#### Financial Vulnerability and Monetary Policy

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#### October 2017

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Introduction

#### Financial Vulnerability and Monetary Policy

Financial vulnerability: Amplification mechanisms in the financial sector

Two questions are hotly debated

- 1. Does monetary policy impact the degree of financial vulnerability?
- 2. Should monetary policy take financial vulnerability into account?

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Motivation

## Financial Variables Predict Tail of GDP Distribution

"Vulnerable Growth" by Adrian, Boyarchenko and Giannone (2016)



Motivation

#### Conditional Mean-Volatility Line for Output Gap Growth



Motivation

#### Conditional Mean-Volatility Relation for Inflation



# Overview of Microfounded Non-Linear Model

- Firms are exactly as in basic New Keynesian model
- Households are as in New Keynesian model but
  - Cannot finance firms directly
  - Trade other financial assets (stocks, riskless desposits) with banks
- Banks
  - Finance firms
  - Trade financial assets among themselves and with households
  - Less risk averse than household
  - Have a preference (risk aversion) shock
  - Subject to Value-at-Risk constraint
- Financial markets are complete but prices are distorted

# Price of Risk and No Arbitrage

- Single source of risk: Browninan motion  $Z_t$
- Real risk-free rate is  $R_t$
- A state price density (SPD) is a process with  $Q_0 \equiv 1$  and

$$\frac{dQ_t}{Q_t} \equiv -R_t dt - \eta_t dZ_t$$

such that for all assets j

$$S_{j,t} = \frac{1}{Q_t} \mathbb{E}_t \left[ \int_t^\infty Q_s D_{j,s} ds \right]$$

where  $\eta_t$  is the "market price of risk"

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# Firms are Standard New Keynesian

- Linear production for good *i*:  $Y_t(i) = N_t(i)$
- Monopolistically competitive on differentiated goods, Calvo pricing
- The FOC for intermediate good producers linearized around deterministic steady sate gives the standard New Keynesian Phillips Curve

$$d\pi_t = (\beta \pi_t - \kappa y_t) \, dt$$

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# The Intermediation Sector Setup

Each "bank" solves a standard Merton portfolio choice problem augmented by a Value-at-Risk constraint and preference shocks

$$V(X_t, t) = \max_{\{\theta_t, \delta_t\}} \mathbb{E}_t \left[ \int_t^\infty e^{-\beta(u-t)} e^{\zeta_u} \log(\delta_u X_u) du \right]$$
  
s.t.  
$$\frac{dX_t}{X_t} = (R_t - \delta_t + \theta_t \mu_t) dt + \theta_t \sigma_t dZ_t$$
  
$$VaR_{\tau,\alpha}(X_t) \leq a_V X_t$$
  
$$d\zeta_t = -\frac{1}{2} s_t^2 dt - s_t dZ_t$$
  
$$ds_t = -\kappa(s_t - \bar{s}) + \sigma_s dZ_t$$

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Intermediation Sector

#### The Intermediation Sector Setup

$$V(X_{t}, t) = \max_{\{\theta_{t}, \delta_{t}\}} \mathbb{E}_{t}^{bank} \left[ \int_{t}^{\infty} e^{-\beta(u-t)} \log(\delta_{u}X_{u}) du \right]$$
  
s.t.  
$$\frac{dX_{t}}{X_{t}} = (R_{t} - \delta_{t} + \theta_{t}\mu_{t} - \theta_{t}\sigma_{t}s_{t}) dt + \theta_{t}\sigma_{t}dZ_{t}^{s}$$
  
$$VaR_{\tau,\alpha}(X_{t}) \leq a_{V}X_{t}$$
  
$$ds_{t} = -\kappa(s_{t} - \bar{s}) + \sigma_{s}dZ_{t}$$

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# The Banks' VaR Constraint and Amplification

- Let  $\hat{X_t}$  be projected wealth with fixed portfolio weights from t to t+ au
- VaR<sub>τ,α</sub> (X<sub>t</sub>) is the α<sup>th</sup> quantile of the distribution of X̂<sub>t+τ</sub> conditional on time-t information



#### **Optimal Portfolio**

The optimal portfolio is characterized by

$$\theta_t = \frac{1}{\gamma_t} (\mu_t / \sigma_t^2 - s_t / \sigma_t)$$
  
$$\delta_t = u(\gamma_t) \beta$$

$$\gamma_t \in (1,\infty)$$
 such that:  $VaR_{ au,lpha}\left(X_t
ight)=X_ta_V$ 

or  $\gamma_t = 1$  otherwise

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#### Representative Household

Household solves

$$\max_{\{C_t, N_t, \omega_t\}_{t \ge s}} \mathbb{E}_s \left\{ \int_s^\infty e^{-\beta(t-s)} \left( \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varsigma}}{1+\varsigma} \right) dt \right\}$$

subject to

$$d(P_tF_t) \le W_t N_t dt - P_t C_t dt + \omega_t d(P_t S_t)$$
$$\omega_{goods,t} = 0$$

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# Euler Equation and Price of Risk

The household's Euler equation gives IS curve

$$dy_t = \frac{1}{\gamma} (i_t - r - \pi_t) dt + \frac{\eta_t}{\gamma} dZ_t$$

 Banks and households trading in complete markets means marginal utilities agree, which together with market clearing give

$$\eta_t = \eta(\gamma_t, V_t, s_t)$$

where

$$V_t = VaR_{\tau,\alpha}(dy_t)$$
  
=  $-\tau \mathbb{E}_t[dy_t/dt] - \mathcal{N}^{-1}(\alpha)\sqrt{\tau} Vol_t(dy_t/dt)$ 

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- Focus on simpler case with no direct impact of monetary policy on γ<sub>t</sub>: Mechanism is through general equilibrium (prices of risk) only
- Abstract from Phillips Curve (fixed prices)
- ► General case still linear-quadratic, can be solved in closed form

Central bank solves

$$L = \min_{\{i_s\}_{s=t}^{\infty}} \mathbb{E}_t \int_t^{\infty} e^{-s\beta} y_s^2 ds$$

#### subject to

$$dy_{t} = \frac{1}{\gamma} (i_{t} - r) dt + \frac{\eta(V_{t}, s_{t})}{\gamma} dZ_{t}$$
$$V_{t} = -\tau \mathbb{E}_{t} [dy_{t}/dt] - \mathcal{N}^{-1}(\alpha) \sqrt{\tau} Vol_{t} (dy_{t}/dt)$$
$$ds_{t} = -\kappa (s_{t} - \overline{s}) + \sigma_{s} dZ_{t}$$

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#### **Optimal Monetary Policy**

- Linearize stochastic part; keeps time variation in risk premium
- Central bank solves

$$L = \min_{\{i_s\}_{s=t}^{\infty}} \mathbb{E}_t \int_t^{\infty} e^{-s\beta} y_s^2 ds$$

subject to

$$dy_{t} = \frac{1}{\gamma} (i_{t} - r) dt + \xi (V_{t} - s_{t}) dZ_{t}$$
$$V_{t} = -\tau \mathbb{E}_{t} [dy_{t}/dt] - \mathcal{N}^{-1}(\alpha) \sqrt{\tau} Vol_{t} (dy_{t}/dt)$$
$$ds_{t} = -\kappa (s_{t} - \overline{s}) + \sigma_{s} dZ_{t}$$

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# **Optimal Monetary Policy**

Using the IS equation

$$dy_t = \frac{1}{\gamma} \left( i_t - r \right) dt + \xi \left( V_t - s_t \right) dZ_t$$

We can plug

$$\mathbb{E}_t[dy_t/dt] = rac{1}{\gamma}(i_t - r)$$
  
/ol<sub>t</sub>(dy<sub>t</sub>/dt) =  $\xi(V_t - s_t)$ 

into

$$V_t = - au \mathbb{E}_t [dy_t/dt] - \mathcal{N}^{-1}(lpha) \sqrt{ au} \, extsf{Vol}_t (dy_t/dt)$$

to see that  $V_t$  and  $i_t$  are one-to-one: The *risk-taking channel* of monetary policy

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## Output Gap Mean-Volatility Tradeoff

- $i_t$  and  $V_t$  are one-to-one, so think of  $V_t$  as central bank's choice
- Eliminating it, dynamics of the economy are

$$dy_t = \xi \left( M \times V_t + \frac{\mathcal{N}^{-1}(\alpha)}{\sqrt{\tau}} s_t \right) dt + \xi \left( V_t - s_t \right) dZ_t$$

where

$$M \equiv -\frac{\xi + \mathcal{N}^{-1}(\alpha)\sqrt{\tau}}{\tau\xi}$$

is the slope of the mean-volatility line for output gap

$$\mathbb{E}_t \left[ dy_t/dt 
ight] = M imes Vol_t \left( dy_t/dt 
ight) - rac{1}{ au} s_t$$

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#### Conditional Mean-Volatility Line for Output Gap Growth



# Tradeoff for Monetary Policy

• Negative slope gives M < 0 and mean variance tradeoff:

$$dy_t = \xi \left( M \times V_t + \frac{\mathcal{N}^{-1}(\alpha)}{\sqrt{\tau}} s_t \right) dt + \xi \left( V_t - s_t \right) dZ_t$$

• Changes in  $V_t$  move the economy along the mean-vol line

- Full stabilization  $(y_t = 0)$  made impossible by vulnerability
- Shocks s<sub>t</sub> shift the line up and down
- ▶ Because M < 0, we have ∂V<sub>t</sub>/∂R<sub>t</sub> < 0: Tighter policy reduces vulnerability</p>

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- Re-introduce Phillips Curve
- Augmented Taylor

$$i_t = \phi_0 + \phi_\pi \pi_t + \phi_y y_t + \phi_v V_t$$

Can be expressed as flexible inflation targeting

$$\pi_t = \psi_0 + \psi_y y_t + \psi_v V_t + \psi_s s_t$$

- Coefficients \u03c6 and \u03c6 are a function of structural parameters that govern vulnerability
- Strict inflation targeting not feasible

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#### Calibration

# Calibration

- Calibration comes directly from a regression of the conditional mean on the conditional vol of output gap growth
- Pick β = 0.01, α = 5%, τ = 1 and match intercept, slope, standard deviation and AR(1) coefficient of residuals to get

 $\xi = 0.36$  $\overline{s} = -0.67$  $\sigma_s = 0.61$  $\kappa = 2.14$ 

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Calibration

## Welfare Gains



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#### Conclusion

# Conclusion

- The NK model can be augmented by
  - A financial sector that intermediates subject to a Value-at-Risk constraint
  - Shocks to financial sector
- > The "second order" linearization approximation
  - Matches the stylized fact that conditional upper GDP quantiles are constant, while lower GDP quantiles move with financial conditions
  - Mathematically tractable
- Optimal monetary policy always depends on vulnerability
  - Optimal monetary policy conditions on vulnerability
  - Vulenrability responds to monetary policy
  - Magnitudes are potentially large quantitatively