-1}R_{W,t+1}^{-\gamma}. \tag{D.2}$$

Substituting in the wealth return, the expectation term in the SDF equals

$$\mathbb{E}_t[R_{W,t+1}^{1-\gamma}]^{-1} = \beta^{1-\gamma}e^{(\gamma-1)\mu-(1-\gamma)^2|\sigma_C|^2/2}(1+p((1-\eta)^{1-\gamma}-1))^{-1}, \tag{D.3}$$

and so the SDF can be written

$$M_{t+1} = \beta e^{-\mu-(1-\gamma)^2|\sigma_C|^2/2-\gamma\sigma_C^\top\varepsilon_{t+1}}\frac{(1-\chi_{t+1})^{-\gamma}}{1+p((1-\eta)^{1-\gamma}-1)}, \tag{D.4}$$

[34] We re-derive this result and the form of the SDF in the production economy. See Appendix G below for the derivation.

and implies the riskfree rate

$$\log R_{ft} = -\log \mathbb{E}_t[M_{t+1}] = -\log\beta + \mu + \frac{1}{2}[(1-\gamma)^2 - \gamma^2]|\sigma_C|^2 \\ - \log(1 + p((1-\eta)^{-\gamma} - 1)) + \log(1 + p((1-\eta)^{1-\gamma} - 1)). \quad \text{(D.5)}$$

The term structure of inflation-protected long-term bonds is flat (that is, all yields equal the riskfree rate).

D.2 Stock prices

Given that the log consumption-dividend ratio evolves as

$$z_{t+1} = (1-\rho_z)\bar{z} + \rho_z z_t + \sigma_z^\top \varepsilon_{t+1} - \xi \log(1-\chi_{t+1}).$$

dividend growth can be expressed as

$$\frac{D_{t+1}}{D_t} = \frac{C_{t+1}}{C_t} e^{z_t - z_{t+1}} = e^{\mu + (1-\rho_z)(z_t - \bar{z}) + (\sigma_C - \sigma_z)^\top \varepsilon_{t+1}} (1-\chi_{t+1})^{1+\xi}. \quad \text{(D.6)}$$

Henceforth, let $\mu_{Dt} = \mu + (1-\rho_z)(z_t - \bar{z})$ and $\sigma_D = \sigma_C - \sigma_z$. For $\sigma_{Cz} \leq 0$, dividends are more volatile than consumption and rise (fall) more when consumption rises (falls). Dividends and consumption are cointegrated, so expected dividend growth is higher when the current consumption-dividend ratio z_t is larger. If we set $\sigma_z^2 = \xi = 0$ and $z_0 = \bar{z} = 1$, then the dividend is just consumption and this becomes the consumption claim.

Consider the claim to the single dividend n periods from now, D_{t+n}, and let P_{Dnt} denote its price. The price-dividend ratio on this claim will be a function of maturity n and the consumption-dividend ratio z_t:

$$\kappa_D(n, z_t) = \frac{P_{Dnt}}{D_t}. \quad \text{(D.7)}$$

The price therefore satisfies

$$\kappa_D(n, z_t) = \mathbb{E}_t\left[M_{t+1}\frac{D_{t+1}}{D_t}\kappa_D(n-1, z_{t+1})\right]. \tag{D.8}$$

Conjecture that

$$\kappa_D(n, z_t) = \exp\{a_D(n) + b_D(n)(z_t - \bar{z})\} \tag{D.9}$$

for some functions $a_D(n)$ and $b_D(n)$ with $a_D(0) = b_D(0) = 0$. Noting that

$$M_{t+1}\frac{D_{t+1}}{D_t} = \beta e^{(1-\rho_z)(z_t-\bar{z})-(1-\gamma)^2|\sigma_C|^2/2+(\sigma_D-\gamma\sigma_C)^\top\varepsilon_{t+1}}\frac{(1-\chi_{t+1})^{1+\xi-\gamma}}{1+p((1-\eta)^{1-\gamma}-1)} \tag{D.10}$$

and

$$\kappa_D(n-1, z_{t+1}) = \exp\left\{a_D(n-1) + b_D(n-1)(\rho_z(z_t-\bar{z}) + \sigma_z^\top\varepsilon_{t+1})\right\}(1-\chi_{t+1})^{-\xi b_D(n-1)}, \tag{D.11}$$

this means that

$$e^{a_D(n)+b_D(n)(z_t-\bar{z})} = \beta e^{a_D(n-1)+[(1-\rho_z)+b_D(n-1)\rho_z](z_t-\bar{z})-(1-\gamma)(1-b_D(n-1))\sigma_{Cz}+(1-b_D(n-1))^2|\sigma_z|^2} \times \frac{1+p((1-\eta)^{1+\xi(1-b_D(n-1))-\gamma}-1)}{1+p((1-\eta)^{1-\gamma}-1)}. \tag{D.12}$$

Taking logs and collecting terms in $z_t - \bar{z}$ implies the recursion

$$b_D(n) = (1-\rho_z) + \rho_z b_D(n-1) = (1-\rho_z)\sum_{j=0}^{n-1}\rho_z^j = 1-\rho_z^n, \tag{D.13}$$

while taking logs and collecting constants implies

$$a_D(n) = a_D(n-1) + \log\beta - (1-\gamma)\rho_z^{n-1}\sigma_{Cz} + \rho_z^{2(n-1)}|\sigma_z|^2 \\ + \log(1 + p((1-\eta)^{1+\rho_z^{n-1}\xi-\gamma} - 1)) - \log(1 + p((1-\eta)^{1-\gamma} - 1)). \quad \text{(D.14)}$$

(Note that in the limit as $n \to \infty$ we have $a_D(n)/n \to \log\beta$ and $b_D(n)/n \to 0$.) Given this solution, it follows that the price-dividend ratio on the whole market is given by

$$\kappa_D(z_t) = \sum_{n=1}^{\infty} \kappa_D(n, z_t). \quad \text{(D.15)}$$

D.3 Bond prices

Consider the n-period zero-coupon nominal bond, which is a claim to the cash flow Π_{t+n}^{-1} (in real terms). This bond has price $P_{\pi nt} = \Pi_t^{-1}\kappa_\pi(n, q_t)$ and satisfies

$$\kappa_\pi(n, q_t) = \mathbb{E}_t\left[M_{t+1}\frac{\Pi_t}{\Pi_{t+1}}\kappa_\pi(n-1, q_{t+1})\right].$$

Conjecture that

$$\kappa_\pi(n, q_t) = \exp\{a_\pi(n) + b_\pi(n)(q_t - \bar{q})\}$$

for some functions $a_\pi(n)$ and $b_\pi(n)$ with $a_\pi(0) = b_\pi(0) = 0$. Noting that

$$M_{t+1}\frac{\Pi_t}{\Pi_{t+1}} = \beta e^{-\mu-(1-\gamma)^2|\sigma_C|^2/2 - q_t - (\sigma_\pi + \gamma\sigma_C)^\top \varepsilon_{t+1}} \frac{(1-\lambda\chi_{t+1})(1-\chi_{t+1})^{-\gamma}}{1 + p((1-\eta)^{1-\gamma} - 1)}$$

and

$$\kappa_\pi(n-1, q_{t+1}) = \exp\left\{a_\pi(n-1) + b_\pi(n-1)(\rho_q(q_t - \bar{q}) + \sigma_q^\top \varepsilon_{t+1})\right\},$$

this means that

$$e^{a_\pi(n)+b_\pi(n)(q_t-\bar{q})} = \beta e^{a_\pi(n-1)-\mu-\bar{q}+[-1+b_\pi(n-1)\rho_q](q_t-\bar{q})-(1-\gamma)^2|\sigma_C|^2/2+|b_\pi(n-1)\sigma_q-\sigma_\pi-\gamma\sigma_C|^2/2} \times \frac{1+p((1-\lambda\eta)(1-\eta)^{-\gamma}-1)}{1+p((1-\eta)^{1-\gamma}-1)}.$$

Taking logs and collecting terms in $q_t - \bar{q}$ implies the recursion

$$b_\pi(n) = -1 + \rho_q b_\pi(n-1) = -\sum_{j=0}^{n-1} \rho_q^j = -\frac{1-\rho_q^n}{1-\rho_q},$$

while taking logs and collecting constants implies

$$a_\pi(n) = a_\pi(n-1) + \log\beta - \mu - \bar{q} - \frac{1}{2}(1-\gamma)^2|\sigma_C|^2 + \frac{1}{2}\left|\frac{1-\rho_q^{n-1}}{1-\rho_q}\sigma_q + \sigma_\pi + \gamma\sigma_C\right|^2 + \log(1+p((1-\lambda\eta)(1-\eta)^{-\gamma}-1)) - \log(1+p((1-\eta)^{1-\gamma}-1)),$$

or, in terms of the riskfree rate,

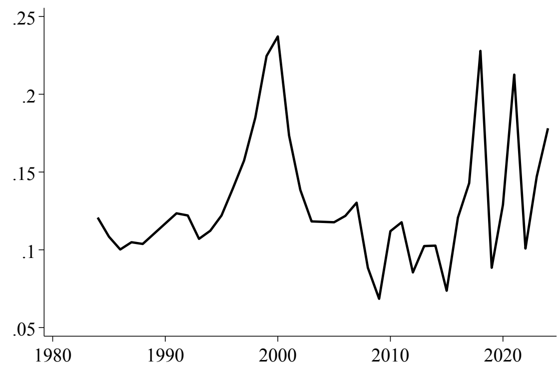
$$a_\pi(n) = a_\pi(n-1) - \log R_f - \bar{q} + \frac{1}{2}\left(\left(\frac{1-\rho_q^{n-1}}{1-\rho_q}\right)^2|\sigma_q|^2 + |\sigma_\pi|^2\right) + \frac{1-\rho_q^{n-1}}{1-\rho_q}(\sigma_{q\pi}+\sigma_{Cq}) + \gamma\sigma_{C\pi} + \log(1+p((1-\lambda\eta)(1-\eta)^{-\gamma}-1)) - \log(1+p((1-\eta)^{-\gamma}-1)).$$

D.4 Calibration

Table D.1 provides additional detail on our calibration. As Section 3.7 explains, we use I/B/E/S forecasts to proxy for expected dividend growth. Specifically, the de-meaned log of our constructed LTG variable is set to match $(1-\rho_z)(z_t - \bar{z})$. The long-run mean $\bar{z}$ is set to the average of the log of consumption divided by earnings. Expected inflation is calibrated to match median one-year-ahead inflation projections from the Survey of Professional

Figure D.2: Analysts expectations for long-term growth

This figure shows the aggregate long-term growth (LTG) measure constructed from I/B/E/S data. Analysts forecast 3–5 year annual growth rates for individual firms, which we aggregate to create an aggregate market forecast for long-term earnings growth.



Forecasters. The persistence and long-run mean of expected inflation are estimated using ordinary least squares on the relationship in Equation (19).

We estimate the variance-covariance matrix of the shocks using the residuals from (16–19), where realized is consumption is taken from real personal consumption expenditures from the Federal Reserve Economic Data (FRED) and where realized inflation is from FRED series CPIAUCSL. The correlation of realized consumption and inflation is allowed to vary between the first and second sample, whereas all other correlations and the volatility remains the same throughout the sample period. We assume no disasters realize in-sample. Note that while μ, for example, appears as a constant in (17), it varies between the first and second sample period.

E International evidence

This appendix applies our model to countries other than the United States. We start by understanding whether a rise in the probability of disaster is an attractive explanation for

Table D.1: Parameters for generalized endowment economy

This table shows parameters for the generalized endowment model which includes rare disasters and inflationary default (see also Table 3). Standard deviations and correlations refer to second moments of shocks. q represents expected inflation outside of disasters. Parameters are in annual terms.

Parameter	Value	Source
Preference parameters		
Risk aversion γ	5	Calibrated
EIS ψ	1	Calibrated
Consumption process		
Disaster probability p	0.0210	Calibrated
Disaster magnitude η	0.30	Calibrated
Consumption shock volatility σ_C	0.016	FRED: A794RX0Q048SBEA
Dividend claim process		
Average consumption-dividend ratio $\bar{z}$	5.072	FRED: PCE and Shiller
Persistence of consumption-dividend ratio ρ_z	0.850	Calibrated
Consumption-dividend ratio shock volatility σ_z	0.261	I/B/E/S
Leverage ξ	2	Calibrated
Inflation process		
Persistence of expected inflation ρ_q	0.942	Survey of Professional Forecasters
Expected inflation shock volatility σ_q	0.003	Survey of Professional Forecasters
Realized inflation shock volatility σ_π	0.014	FRED: CPIAUCSL
Correlations		
Corr(π, q)	0.373	Various
Corr(C, z)	0.307	Various
Corr(C, q)	0.114	Various

the decline in government bond yields in these other countries with results presented in Table E.2. Note that we do not have survey expectations data for these countries, so instead of the ex ante inflation-adjusted yield $\bar{y}_b$ from (14), we calibrate to the ex post inflation-adjusted yield, defined in (15).

In all three calibrations, we see that both the discount factor β and the probability of disaster must rise substantially in order to match the data. The discount factor rises by 1.8 pp in the United Kingdom, 0.5 pp in Japan, and 1.9 pp in the other countries. Similarly, the probability of disaster rises from 1.3% to 5.1% in the United Kingdom, 0.6% to 2.4% in Japan, and 0.7% to 3.2% for the other countries. This is consistent with the results in the United States.

Next, we seek to understand whether the model with disasters and inflationary default can match the evidence from abroad. Here, we find quite similar results to those in the United States. The results are presented in Table E.3. Again, crucially, we see that substantially smaller rises in the discount factor β are needed to match the data. Instead, small declines in the risk of inflationary default are sufficient. Again, this is quite consistent with the results in the United States.

Table E.2: Disaster model calibrated to international data

This table shows parameters necessary to match the data, assuming an endowment economy with rare disasters. We take average consumption growth from the data in each sample. We calibrate the discount factor β and the probability of disaster p to match the average price-dividend ratio and the average inflation-adjusted Treasury bill. We do this for the United Kingdom, Japan, and for countries other than Japan, United Kingdom, and the U.S. that are present in the Jorda-Schularick-Taylor Macrohistory Database, using a population-weighted average of the price-dividend ratio, interest rate, and GDP growth rate. The countries are Australia, Belgium, Canada, Germany, Denmark, Spain, Finland, France, Ireland, Italy, the Netherlands, Norway, Portugal, and Sweden. Parameters and yields are in annual terms.

		Values	
	Parameter	1984–2000	2001–2020
Panel A: United Kingdom calibration			
Data:			
Price-dividend ratio	κ	27.78	29.46
Ex post inflation-adjusted Treasury yield	$\hat{y}_b$	0.0525	0.0028
Average consumption growth	μ	0.0303	0.0114
Model:			
Discount factor	β	0.954	0.972
Probability of disaster	p	0.0134	0.0510
Panel B: Japan calibration			
Data:			
Price-dividend ratio	κ	138.26	62.80
Ex post inflation-adjusted Treasury yield	$\hat{y}_b$	0.0268	-0.0006
Average consumption growth	μ	0.0252	0.0056
Model:			
Discount factor	β	0.982	0.987
Probability of disaster	p	0.0055	0.0235
Panel C: All other countries calibration			
Data:			
Price-dividend ratio	κ	42.56	51.22
Ex post inflation-adjusted Treasury yield	$\hat{y}_b$	0.0462	0.0008
Average consumption growth	μ	0.0303	0.0099
Model:			
Discount factor	β	0.964	0.983
Probability of disaster	p	0.0073	0.0316

Table E.3: Inflationary default model calibrated to international data

This table shows parameters necessary to match the data, assuming an endowment economy with rare disasters and inflationary default. We take average consumption growth from the data in each sample. We calibrate the discount factor β and the decline in bond value $\lambda\eta$ to match the average price-dividend ratio and the average inflation-adjusted Treasury bill, assuming no disasters in-sample. We assume the disaster probability equals 2.10%. We do this for the United Kingdom, Japan, and for countries other than Japan, United Kingdom, and the U.S. that are present in the Jorda-Schularick-Taylor Macrohistory Database, using a population-weighted average of the price-dividend ratio, interest rate, and GDP growth rate. The countries are Australia, Belgium, Canada, Germany, Denmark, Spain, Finland, France, Ireland, Italy, the Netherlands, Norway, Portugal, and Sweden. Parameters and yields are in annual terms.

		Values	
	Parameter	1984–2000	2001–2020
Panel A: United Kingdom calibration			
Data:			
Price-dividend ratio	κ	27.78	29.46
Ex post inflation-adjusted Treasury yield	$\hat{y}_b$	0.0525	0.0028
Average consumption growth	μ	0.0303	0.0114
Model:			
Discount factor	β	0.958	0.969
Fraction of bond value lost	$\lambda\eta$	0.187	-0.065
Panel B: Japan calibration			
Data:			
Price-dividend ratio	κ	138.26	62.80
Ex post inflation-adjusted Treasury yield	$\hat{y}_b$	0.0268	-0.0006
Average consumption growth	μ	0.0252	0.0056
Model:			
Discount factor	β	0.988	0.990
Fraction of bond value lost	$\lambda\eta$	0.252	0.111
Panel C: All other countries calibration			
Data:			
Price-dividend ratio	κ	42.56	51.22
Ex post inflation-adjusted Treasury yield	$\hat{y}_b$	0.0462	0.0008
Average consumption growth	μ	0.0303	0.0099
Model:			
Discount factor	β	0.970	0.984
Fraction of bond value lost	$\lambda\eta$	0.237	0.055

F Government debt and inflationary default risk

Another question that arises from our model is how it can be possible that sovereign default risk could go down when government debt-to-GDP has risen so much over the same period. To answer this question, we use the fiscal theory of the price level (Cochrane, 2023) to endogenize the inflation process in terms of fiscal policy. This allows us to show how the risk of sovereign default (the parameter λ) and the present value of government debt depend on beliefs about future surpluses.

Suppose the government runs surpluses $S_t = T_t - G_t$ and issues one-period debt with total nominal value $Q_t^{\$}B_t$. The government budget constraint is

$$B_{t-1} = \Pi_t S_t + Q_t^{\$} B_t. \tag{F.1}$$

The left-hand side is the face value of bonds owed to investors (each bond issued at $t-1$ promises \$1 at time t). The government fulfills this obligation by a combination of paying off the debt (the term $\Pi_t S_t$, representing the nominal value of the surplus) and issuing new debt B_t at price $Q_t^{\$}$. Iterating this constraint forward and taking expectations implies the present-value relation

$$\frac{B_{t-1}}{\Pi_t} = \mathbb{E}_t \left[\sum_{j=0}^{\infty} M_{t+j} S_{t+j} \right]. \tag{F.2}$$

The price level Π_t, and hence the inflation process, is determined by this equation.

For convenience, let us write the surplus as the product of current consumption and the surplus-consumption ratio,

$$S_t = s_t C_t. \tag{F.3}$$

To highlight the importance of disaster default in our model, we assume that the surplus-consumption ratio follows a three-state process that depends on the occurrence of a disaster:

$$s_t = \begin{cases} \bar{s} & \text{if} \quad \chi_{t-1} = \chi_t = 0, \\ \bar{s} - s^- & \text{if} \quad \chi_{t-1} = 0 \quad \text{and} \quad \chi_t = \eta, \\ \bar{s} + s^+ & \text{if} \quad \chi_{t-1} = \eta. \end{cases} \tag{F.4}$$

In non-disaster times, the government raises a constant fraction $\bar{s}$ of the endowment as surpluses. When a disaster occurs, the government runs a deficit s^-, but commits to repaying that deficit at a rate s^+ thereafter. The higher the disaster-contingent repayment rate s^+, the less the government will need to default by inflation in a disaster.

Let us now define the debt-consumption (i.e., debt-to-GDP) ratio

$$b_t \equiv \frac{B_{t-1}}{\Pi_t C_t}.$$

We can rewrite the valuation equation (F.2) in terms of this ratio recursively as

$$b_t = s_t + \mathbb{E}_t \left[M_{t+1} \frac{C_{t+1}}{C_t} b_{t+1} \right]. \tag{F.5}$$

Assuming the agent has risk aversion γ and a unit EIS, we can next substitute in the stochastic discount factor (SDF), and this relation becomes

$$b_t = s_t + \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{1-\gamma} \right]^{-1} \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{1-\gamma} b_{t+1} \right]. \tag{F.6}$$

Finally, substituting in the endowment process,

$$b_t = s_t + \beta \mathbb{E}_t \left[\frac{(1-\chi_{t+1})^{1-\gamma}}{(1-p) + p(1-\eta)^{1-\gamma}} b_{t+1} \right]. \tag{F.7}$$

Now, because $s_t = s(\chi_{t-1}, \chi_t)$, we have a solution of the form $b_t = b(\chi_{t-1}, \chi_t)$, which solves the system of equations

$$b(\chi_{t-1}, \chi_t) = s(\chi_{t-1}, \chi_t) + \beta \mathbb{E}_t[\tilde{p} b(\chi_t, \eta) + (1 - \tilde{p}) b(\chi_t, 0)], \tag{F.8}$$

where

$$\tilde{p} \equiv \frac{p(1-\eta)^{1-\gamma}}{(1-p) + p(1-\eta)^{1-\gamma}} \tag{F.9}$$

is a risk-adjusted disaster probability (equal to the physical probability p if $\gamma = 1$). This is a linear system of four equations in four unknowns $b(\chi_{t-1}, \chi_t)$, the solution to which is

$$\begin{bmatrix} b(\eta,0) \\ b(0,\eta) \\ b(\eta,\eta) \\ b(0,0) \end{bmatrix} = \begin{bmatrix} 1 & -\beta\tilde{p} & 0 & -\beta(1-\tilde{p}) \\ -\beta(1-\tilde{p}) & 1 & -\beta\tilde{p} & 0 \\ -\beta(1-\tilde{p}) & 0 & 1-\beta\tilde{p} & 0 \\ 0 & -\beta\tilde{p} & 0 & 1-\beta(1-\tilde{p}) \end{bmatrix}^{-1} \begin{bmatrix} \bar{s} + s^+ \\ \bar{s} - s^- \\ \bar{s} + s^+ \\ \bar{s} \end{bmatrix}. \tag{F.10}$$

Computing the matrix inverse and multiplying by the surplus vector, we then get the explicit solution

$$\begin{bmatrix} b(\eta,0) \\ b(0,\eta) \\ b(\eta,\eta) \\ b(0,0) \end{bmatrix} = \frac{1}{1-\beta}\bar{s} + \begin{bmatrix} \beta^2\tilde{p} - \beta + 1 \\ \beta(\beta\tilde{p} - \beta + 1) \\ \beta^2\tilde{p} - \beta^2 + 1 \\ \beta^2\tilde{p} \end{bmatrix} \frac{1}{1-\beta} s^+ - \begin{bmatrix} \beta\tilde{p}(1-\beta\tilde{p}) \\ \beta^2\tilde{p}(1-\tilde{p}) + 1 - \beta \\ \beta^2\tilde{p}(1-\tilde{p}) \\ \beta\tilde{p}(1-\beta\tilde{p}) \end{bmatrix} \frac{1}{1-\beta} s^-. \tag{F.11}$$

Now let us use this solution to determine the equilibrium inflation process. Suppose there have been no recent disasters, so $b_t = b(0,0)$. In this case, we have

$$b(0,0) = \frac{1}{1-\beta}\bar{s} + \frac{\beta^2\tilde{p}}{1-\beta} s^+ - \frac{\beta\tilde{p}(1-\beta\tilde{p})}{1-\beta} s^-. \tag{F.12}$$

Next period's debt-to-consumption will either remain at $b(0,0)$ or become

$$b(0,\eta) = \frac{1}{1-\beta}\bar{s} + \frac{\beta^2\tilde{p} + \beta(1-\beta)}{1-\beta}s^+ - \frac{(1-\beta(1-\tilde{p}))(1-\beta\tilde{p})}{1-\beta}s^- \tag{F.13}$$

if there is a disaster. Thus, we can write

$$b(0,\eta) = b(0,0) + \beta s^+ - (1-\beta\tilde{p})s^-, \tag{F.14}$$

or, in terms of the disaster shock,

$$b_{t+1} = b(0,0) - \left[(1-\beta\tilde{p})s^- - \beta s^+\right]\frac{\chi_{t+1}}{\eta}. \tag{F.15}$$

To solve for the inflation rate, note that the flow budget constraint can be rewritten

$$Q_t^{\$}\frac{\Pi_{t+1}}{\Pi_t} = \left(\frac{C_{t+1}}{C_t}\right)^{-1}\frac{b_t - s_t}{b_{t+1}}. \tag{F.16}$$

Now conjecture that there exists a constant λ and a deterministic process $\mu_{\pi t}$ such that

$$\frac{\Pi_{t+1}}{\Pi_t} = e^{\mu_{\pi t}}(1-\lambda\chi_{t+1})^{-1}. \tag{F.17}$$

Substituting this conjecture in (note that $Q_t^{\$} = e^{-\mu_{\pi t} - \bar{y}_b}$) and collecting disaster terms implies

$$1 - \lambda\chi_{t+1} = (1-\chi_{t+1})\left(1 - \frac{(1-\beta\tilde{p})s^- - \beta s^+}{\eta b(0,0)}\chi_{t+1}\right). \tag{F.18}$$

Expanding the right-hand side and using the fact that $\chi_{t+1}^2 = \eta\chi_{t+1}$,[35] this implies

$$1 - \lambda\chi_{t+1} = 1 - \left(1 - \left(\frac{1-\eta}{\eta}\right)\left(\frac{\beta s^+ - (1-\beta\tilde{p})s^-}{b(0,0)}\right)\right)\chi_{t+1}, \tag{F.19}$$

[35]Recall that χ_{t+1} equals either 0 or η. If $\chi_{t+1} = 0$, then $\chi_{t+1}^2 = 0$. If $\chi_{t+1} = \eta$, then $\chi_{t+1}^2 = \eta^2 = \eta\chi_{t+1}$. Thus, $\chi_{t+1}^2 = \eta\chi_{t+1}$ for both possible outcomes.

or, equivalently, that

$$\lambda = 1 - \left(\frac{1-\eta}{\eta}\right)\left(\frac{\beta s^{+} - (1-\beta\tilde{p})s^{-}}{b(0,0)}\right). \tag{F.20}$$

Finally, noting that (F.12) implies

$$\beta s^{+} - (1-\beta\tilde{p})s^{-} = \frac{(1-\beta)b(0,0) - \bar{s}}{\beta\tilde{p}}, \tag{F.21}$$

we can write λ in terms of the debt-to-GDP ratio $b(0,0)$:

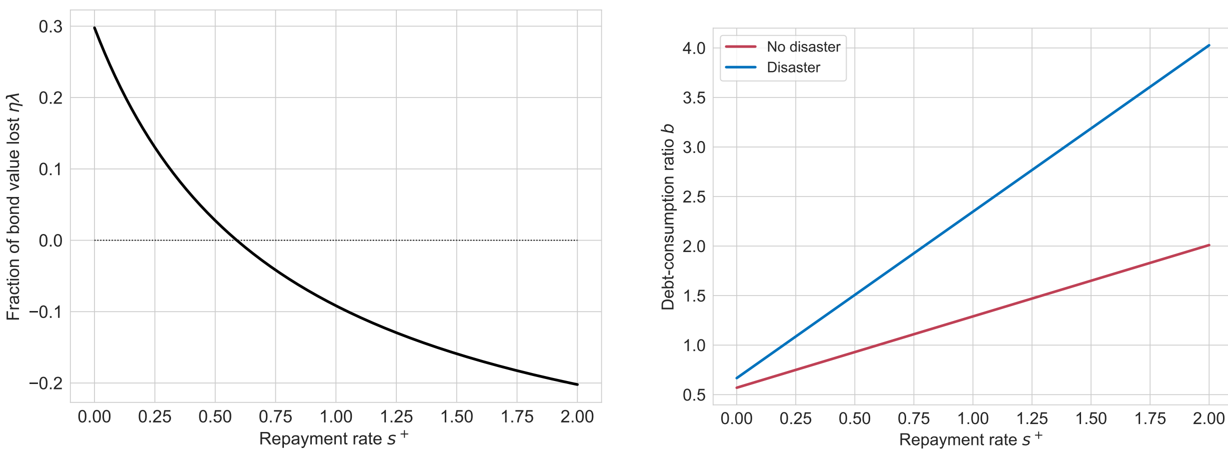
$$\lambda = \left(1 - \left(\frac{1-\eta}{\eta}\right)\frac{1-\beta}{\beta\tilde{p}}\right) + \left(\frac{1-\eta}{\eta}\right)\frac{1}{\beta\tilde{p}}\frac{\bar{s}}{b(0,0)}. \tag{F.22}$$

For any set of parameters, the default size λ is inversely related to the debt-to-GDP ratio. Intuitively, this is because the government's debt is more valuable when investors believe that the risk of default is lower.

All else equal, the debt-to-GDP ratio b is decreasing in the size of the disaster deficit s^{-}, but increasing in the repayment rate s^{+}. By (F.22), the magnitude of the inflation in a disaster state, λ, is therefore increasing in s^{-} and decreasing in s^{+}. Figure F.3 plots a comparative static for the equilibrium default size $\eta\lambda$ and the debt-to-GDP ratio b_t as a function of the expected repayment rate s^{+}. Panel A shows that, as expected repayment s^{+} increases, $\eta\lambda$ falls. This suggests one possible explanation for the decline in λ we estimate in the data: growing confidence that the government will pay back investors should a disaster occur. Importantly, what matters for this fiscal-theoretic view of the price level is *beliefs* about future surpluses. In particular, what matters here is not so much the normal-times surplus rate $\bar{s}$, but beliefs about the surplus rate s^{+} following disasters. Data on beliefs about disaster-contingent repayment are not available to directly test this hypothesis, but Jiang et al. (2024a) do present evidence that expectations of future surpluses in the twenty-first century were high, and indeed much higher than the surpluses that actually materialized, consistent with this story.

Figure F.3: Government debt and disaster inflation in the fiscal theory

The figure shows comparative statics with respect to the post-disaster repayment rate s^+ from the surplus process (F.4). Both panels hold fixed all other model parameters, including the other surplus parameters $\bar{s}$ and s^-. Panel A shows the fraction of nominal bond value lost in a disaster, $\lambda\eta$, as a function of this parameter. Panel B shows the debt-consumption ratio in the state without any disasters ($b(0,0)$, the red line) and during a disaster ($b(0,\eta)$, the blue line).



To address the question of how high debt levels could co-occur with low default risk $\eta\lambda$, Panel B plots debt-to-GDP both in the non-disaster ($b(0,0)$) and disaster ($b(0,\eta)$) states as a function s^+. In both states, the value of debt-to-GDP is increasing in the expected repayment rate for two reasons. First is a cashflow effect, whereby higher repayment means higher surpluses on average. Second is a risk premium effect, whereby higher repayment means a larger increase in the value of government debt in the disaster state (the blue line), rendering government debt a hedge against disaster risk and increasing its value ex ante. Notably, the increased fiscal capacity from high s^+ could allow the government some flexibility to reduce its normal-times surplus rate $\bar{s}$, which could also help explain how debt continued to rise amid unexpectedly low surpluses.

Figure F.4: Government debt level and returns in the data

The figure shows the time series of total real returns on government debt and the government debt-to-GDP ratio from 1984 to 2016. The solid green line (left axis) is the five-year-ahead realized real return on the portfolio of all U.S. Treasuries. This blended return series is computed as in Jiang et al. (2024a). The dashed purple line (right axis) is the ratio of total debt to GDP in the U.S.

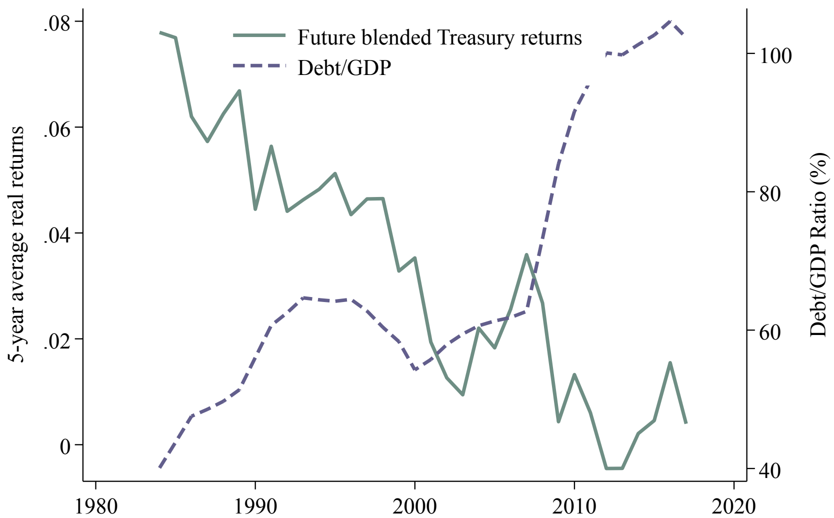


Figure F.4 plots the trends in government debt-to-GDP (the right axis) and real returns on government debt (the left axis) over our sample period. For returns, we compute the total real return on all Treasuries following Jiang et al. (2024a) and then average those returns over the next five years. Clearly, as debt-to-GDP has increased, real returns on debt have fallen, suggesting a decline in the overall discount rate (including the default risk premium). Importantly, as Jiang et al. (2024a) show, the decline in the discount rate alone is not enough to explain the rise in debt, and so it remains a puzzle that debt has risen amid persistent deficits. As we discuss in Section 3.5, our main point in doing this analysis is to say that rising debt and declining risk of default are not in contradiction. It is not to say that lower inflation risk alone explains the rise in debt. The stylized model above assumes a simple, stationary model of surpluses and imposes rational expectations; in reality, investors' beliefs about the surplus process may be non-stationary and may not accord with the data-generating process, and these deviations are likely to be important for explaining the rise in debt along with the decline in default risk.

The only substantive assumptions on the surplus process that we require is that, in the event of a consumption disaster, news is revealed about the path of future surpluses. If this news is negative (namely unexpected deficits that will not be repaid), then default (inflation) will occur if and when a disaster occurs. Aside from this assumption, the surplus process can have arbitrary, even non-stationary, dynamics outside of disasters, and our main point will still stand.

G Production model

G.1 Solution to the no-inventory case

Consider the model in Section 4.1. The agent maximizes (28), subject to (27). Conjecture that

$$V(W_t) = \nu W_t, \tag{G.1}$$

for some constant $\nu > 0$. Substituting this conjecture into (28), with $R_{W,t+1} \equiv R_{f,t+1} + \alpha_t(R_{K,t+1} - R_{f,t+1})$ implies

$$(1-\beta)\log\nu + \log W_t = \max_{C_t,\alpha_t}\left\{(1-\beta)\log C_t + \beta\log\left(W_t - C_t\right) + \frac{\beta}{1-\gamma}\log\left(\mathbb{E}_t\left[R_{W,t+1}^{1-\gamma}\right]\right)\right\}. \tag{G.2}$$

At the optimum, the derivative of the right-hand side with respect to C_t equals zero. Thus:

$$\frac{1-\beta}{C_t} - \frac{\beta}{W_t - C_t} = 0$$

yielding the result $C_t/W_t = 1-\beta$. Moreover, setting the derivative of the right hand side with respect to α equal to zero yields

$$\mathbb{E}_t\left[R_{W,t+1}^{1-\gamma}\right]^{-1}\mathbb{E}_t\left[R_{W,t+1}^{-\gamma}(R_{K,t+1}-R_{f,t+1})\right]=0. \tag{G.3}$$

From these conditions, we can derive growth and asset prices on a balanced-growth path (i.e., when consumption, investment, and output grow at the same rate). First, note that wealth grows at rate:

$$\frac{W_{t+1}}{W_t}=\frac{W_t-C_t}{W_t}\frac{W_{t+1}}{W_t-C_t}=\beta R_{K,t+1}. \tag{G.4}$$

(We have used the constant consumption-wealth ratio and the equilibrium condition $\alpha = 1$.) This must also be the growth rate of consumption. Substituting in for $R_{K,t+1}$ implies

$$\frac{C_{t+1}}{C_t}=\beta(1-\delta+A)(1-\chi_{t+1}). \tag{G.5}$$

This is then also the growth rate of planned capital, lagged one period. In equilibrium, all investment is in planned capital, so $\tilde{K}_{t+1}/\tilde{K}_t = W_t/W_{t-1}$. From (G.4), then, it follows that[36]

$$\frac{K_{t+1}}{K_t}=\frac{\tilde{K}_{t+1}}{\tilde{K}_t}\frac{1-\chi_{t+1}}{1-\chi_t}=\beta R_{K,t}\frac{1-\chi_{t+1}}{1-\chi_t}=\beta(1-\delta+A)(1-\chi_{t+1}).$$

The result for output then follows from $Y_t = AK_t$ and the result for investment follows from (29). As a consequence, the price-dividend ratio κ^Y on the claim to output equals the price-dividend ratio κ on the consumption claim: $\kappa^Y = \kappa = \beta/(1-\beta)$.

We now turn to the implications of this model for the interest rate and for stock returns. The equilibrium condition $\alpha = 1$ implies $R_{W,t+1} = R_{K,t+1}$. Substituting $R_{K,t+1}$ into (G.3)

[36]Here we have used the fact that planned capital $\tilde{K}_{t+1}$ is a constant fraction of wealth W_t (in this model, this fraction is one), so that $\tilde{K}_{t+1}/\tilde{K}_t = W_t/W_{t-1} = \beta R_{K,t}$.

implies a value for the log riskfree rate:

$$\begin{aligned}\log R_f &= \log \mathbb{E}_t \left[R_{K,t+1}^{1-\gamma}\right] - \log \mathbb{E}_t \left[R_{K,t+1}^{-\gamma}\right] \\ &= \log(1-\delta+A) + \log(1+p((1-\eta)^{1-\gamma}-1)) - \log(1+p((1-\eta)^{-\gamma}-1)) \\ &\approx A - \delta + p((1-\eta)^{1-\gamma} - (1-\eta)^{-\gamma}).\end{aligned} \tag{G.6}$$

Equations (G.3) and (G.6) imply the following expression for the SDF:

$$M_{t+1} = \mathbb{E}_t \left[R_{W,t+1}^{1-\gamma}\right]^{-1} R_{W,t+1}^{-\gamma}. \tag{G.7}$$

Furthermore, the risk premium equals

$$\begin{aligned}\log \mathbb{E}_t[R_{K,t+1}] - \log R_f = \log(1-p\eta) \\ + \log(1+p((1-\eta)^{-\gamma}-1)) - \log(1+p((1-\eta)^{1-\gamma}-1)),\end{aligned} \tag{G.8}$$

exactly as in the endowment economy.

Note that, as discussed in the main text, asset prices are identical to those in the endowment economy if $\beta^{-1}e^{\mu} = (1-\delta+A)$. The key difference between the models, however, is that there are two margins of adjustment in the production economy: quantities and prices. This is why, for example, the patience parameter β does not show up in (G.6). Instead, β influences quantities through the investment-capital ratio, which in turn affects prices. In the standard endowment economy, quantities cannot adjust, as the representative investor consumes whatever is produced in a given period.

G.2 Solution to the general case

The agent can invest in an inventory asset with net return $r_I = 0$, a riskfree bond with net return $r_{f,t+1}$, and a risky capital asset with net return $r_{K,t+1}$. Let $r_{j,t+1}$, $j \in \mathcal{J} =$

$\{I, f, K\}$, represent net returns, and let $\alpha_{j,t}$ denote the percent allocation of savings to asset j. Note that, in our setting with a binary shock χ_{t+1}, markets are complete, so the agent will be able to construct any state-contingent portfolio return $r_{i,t+1}$. Inventory and capital are the only securities in positive net supply; furthermore, we restrict inventory to be in non-negative supply ($I_t \geq 0$). It follows from this setup that the return on wealth $R_{W,t+1} = \sum_{j\in\mathcal{J}} \alpha_{j,t}(1 + r_{j,t+1})$, where $\sum_{j\in\mathcal{J}} \alpha_{j,t} = 1$.

Suppose that the agent has Epstein-Zin utility with unit EIS. The agent's optimization problem is therefore

$$\max_{C_t, \{\alpha_{j,t}\}_{j\in\mathcal{J}}} \left(C_t^{1-\beta} \left(\mathbb{E}_t \left[V(W_{t+1})^{1-\gamma} \right] \right)^{\frac{\beta}{1-\gamma}} \right), \tag{G.9}$$

subject to the dynamic budget constraint

$$W_{t+1} = (W_t - C_t)R_{W,t+1} = (W_t - C_t) \sum_{j\in\mathcal{J}} \alpha_{j,t}(1 + r_{j,t+1}), \tag{G.10}$$

the portfolio weight restriction

$$\sum_{j\in\mathcal{J}} \alpha_{j,t} = 1, \tag{G.11}$$

and the inventory non-negativity constraint

$$\alpha_{I,t} \geq 0. \tag{G.12}$$

Let ζ_t and ξ_t denote the Lagrange multipliers on the constraints (G.11) and (G.12), respectively.

Substituting (G.1) and the budget constraint (G.10) into (G.9), then taking logs, we again obtain (G.2) and the identical first-order condition for consumption as above. The first-order condition with respect to asset allocation $\alpha_{j,t}$, $j \neq I$, is

$$\beta \mathbb{E}_t \left[R_{W,t+1}^{1-\gamma} \right]^{-1} \mathbb{E}_t \left[R_{W,t+1}^{-\gamma} (1 + r_{j,t+1}) \right] = \zeta_t, \tag{G.13}$$

and the first-order condition with respect to the inventory allocation $\alpha_{I,t}$ is

$$\beta \mathbb{E}_t \left[R_{W,t+1}^{1-\gamma} \right]^{-1} \mathbb{E}_t \left[R_{W,t+1}^{-\gamma} \right] + \xi_t = \zeta_t. \tag{G.14}$$

Multiply both sides of (G.13) by $\alpha_{j,t}$, take the sum over $j \in \mathcal{J} \setminus \{I\}$, and substitute in (G.14) to see that

$$\zeta_t = \beta + \xi_t \alpha_{I,t} = \beta, \tag{G.15}$$

by complementary slackness. This implies the Euler equation for gross returns

$$\mathbb{E}_t \left[R_{W,t+1}^{1-\gamma} \right]^{-1} \mathbb{E}_t \left[R_{W,t+1}^{-\gamma} R_{j,t+1} \right] = 1 \tag{G.16}$$

and the Euler equation for inventory

$$\mathbb{E}_t \left[R_{W,t+1}^{1-\gamma} \right]^{-1} \mathbb{E}_t \left[R_{W,t+1}^{-\gamma} \right] + \frac{\xi_t}{\beta} = 1. \tag{G.17}$$

Note the market clearing condition $\alpha_{I,t} = 1 - \alpha_{K,t}$, where $\alpha_{K,t}$ is simply denoted α_t in our setup in the main text. We thus have that $\xi_t > 0$ if and only if $\alpha_t < 1$.

We now show formally that inventory imposes a zero lower bound. Throughout, we assume that the bond is in zero net supply.

Lemma 1. *If $\alpha_t < 1$, then the gross real riskfree rate $R_{f,t+1} = 1$. If $\alpha_t = 1$, then $R_{f,t+1} \geq 1$ and is equal to the real riskfree rate in a no-inventory economy $R^*_{f,t+1}$.*

Proof. If $\alpha_{I,t} > 0$, then $\xi_t = 0$ and (G.16) and (G.14) combine to give us $R_{f,t+1} = 1$. If $\alpha_{I,t} = 0$, then $\xi_t \geq 0$ and

$$R_{f,t+1} = \frac{\beta}{\beta - \xi_t}, \tag{G.18}$$

which is greater than or equal to 1. Moreover, if $\alpha_{I,t} = 0$, then market clearing implies

$R_{W,t+1} = R_{K,t+1}$ and the Euler equation (G.16) yields

$$R_{f,t+1} = \mathbb{E}_t \left[R_{K,t+1}^{1-\gamma}\right] \mathbb{E}_t \left[R_{K,t+1}^{-\gamma}\right]^{-1}, \tag{G.19}$$

which is the same as the riskfree rate $R_{f,t+1}^*$ in the no-inventory economy. ■

We next show that the unconstrained riskfree rate determines α.

Theorem 1. *If the unconstrained gross riskfree rate $R_{f,t+1}^* < 1$, then $\alpha_t < 1$ and the constrained riskfree rate $R_{f,t+1} = 1$. If $R_{f,t+1}^* \geq 1$, then $\alpha_t = 1$ and the equilibrium is as in a standard no-inventory production economy with $R_{f,t+1} = R_{f,t+1}^*$.*

Proof. We will prove the theorem by contradiction using Lemma 1.

Suppose $R_{f,t+1}^* < 1$ and $\alpha_{I,t} = 0$. Then $R_{f,t+1} = R_{f,t+1}^* < 1$, which contradicts Lemma 1. It must therefore be the case that $R_{f,t+1}^* < 1$ implies $\alpha_{I,t} > 0$, which implies $R_{f,t+1} = 1$.

Now suppose $R_{f,t+1}^* > 1$ and $\alpha_{I,t} > 0$. Then $R_{f,t+1} = 1 < R_{f,t+1}^*$, which contradicts Lemma 1. Moreover, in the knife-edge case $R_{f,t+1}^* = 1$, the equilibrium conditions (G.16) and (G.14) imply $\xi_t = 0$, which implies that $\alpha_{I,t} = 0$ and $R_{f,t+1} = R_{f,t+1}^* = 1$. Thus, it must be that $R_{f,t+1}^* \geq 1$ implies $\alpha_{I,t} = 0$, which implies $R_{f,t+1} = R_{f,t+1}^* \geq 1$. ■

We conjecture that the price-dividend ratio depends only on the current state χ_t (i.e., whether the disaster occurred or not). The intuition for this is that output growth Y_{t+1}/Y_t is a function of χ_t only. Thus,

$$1 = \mathbb{E}_t \left[R_{W,t+1}^{1-\gamma}\right]^{-1} \mathbb{E}_t \left[R_{W,t+1}^{-\gamma} \left(\frac{\kappa^Y(\chi_{t+1}) + 1}{\kappa^Y(\chi_t)}\right) \left(\frac{Y_{t+1}}{Y_t}\right)\right]. \tag{G.20}$$

This implies that we have two equations, one for the non-disaster state,

$$\kappa^Y(0) = \hat{\beta}\Big[(1-p)(1+\alpha r_{K,0})^{1-\gamma}(\kappa^Y(0)+1) \\ + p(1+\alpha r_{K,\eta})^{-\gamma}(\kappa^Y(\eta)+1)(1-\eta)(1+\alpha r_{K,0})\Big], \quad \text{(G.21)}$$

and one for the disaster state,

$$\kappa^Y(\eta) = \hat{\beta}\Big[(1-p)(1+\alpha r_{K,0})^{-\gamma}(\kappa^Y(0)+1)(1-\eta)^{-1}(1+\alpha r_{K,\eta}) \\ + p(1+\alpha r_{K,\eta})^{1-\gamma}(\kappa^Y(\eta)+1)\Big]. \quad \text{(G.22)}$$

In these equations, $\hat{\beta} \equiv \beta\Big[(1-p)(1+\alpha r_{K,0})^{1-\gamma} + p(1+\alpha r_{K,\eta})^{1-\gamma}\Big]^{-1}$, $r_{K,0} \equiv (1-\delta+A)-1$, and $r_{K,\eta} \equiv (1-\delta+A)(1-\eta)-1$. The solution to this system is as stated in the main text (after defining the weights ν).

Although the price-dividend ratio is state-dependent when the agent chooses to hold inventory, the risk premium is not. The risk premium at time t when the agent holds inventory is given by $\log \mathbb{E}_t[R^Y_{t+1}] - \log R_f$, for the expected return on the output claim

$$\mathbb{E}_t[R_{Y,t+1}] = \mathbb{E}_t\left[\left(\frac{\kappa^Y(\chi_{t+1})+1}{\kappa^Y(\chi_t)}\right)\left(\frac{Y_{t+1}}{Y_t}\right)\right]. \quad \text{(G.23)}$$

If the expected return on the output claim is the same across states, then so is the risk premium. In the no-disaster state, the expected return on the output claim is

$$\mathbb{E}_t[R_{Y,t+1}|\chi_t = 0] = \left(\frac{(1-p)\kappa^Y(0) + p\kappa^Y(\eta)+1}{\kappa^Y(0)}\right)\times \\ \Big(\beta(1-p\eta)\left(\alpha(1-\delta+A)+1-\alpha\right)\Big) \quad \text{(G.24)}$$

and in the disaster state by

$$\mathbb{E}_t[R_{Y,t+1}|\chi_t = \eta] = \left(\frac{(1-p)\kappa^Y(0) + p\kappa^Y(\eta) + 1}{\kappa^Y(\eta)}\right)\times \left(\beta(1-p\eta)\left(\alpha(1-\delta+A) + \left(\frac{1-\alpha}{1-\eta}\right)\right)\right). \quad \text{(G.25)}$$

Examining the two expressions, we see that the expected return in both states are the same if and only if

$$\kappa^Y(\eta)(1-\eta)\Big(\alpha(1-\delta+A)+1-\alpha\Big) = \kappa^Y(0)\Big(\alpha(1-\delta+A)(1-\eta)+1-\alpha\Big).$$

The terms inside the parentheses can be written so that

$$\kappa^Y(\eta)(1-\eta)(1+\alpha r_{K,0}) = \kappa^Y(0)(1+\alpha r_{K,\eta}),$$

which is true if we substitute in the expressions for $\kappa^Y(\chi_t)$. This implies that, while the price-dividend ratio is time-varying, the risk premium is not.

Finally, let us solve for the ex ante inflation-adjusted Treasury yield. The price of the one-period nominal bond in this economy is

$$Q_t^{\$} = \mathbb{E}_t\left[R_{W,t+1}^{1-\gamma}\right]^{-1}\mathbb{E}_t\Big[R_{W,t+1}^{-\gamma}(1-L_{t+1})\Big]e^{-q_t+\sigma_\pi^2/2},$$

which evaluates to

$$Q_t^{\$} = \frac{p(1+\alpha r_{K,\eta})^{-\gamma}(1-\lambda\eta) + (1-p)(1+\alpha r_{K,0})^{-\gamma}}{p(1+\alpha r_{K,\eta})^{1-\gamma} + (1-p)(1+\alpha r_{K,0})^{1-\gamma}}e^{-q_t+\sigma_\pi^2/2}.$$

Thus, the nominal yield is

$$y_{b,t}^{\$} = -\log Q_t^{\$} = \log(p(1+\alpha r_{K,\eta})^{1-\gamma} + (1-p)(1+\alpha r_{K,0})^{1-\gamma}) \\ - \log(p(1+\alpha r_{K,\eta})^{-\gamma}(1-\lambda\eta) + (1-p)(1+\alpha r_{K,0})^{-\gamma}) + q_t - \frac{1}{2}\sigma_\pi^2,$$

and, by (13), the ex ante inflation-adjusted Treasury yield is

$$\bar{y}_b = \log(p(1+\alpha r_{K,\eta})^{1-\gamma} + (1-p)(1+\alpha r_{K,0})^{1-\gamma}) \\ - \log(p(1+\alpha r_{K,\eta})^{-\gamma}(1-\lambda\eta) + (1-p)(1+\alpha r_{K,0})^{-\gamma}) - \sigma_\pi^2 - \log(1+p\lambda\eta(1-\lambda\eta)^{-1}). \tag{G.26}$$