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## The (ir)relevance of the nominal lower bound for real yield curve analysis

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**Abstract**

I propose a new term structure model for euro area real and nominal interest rates which explicitly incorporates a time-varying lower bound for nominal interest rates. Results suggest that the lower bound is of importance in structural analyses implying time-varying impulse responses of yield components. With short-term rate expectations at or close to the lower bound, premium components are less reactive to a typical 10 bp increase in inflation, while real rate responses change their sign from positive to negative. However, it is further shown that the lower bound is of only little relevance for decomposing yields into their expectations and premium components once survey information is incorporated. Overall, results support the conclusion that reaching the effective lower bound may change the way macroeconomic shocks propagate along the term structure of nominal as well as real interest rates.

**Keywords:** Joint real-nominal term structure modelling, lower bound, inflation expectations, inflation risk premium, monetary policy, euro area.

**JEL classification:** E31, E43, E44, E52.

## Non-technical summary

With short-term interest rates at or close to their effective lower bound (ELB), central banks around the globe have increasingly resorted to unconventional monetary policy measures. In the case of the European Central Bank (ECB), amongst other things, these include large-scale asset purchases like the public sector purchase programme (PSPP) launched in January 2015, forward guidance and the introduction of negative interest rates. Commonly, these measures are considered to have been an effective tool that allowed the ECB to lower nominal yields of short and long-term maturities alike.

To assess whether the decline in nominal yields supported the return of euro area inflation towards the ECB's inflation aim of close to, but below, 2%, it is important to understand to what extent these nominal rate declines reflected a decline in real yields rather than prospects of lower future inflation. After all, it is the level of real rates that according to economic theory matters for consumption and investment and thus ultimately drives inflation.

This paper presents an empirical analysis of this matter by decomposing observed nominal overnight index swap (OIS) yields in their inflation and real components. In doing so, it pays special attention to the low-interest rate environment in which the ELB introduces important non-linearities into the dynamics of nominal yields. The paper seeks to answer the question if these non-linearities also matter for the analysis of real yields and market-based indicators of inflation expectations.

Specifically, this paper introduces a dynamic term structure model (DTSM) that jointly models the overnight index swap (OIS) yield curve and market-based indicators of inflation (i.e., inflation-linked swap rates [ILS]), while explicitly accounting for the ELB of nominal yields. This allows for a quantification of the extent to which the non-linearity in nominal yields is of relevance to estimates of the dynamics of real rates and the inflation component. Based on this model, the inflation and real component of nominal yields are recovered and further decomposed in what is called their expectation component (i.e. average expected future inflation and real rates) and their risk premium component (i.e. the inflation risk premium and the real risk premium).

A key result of the analysis is that the ELB is an important factor in structural analyses of the yield curve. For instance, when analysing the response of yield components to a typical increase in inflation, it shows that the closer rates are to the ELB, the smaller is the impact of such increases across all maturities. The response of real rates is also non-linear: While nominal rates are distant from the ELB, real rates show a positive response to an increase in inflation; they react negatively when nominal rates are at or close to the ELB. Overall, these results suggest that the ELB introduces non-linearities with a pronounced impact on structural relationships in the economy. At the same time, the ELB is of minor relevance to yield decompositions if survey information about yield and inflation expectations is incorporated into the model estimation.

Regarding the effectiveness of the ECB's policy measures, the analysis supports the conclusion that they have contributed to lower real rate expectations and real risk premiums – both of which together account for a fair share of the decline observed around the introduction of PSPP. At the same time, the model-implied decomposition of the decline in nominal yields also reveals that at least until early 2016 it was equally driven by lower inflation expectations and inflation risk premiums. However, while real rate expectations and premiums remained low throughout the further course of asset purchases, inflation expectation and premiums eventually recovered around two years into the programme. With regard to the ECB's negative interest rate policy, model simulations indicate that the consecutive interest rate cuts to levels below 0% have primarily worked through lowering real rate expectations at the short end of the yield curve and can therefore be expected to have been supportive of the ECB's inflation aim.

# 1 Introduction

The analysis of the term structure of interest rates is a central focus of the assessment of the transmission and effectiveness of monetary policy. By definition, any move in nominal rates is driven by either the inflation component or the real rate component. While central banks aim to steer the level and expectations of nominal rates, it is essential for monetary authorities that they manage to effectively influence real rates in the intended manner, as according to economic theory, it is the level of real rates that matters for consumption and investment and thus ultimately drives inflation.

A standard tool for the analysis of yields are dynamic term structure models (DTSMs). Most commonly, they are used to decompose the yield curve into genuine expectations about future short rates and the term premium, which compensates the investor for the uncertainty about future short rate realizations. However, beyond the decomposition of nominal yields, they can also be applied for a further decomposition of these expectations and premia into their real and inflation components. While earlier models generally assumed that yields are linear functions of a small set of pricing factors, the literature has moved towards more complicated models – so-called shadow rate term structure models (SRTSMs) – since yields have been considered close to or at their effective lower bound (ELB). As it is assumed that yields cannot fall below their ELB, this implies that they follow a non-linear distribution.

The literature has argued that failing to take this non-linearity into account may otherwise lead to implausible estimates for rate expectations and premia (see e.g. Krippner (2015a), Priebsch (2013), Lemke and Vladu (2016), Wu and Xia (2016), Geiger and Schupp (2018)) and consequently also to a non-reliable inference of the dynamics of inflation expectations and real rates embedded in observed nominal rates (Carriero, Mouabbi, and Vangelista (2018)). Against this backdrop, I propose a joint model for euro area nominal rates and inflation-linked swap (ILS) rates, which allows the aforementioned components to be isolated. Unlike earlier models focusing on the euro area (see Hördahl and Tristani (2014), García and Werner (2012)), the model in this paper explicitly takes into account the ELB of nominal interest rates, which for the euro area is considered to be time-varying (Lemke and Vladu (2016), Wu and Xia (2017)). This implies a non-linear interest rate distribution close to or at the ELB, as substantially lower interest rates are considered to be unlikely if not ruled out.

Indeed, results suggest that modelling the ELB is of relevance for two reasons. First, an analysis of responses by yield components to a shock-induced typical 10 bps increase in inflation shows that the magnitude and sign of these responses are conditional on the degree to which the ELB is binding. For nominal yields, we observe a decreasing impact of a typical increase in inflation across all maturities, the closer rates are to the ELB. The response of real rates is non-linear. While nominal rates are distant from the ELB, real rates show a positive response to a shock-induced increase in inflation; they react negatively when nominal rates are close to or at the ELB. Overall, these results suggest that the ELB introduces non-linearities with a meaningful impact on structural relationships in the economy. The finding of non-linear or time-varying impulse responses relates to findings of Mertens and Williams (2018) who, in a small structural model, find that the lower bound alters the distributions of both interest rates and inflation by restricting the central bank's scope for action. The findings further relate to work by King (2019) and Geiger and Schupp (2018) who likewise attest a decreasing effectiveness of conventional monetary policy at the ELB due to a receding reactiveness of interest rates, in particular, at shorter maturities.

Second, isolated changes in the ELB impact, in particular, nominal and real forward rates mainly through their expectations component. In our analysis, a 10-bp cut in the ELB yields an average impact of -5 (-3) bps on 24-month (120-month) nominal forward rates. These impacts are almost entirely transmitted through real rate expectations and only to a very small extent through real or inflation risk premia. Thus, these results imply that the central bank can lower real rate expectations by solely changing the effective lower bound of interest rates. These results build upon work of Lemke and Vladu (2016) who have shown that the perceived lower bound by itself can be considered a monetary policy tool to lower yields across all horizons.

While the above results stress the importance of incorporating the ELB, another finding of this paper is that similar yield decompositions are obtained from the model whether or not it incorporates a lower bound, conditional of the inclusion of survey information on expected rates and inflation. In fact, the model in both cases achieves a similar fit of these surveys, the expectations components do not differ markedly, even though both models do yield somewhat different results in terms of persistence and the unconditional means of the nominal short rate, real short rate and inflation.

The proposed model is further applied to a decomposition of the change in nominal long-term rates between mid-2014 and mid-2016. This decline is often considered to have been initiated in anticipation of the Eurosystem's unconventional monetary policy measures, in particular, its large-scale asset purchases. Commonly, such programmes are considered to affect yields mainly through two channels: 1) the duration extraction or portfolio rebalancing channel affecting risk premia (see Vayanos and Vila (2009)), and 2) the rate signalling channel affecting rate expectations (see Bauer and Rudebusch (2014)). Indeed, the results show that both, nominal rate expectations and premia contributed to the decline which is principally in line with both these transmission channels mentioned above. At the same time, however, the reduction to a large extent also reflected declines in inflation expectations and inflation risk premia, which may be an expression of market's anticipating an increased probability of low inflation or even deflation scenarios (see also Camba-Mendez and Werner (2017), García and Werner (2012)). Overall, this lays the ground for the supposition that monetary policy may have had adverse effects through negative information effects (see Christensen and Spiegel, 2019).

The paper is also closely related to the vast body of literature on joint real-nominal yield curve modelling. For work that focuses on the US, see Ang, Bekart, and Wei (2008), Adrian and Wu (2009), Campbell, Sunderam, and Viceira (2016), Christensen, Lopez, and Rudebusch (2010), D'Amico, Kim, and Wei (2018), Chen, Liu, and Cheng (2010) and Roussellet (2020). Hördahl and Tristani (2014) jointly model the real and nominal term structure of interest rates for the euro area. Other related work with a focus on the euro area, which, however, does not jointly model real and nominal yields, but focuses on the term structure of inflation and the inflation risk premium based on data for euro area inflation-linked swap rates, can be found in García and Werner (2012) and Camba-Mendez and Werner (2017). For an analysis for the UK, see Barr and Campbell (1997) and Carriero et al. (2018), while Christensen and Spiegel (2019) cover the topic with a focus on Japan. Among all related work, Carriero et al. (2018) and Roussellet (2020) are, to the best of my knowledge, the only ones who also incorporate a lower bound for nominal rates. Unlike the model presented in this paper, both, however, do not model a time-varying lower bound. While Carriero et al. (2018) assume a constant lower bound at zero, Roussellet (2020) ensures ELB-consistency by building on a standard quadratic term structure framework. The model presented here is further distinct from both these works in its identification scheme and the assumed pricing factors.

My paper is further related to the literature on yield curve modelling in lower bound environments. For the US, see Krippner (2015a), Christensen and Rudebusch (2015), Bauer and Rudebusch (2014), Wu and Xia (2016), Priebsch (2013). Applications for Japan and the UK comprise Ichiue and Ueno (2013) and Andreasen and Meldrum (2015). Other papers estimating shadow rate term structure models (SRTSMs) on euro area data are, e.g., Lemke and Vladu (2016), Kortela (2016), Wu and Xia (2017), Geiger and Schupp (2018).

## 2 Model

The model assumes that the term structure of interest rates is explained by $N = 4$ factors $X_t^j$, with $j = 1, 2, 3, \Pi$.[1] The factors are defined such that the first three factors may be interpreted as three latent real yield curve factors, while observed monthly inflation constitutes the fourth factor. Factor dynamics follow a first-order Gaussian vector autoregressive process both under the risk-neutral ($\mathbb{Q}$) and the historical ($\mathbb{P}$) probability measure.

$$X_t = \mu^{\mathbb{Q}} + \rho^{\mathbb{Q}} X_{t-1} + \Sigma u_t, \qquad u_t \sim N(0, I) \tag{1}$$

$$X_t = \mu^{\mathbb{P}} + \rho^{\mathbb{P}} X_{t-1} + \Sigma u_t, \qquad u_t \sim N(0, I). \tag{2}$$

### 2.1 Real and Nominal Shadow Short Rates

Following the standard literature on SRTSMs it is assumed that the actual nominal short rate is constrained by a (time-varying) ELB $l_t$, which serves as hard floor. Thus, by assumption, the short rate corresponds to the shadow short rate as long as the latter is above the ELB and equals the ELB otherwise. This specification also allows for forward rates and the expected path of the short rate to remain at this ELB for an extended period of time, as has been observed in the euro area since interest rates have reached the ELB. Specifically, it holds that

$$i_{1,t} = \max(si_{1,t}, l_t), \tag{3}$$

where $si_{1,t}$ is the shadow short rate. From an economic perspective, the nominal shadow rate can be interpreted as the short rate which would prevail in the absence of the ELB, and thus describes the hypothetical value of the option to hold cash (see Black, 1995). Essentially, the existence of the nominal shadow rate also implies the existence of a real shadow short rate $si^*_{t,1}$ which Black (1995) describes as the difference between the nominal shadow rate $si_t$ and

[1] This specification follows Ajello, Benzoni, and Chyruk (2012).

inflation:[2]

$$si^*_{t,1} = si_{t,1} - E^{\mathbb{Q}}_t(\Pi_{t+1}). \tag{4}$$

Another central assumption of the model proposed here is that $si^*_{t,1}$ is linear in the pricing factors:

$$si^*_{1,t} = \delta_0 + \delta_1^{'} X_t, \tag{5}$$

where $\delta_1 = [1; 1; 1; \delta_\Pi]$.[3]

### 2.2 Nominal Bond Prices

Central to any term structure model is the assumption of no arbitrage, which implies the existence of a unique stochastic discount factor (SDF) $m$ which prices all bonds of any maturity $n$. For the price of a nominal zero-coupon bond of maturity $n$, it then holds that

$$P_{n,t} = E^{\mathbb{P}}_t[m_{t+1} P_{n-1,t+1}] \tag{6}$$

In general, the SDF can be defined in real and nominal terms depending on which kind of bond is to be priced. For real bonds, the real pricing kernel in the following is defined as proposed by Ang et al. (2008):

$$m^*_{t+1} = exp(-i^*_{1,t} - \frac{1}{2}\lambda'_t \lambda_t - \lambda'_t \epsilon_{t+1}) \tag{7}$$

[2]The real shadow short rate will differ from the actual real rate to the extent that the nominal shadow short rate differs from the actual nominal short rate. Further note that while actual real rates are technically not constrained by a lower bound, the nominal lower bound implies that the space of feasible real rate realizations is still constrained to the extent that they emerge as the difference of ex-ante expected inflation and constrained nominal rates. Thus, at the ELB the lowest feasible real rate realizations decisively depend on the upper tail of the inflation distribution.

[3]The choice of $\delta_\Pi$ does have theoretical implications. As Ang et al. (2008) argue, a zero loading of the real short rate on expected inflation implies money neutrality. A possible Mundell-Tobin effect would call for a negative correlation, and an activist Taylor rule, on the other hand, would predict a positive correlation. We decide to leave this parameter unrestricted.

with $i^*_t$ representing the real short rate. Subsequently the nominal pricing kernel is

$$\begin{aligned} m_{t+1} =& m^*_{t+1}\frac{Q_t}{Q_{t+1}} = m^*_{t+1}exp(-\pi_{t+1}) \\ =& exp(-i^*_{1,t} - \pi_{t+1} - \frac{1}{2}\lambda'_t\lambda_t - \lambda'_t\epsilon_{t+1}). \end{aligned} \tag{8}$$

Note, that $\lambda_t$ constitute the prices of risk investors demand in the market. Following Dai and Singleton (2002), these are themselves linear functions of the factors $X_t$ and thus time-varying. Their function takes the form

$$\lambda_t = \lambda_0 + \lambda'_1 X_t. \tag{9}$$

Following Wu and Xia (2017), shadow real yields are also assumed to be linear functions of the pricing factors:

$$si^*_{n,t} = a_n + b'_n X_t, \tag{10}$$

with $a_n = -A_n/n$ and $b_n = -B_n/n$.

Subsequently, expressions for $a_n$ and $b_n$ are obtained via recursive solutions[4]:

$$A_{n+1} = -\delta_0 + A_n + B_n(\rho^{\mathbb{P}}_0 - \Sigma\lambda_0) - \rho^{\pi,\mathbb{P}}_0 + \frac{1}{2}B_n\Sigma\Sigma' B_n + \frac{1}{2}\Sigma^\pi\Sigma^{\pi'} \tag{11}$$

$$B_{n+1} = -\delta_1 - \rho^{\pi,\mathbb{P}}_1 + B^{*'}_n(\rho^{\mathbb{P}}_1 - \Sigma\lambda_1) + \Sigma^\pi\lambda_1 \tag{12}$$

Note that $\rho^{\mathbb{Q}}_0 = \rho^{\mathbb{P}}_0 - \Sigma\lambda_0$ and $\rho^{\mathbb{Q}}_1 = \rho^{\mathbb{P}}_1 - \Sigma\lambda_1$.

$$A_1 = -\delta_0 - \rho^{\pi,\mathbb{P}}_0 + \Sigma^\pi\lambda_0 + \frac{1}{2}\Sigma^\pi\Sigma^{\pi'} \tag{13}$$

$$B'_1 = -\delta'_1 - \rho^{\pi,\mathbb{P}}_1 + \Sigma^\pi\lambda_1 \tag{14}$$

[4]For details see Appendix A.

so that here the nominal shadow short rate is defined as

$$si_{t,1} = \delta_0 + \rho^{\pi,\mathbb{P}} - \Sigma^{\pi}\lambda_0 - \frac{1}{2}\Sigma^{\pi}\Sigma^{\pi'} + (\delta_1' + \rho_1^{\pi,\mathbb{P}} - \Sigma^{\pi}\lambda_1)X_t \tag{15}$$

Finally, it then follows for shadow forward rates:

$$sf_{n,t} = (A_{n+1} - A_n) + (B_{n+1}' - B_n')X_t. \tag{16}$$

Given the lower bound restriction, the mapping of pricing factors into interest rates is non-linear, and in this case no closed-form solutions for bond prices exist. However, Wu and Xia (2017) show that generally implied one-period forward rates $n$ periods ahead, $f_{n,t}$, can be expressed as

$$f_{n,t} \approx \int \left( l_{t+n} + \sigma_n^{\mathbb{Q}} g\left( \frac{sf_{n,t} - l_{t+n}}{\sigma_n^{\mathbb{Q}}} \right) \right) P_t^{\mathbb{Q}}(l_{t+n})\, \mathrm{d}x \tag{17}$$

where $g(x) = x\Phi(x) + \phi(x)$ with $\Phi(x)$ the standard normal cdf, $\phi(x)$ the standard normal pdf, and $\sigma_n^{\mathbb{Q}}$ the conditional variance of future shadow short rates. Note that in this general form, the forward rate is calculated as the average of future short rates with $l_{t+n}$ weighted by the risk-neutral probability of $l_{t+n}$.

In euro area term structure literature, the lower bound is typically regarded as time-varying (see i.a. Lemke and Vladu (2016), Kortela (2016), Wu and Xia (2017), Geiger and Schupp (2018)). Allowing the ELB to change over time acknowledges that before interest rates were actually lowered to negative levels, it was reasonable to assume that zero would have constituted the ELB for interest rates. Indeed, Lemke and Vladu (2016) present survey evidence that the first cut to negative levels in the euro area was widely unanticipated at that point in time. Later, even after interest rates were lowered into negative territory, it was not clear that the ECB would lower rates even further as statements by ECB President Mario Draghi attest. After the cut to -10 bps as well as after the move to -20 bps, he declared that the technical lower bound of interest rates had been reached.[5] Thus, allowing for discrete changes of $l_t$ accommodates the

[5] E.g. after the ECB Governing Council lowered the DFR to -10 bps on 5 June 2014, President Draghi at the press conference emphasized that "[...] for all the practical purposes, we have reached the lower bound". While saying that this would not exclude some "little technical adjustments, which could lead to some lower interest

notion that the market adapted perceptions of where the ELB with each subsequent cut in the course of 2014-2016.[6]

The literature offers a number of alternatives to mirror this ELB dynamic in a otherwise standard SRTSM. The simplest calibration would hardwire the ELB to equal the level of the Deposit Facility Rate (DFR) during the lower bound period beginning in summer 2012 (see e.g. Kortela, 2016)). However, this omits that downward sloping forward rate constellations, in particular, during the years 2014 to 2016 indeed signalled that the market did not necessarily consider the current DFR to be the actual ELB. While it then still would be plausible to assume that the DFR is the lower bound for the short rate, this does not necessarily hold for all future expected short rates. This was first addressed by Wu and Xia (2016), who at each point in time allow for expected changes in the lower bound, which is modelled to follow a Markov-chain process. Here, we follow Geiger and Schupp (2018), who assume that for all future dates $t+n$ the perceived lower bound equals the minimum forward rate, while the current short rate remains bound by the DFR. This allows the model to fit a downward sloping yield curve also at the current ELB.[7]

More precisely, the ELB is the specified in the following way:

$$l_{t+n} = \begin{cases} 0 & \text{if prior to ELB period and } \forall n = 0,1,2,\ldots \\ \gamma_t i_t^{DFR} + (1-\gamma_t) i_{t+1}^{DFR} + sp_t & \text{if ELB period and } n = 0 \\ \min(l_t, \bar{f}_t) & \text{if ELB period and } \forall n = 1,2,\ldots \end{cases} \quad (18)$$

with $\bar{f}_t = \min(f_{t,n})$ for $n = [1, 2, \ldots, N]$. In the period before reaching the ELB, the current and expected ELB is set to zero. Following Wu and Xia (2016) this leads to the following analytical approximation of Equation 19:

rates", he then repeated that "from all practical purposes, I would consider having reached the lower bound today". Then, after the DFR was cut to -20 bps after all in September 2014, President Draghi again said that the lower bound had now been finally reached. He announced this cut as a technical adjustment, now even ruling out further adjustments. After the consecutive cuts to -30 and -40 bps, the President avoided any statements on the lower bound of interest rates.

[6] OIS rates are in general considered to be bound by the Eurosystem's DFR as transactions underlying the computation of EONIA take place between counterparties that all have access to the deposit facility of the Eurosystem. Thus, they are expected to have no incentive to lend below that rate.

[7] The lower bound specification of Geiger and Schupp (2018) also accounts for calender effects by setting the current ELB, $l_t$, equal to the weighted average of the DFR in period $t$ and the expected DFR in period $t+1$, which in their specification is treated as being known in period $t$, where $\gamma_t$ is the fraction of days between the end of the month and the next Governing Council meeting in the following month. Moreover, expectations of future changes in the lower bound are accounted for by separately defining a lower bound for all future periods $t+h$ as the minimum of the current lower bound $l_t$ and the observed 1-month forward rates over the next 24 months.

$$f_{h,t} \approx l_{t+n} + \sigma_n^{\mathbb{Q}} g\left(\frac{sf_{n,t} - l_{t+n}}{\sigma_n^{\mathbb{Q}}}\right). \tag{19}$$

## 3 Estimation

For the estimation, the model is cast into state space form with the transition equation given by Equation 2:

$$X_t = \mu^{\mathbb{P}} + \rho^{\mathbb{P}} X_{t-1} + \Sigma u_t, \qquad u_t \sim N(0, I). \tag{20}$$

The measurement equation takes the form of

$$\hat{Z}_t = Z_t + e_t \tag{21}$$

where $Z_t$ contains observed yields, ILS rates, observed 1-month Inflation $\Pi^o$ and the survey information on short-rate and inflation expectations as explained above with model-implied yields $Y_t = g(X_t, \rho_0^{\mathbb{Q}}, \rho_1^{\mathbb{Q}}, \Sigma, \delta_0, \delta_1, \lambda_0, \lambda_1)$.

Without further restrictions, the latent state is not uniquely determined. In general, identifying the model and preventing the latent factors from shifting, rotating or scaling, only a small number of restrictions are needed. Here, the identification follows Joslin, Singleton, and Zhu (2011) who develop a maximally-flexible model in which all identifying restrictions are imposed on the cross-section of yields, while time series dynamics of yields are described by an unrestricted VAR(1) process.[8]

An additional measurement equation is formulated for ILS rates, which we interpret as the observed break-even inflation rate ($BEIR^o$). The model-implied $BEIR$ itself is defined as

$$BEIR_{t,n} = i_{n,t+j} - i^*_{n,t+j} \tag{22}$$

[8]Specifically, it is imposed that $\rho^{\mathbb{Q}} = diag(\rho_1^{1,\mathbb{Q}}, \rho_1^{2,\mathbb{Q}}, \rho_1^{3,\mathbb{Q}}, \rho_1^{\Pi,\mathbb{Q}})$ and is in Jordan form, $\rho_0^{\mathbb{Q}} = [\kappa_\infty^{\mathbb{Q}}, 0, 0, \rho_0^{\Pi}]$, $\delta_0 = 0$ and $\delta_1 = [1, 1, 1, \Pi]$. Deviating from Joslin et al. (2011), $\Sigma$ is restricted to be diagonal, so that shocks $u_t$ are orthogonal to each other. On the one hand, this implies that shocks to inflation do not directly impact real factors, which seems to be a plausible assumption to make. On the other hand, the diagonality assumption on the latent block follows Christensen, Diebold, and Rudebusch (2011) who suggest that this assumption improves upon the forecast performance of the model. In addition, it has been shown that allowing for correlation among the shocks significantly reduced the model's ability to fit survey-based inflation expectations.

As our model does not directly observe the actual real rate $i^*_{n,t+j}$, we re-formulate the above as

$$BEIR_{t,n} = i^*_{n,t+j} + E^{\mathbb{P}}_t(\Pi_{t,n}) + IRP_{t,n} - i^*_{n,t+j} \tag{23}$$

$$BEIR_{t,n} = E^{\mathbb{P}}_t(\Pi_{t,n}) + IRP_{t,n} \tag{24}$$

where $E^{\mathbb{P}}_t(\Pi_{t,n})$ and $IRP_{t,n}$ are genuine inflation expectations and the inflation risk premium. To properly account for convexity effects, which in our model partly depend on the inflation risk prices $\lambda_{0/1,\Pi}$, we determine the $IRP$ as

$$IRP_{n,t} = i_{n,t} - i^{w/oIRP}_{n,t} \tag{25}$$

where the latter term is determined by computing nominal yields while setting all $\lambda_{0/1,\Pi}$ to zero.

Further, as discussed above, the high persistence of yields which are only available in short samples for the euro area leaves the model with only little information about the data generating process $\mathbb{P}$ as well as the drift in distant short-rate expectations. To arrive at more precise estimates of the parameters under the $\mathbb{P}$-measure, we link model-implied expectations to survey forecasts on short rate expectations as a further central feature of our model as advocated by Kim and Orphanides (2012).[9] When including survey information, it is crucial to allow for measurement errors when aligning model-implied expectations with corresponding survey forecasts as there is little evidence that these surveys perfectly reflect actual expectations embedded in the yield curve. For any given survey interest rate forecast with residual maturity $n$ in $j$-periods ahead, we add the following equation to our model set-up:

$$i^{survey}_{n,t+j} = E^{\mathbb{P}}_t\left[i_{n,t+j}\right] + e^{i_{survey}}_{n,t} \tag{26}$$

where $e^{i_{survey}}_{n,t}$ is the survey expectation measurement error with standard deviation $\sigma^{\Pi_{survey}}$.

The model further incorporates survey information on inflation expectations, which enter

[9] Further applications of term structure models including surveys are, e.g., Priebsch (2017), Guimarães (2014), Chernov and Mueller (2012) and Geiger and Schupp (2018).

the model via the following measurement equation:

$$\Pi_{n,t+j}^{survey} = E_t^{\mathbb{P}}\left[i_{n,t+j}\right] + e_{n,t}^{\Pi_{survey}} \tag{27}$$

and $e_{n,t}^{\Pi_{survey}}$ is the survey expectation measurement error with standard deviation $\sigma^{\Pi_{survey}}$.[10]

As the mapping between interest rates and pricing factors in the measurement equation is non-linear, we apply the non-linear extended Kalman filter when maximizing the likelihood function.[11]

As regards the data, we use monthly overnight index swap (OIS) rates based on EONIA and euro area inflation-linked swap (ILS) rates spanning a sample from June 2005 to December 2019. The length of the sample is determined by the availability of reliable euro area ILS rates. OIS rates are included for maturities of 1,3, and 6 months as well as 1,2,3,5,7 and 10 years, while the ILS maturities included are 1,2,3,4,5,7,9 and 10 years. Hence, our sample consists of $T = 175$ months for $J_i = 9$ and $J_\Pi = 8$ maturities. In addition to OIS and ILS rates, we further included survey information on interest rate and inflation expectations provided by Consensus Economics and the ECB's Survey of Professional Forecasters (SPF). In particular, we include Consensus economics 3-month interest rate forecasts for 1 to 7 quarters as well as 6 to 10 years ahead. As regards inflation expectations, SPF average forecasts of year-on-year inflation 1, 2 and 5 years ahead are included.

# 4 Results

## 4.1 Goodness of Fit

Overall, the model delivers a satisfying fit of yields across all maturities, with an average mean absolute error (MAE) of 4 bps over the entire sample. ILS rates are fitted with MAE of 2 bps and short-term interest rate and inflation surveys with 16 and 30 bps, respectively (see Figure 1 and Tables B.1 and B.4).

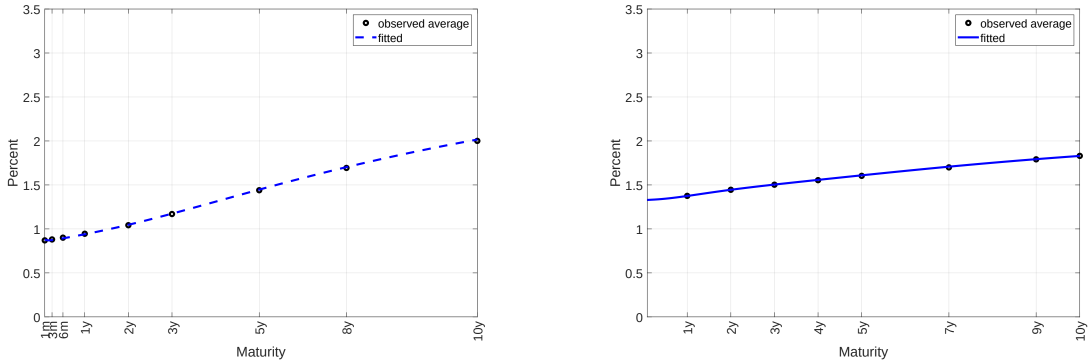
As has already been observed for nominal yields in Geiger and Schupp (2018), the fit improves notably after the ELB is reached, especially at the very short end. For nominal yields this result

[10] We allow the standard deviation of measurement errors of surveys to differ between the short- and long-term interest rate and inflation surveys.

[11] Alternative non-linear filters include the iterated extended Kalman Filter as well as the unscented Kalman filter (Kim and Singleton (2012),Priebsch (2013), Krippner (2015b)).

Figure 1: Average model fit

(a) OIS yields (b) ILS rates




Note: Panel (a) depicts the average fit of OIS yields for the maturities included in the model. Panel (b) depicts the average fit of ILS rates for the maturities included in the model.

seems straightforward, as the chosen lower bound specification almost guarantees a very good fit of the short rate, ensuring that the ELB is binding at least for the short rate.

The larger fitting errors with respect to surveys are in line with other results from the literature (see e.g. Priebsch (2017) and Geiger and Schupp (2018)). This mainly reflects that the model incorporates considerably less information in terms of the number of observations on interest rate expectations compared to observed yields. However, to some extent, this may also signal that expectations embedded in market prices deviate from those expressed by survey participants.[12] In this regard, it is interesting that fitting errors for surveys decrease to a similar extent once entering the ELB period in mid-2012. Since then, the installment of forward guidance by the ECB's Governing Council in 2013 (see Hattori, Schrimpf, and Sushko, 2016) and also the strong deterioration in the inflation outlook since mid-2014 may have further increased certainty about the rate outlook.[13] In light of these events, the forward curve as well as the

[12]Potential sources of such deviations are numerous, and many have been discussed in the literature before. First, as pointed out by Kim and Orphanides (2012), surveys report average expectations, while market prices are driven by marginal expectations on interest rates – a problem that might be exacerbated by relatively low numbers of survey participants compared to the overall number of market participants. Further, there is a potential discrepancy between the information sets available to survey and market participants, given that surveys are collected over a particular reporting period, rather than at the point in time at which we observe the end-of-month interest rate. Therefore, it can well be assumed that the subjective expectations of survey participants deviate from the objective statistical $\mathbb{P}$-measure expectation. Also, survey participants are potentially not interested in revealing their true expectations, leaving surveys biased themselves, making them an inaccurate measure of participants' true expectations (Cochrane and Piazzesi, 2008; Chernov and Mueller, 2012)

[13]This increase in certainty, for example, manifests in an observed lower realized yield volatility (see Figure).

path of survey expectations for the 3-months rate up to 7 quarters ahead flattened considerably. This seems likely to have contributed to some convergence in expectations by market and survey participants which in turn may partly explain these lower fitting errors.

## 4.2 The Real-Nominal Decomposition of Interest Rates and the Nominal Effective Lower Bound

The following section will explore in more detail the ways in which the ELB is affecting the decomposition of the nominal yield curve in its real and inflation components. To begin with, this analysis is based on a comparison of the yield decomposition implied by the proposed lower bound model ($RTSM_{LB}$) and an affine version of this model ($RTSM_{woLB}$). Table 1 summarizes estimates of the unconditional means of the nominal and real short rate as well of inflation. In addition, it also depicts information about the estimated persistence of factor dynamics. In both models, the unconditional mean of the inflation factor is estimated to be around 1.9%. This is in line with the what would be expected according the Eurosystem's inflation aim of close to, but below, 2% over the medium term. At the same time, the models disagree on the unconditional mean of nominal and real rates. The lower bound model estimates imply them to be 3% and 1.6%, while they are estimated to be somewhat higher in the affine model, at 3.7% and 3.2%. This mainly reflects differences in the persistencies of the state dynamics under the $\mathbb{P}$-measure.

Nevertheless, and despite the differences in the estimated dynamics under the $\mathbb{P}$-measure, average decompositions of the nominal yield curve hardly differ across the lower bound and affine version of the model. Likewise, average decompositions of the nominal and real components do not differ by much (see Figure 2).[14] Somewhat larger but still contained differences emerge in both decompositions of the inflation component and real yields, in particular, at the short to medium maturities (see Panel (b) and Panel (c) of Figure 2). On average, inflation expectations are around 10 bps lower across maturities shorter than 5 years, while differences are less pronounced at longer horizons. Naturally, the opposite is true for the inflation risk premium, which is higher on average in the lower bound model. Mirroring observations for the decomposition of the inflation component, Panel (c) of Figure 2 shows that the real rate expectations in the lower bound model tend to be somewhat higher across the term structure, with both average

[14] In both model versions, average premia of around zero at shorter maturities emerge from an implied convergence of all premium components towards zero at horizons of up to 1 year since around 2013, potentially driven by the ECB's forward guidance. These results are in line with Geiger and Schupp (2018) and Priebsch (2017), who finds similar results for the US.

Table 1: (Shadow) short rate summary statistics – $\mathbb{P}$-estimates

| Model | $RTSM_{LB}$ | $RTSM_{woLB}$ |
|---|---|---|
| Unconditional mean $E^{\mathbb{P}}i_1$: | 2.994 | 3.710 |
| Unconditional mean $E^{\mathbb{P}}i_1^*$: | 1.090 | 1.896 |
| Unconditional mean $E^{\mathbb{P}}\Pi_1$: | 1.903 | 1.814 |
| Eigenvalues under $\mathbb{P}$-measure: | 0.985 | 0.987 |
| | 0.967 | 0.987 |
| | 0.889 | 0.931 |
| | 0.889 | 0.931 |
| **Model** | $RTSM_{LB}^{noIRSurveys}$ | $RTSM_{woLB}^{noIRSurveys}$ |
| Unconditional mean $E^{\mathbb{P}}i_1$: | 1.587 | 3.240 |
| Unconditional mean $E^{\mathbb{P}}i_1^*$: | −0.353 | 1.366 |
| Unconditional mean $E^{\mathbb{P}}\Pi_1$: | 1.940 | 1.873 |
| Eigenvalues under $\mathbb{P}$-measure: | 0.985 | 0.982 |
| | 0.967 | 0.982 |
| | 0.878 | 0.916 |
| | 0.878 | 0.916 |
| Sample mean ($i_1$): 0.870 | | |

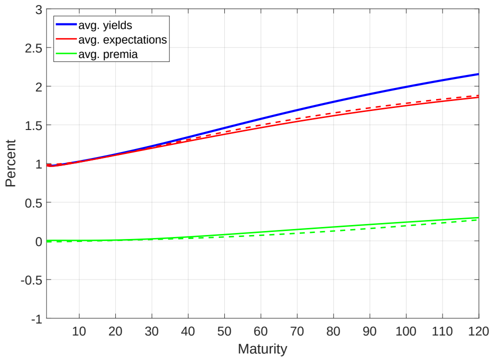
decompositions converging at longer horizons.

While rate volatility will by definition be constant in the affine model, one important feature of lower bound models is that they can retrace the dynamics of realized interest rate volatility. At the ELB this reduced interest rate volatility is also an expression of the fact that interest rates were not expected to go much lower, while on the other hand monetary policy amid persistently low inflation was not signalling any rate hikes in the near future. This, the affine model fails to reproduce. In fact, its constant volatility assumption to the contrary implies that even at the ELB there is equal chance of even lower rates as there is of an increase in rates, as the implied distribution is strictly symmetric. Thus, with respect to these considerations, modelling the ELB is essential despite the fact that both model-implied yield decompositions are very similar (see Figure 3).[15]

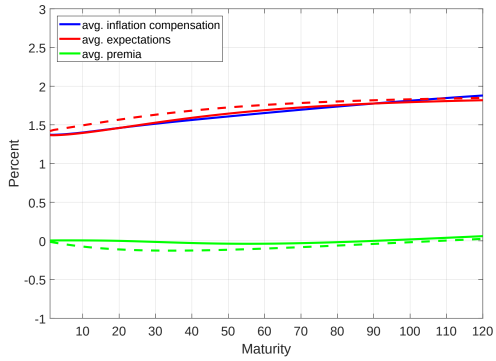
[15] Realized volatility has been computed by considering all daily changes over a 3-months window: $RealizedVol_{t,3m}(y_t^n) = \sqrt{\sum(\Delta y_{t+n}^n)^2}$. For the affine model the 3-months conditional volatility can be expressed in closed-form: $Vol_t^{RTSM_{woLB}}(y_{t+n}^n) = \sqrt{Var_t(y_{t+n}^n)} = \sqrt{B_n' Var_t(X_{t+n}) B_n}$. No closed-form expression exists in for the lower bound model. Therefore, we follow Lemke and Vladu (2016) and at each point in time conduct a Monte Carlo simulation computing 5,000 draws of $X_{t+n}$ based on their $\mathbb{P}$-parameters. For each draw, we compute the corresponding yields and subsequently compute the standard deviation of these 5,000 draws of yields.

Figure 2: Average decomposition yield components

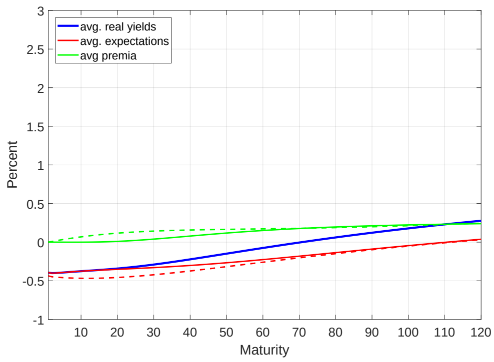
(a) Nominal yields




(b) Inflation-linked swap rates




(c) Real yields



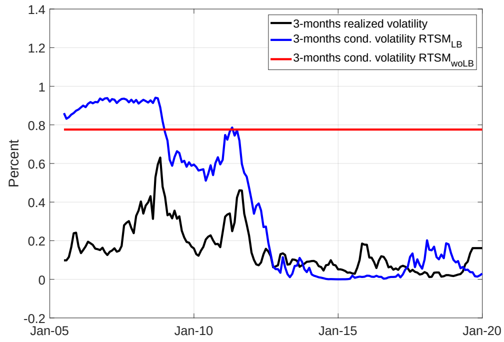

Note: Panel (a) depicts the average model-implied decomposition of the term structure of nominal yields. Panel (b) depicts the average model-implied decomposition of the term structure of inflation-linked swap rates. Panel (c) depicts the average model-implied decomposition of the term structure of real yields. In the panels, solid lines are based on the lower bound model, while dashed lines depict results from the affine model.

At first sight, the observed differences in yield decompositions seem quite surprising as they challenge the view in the literature that shadow rate models are essential for plausible yield decompositions at or close to the ELB.[16] One common observation is that affine models fail to produce stickiness in short-term short rate expectations, which tend to rise from the ELB rather quickly, mean-reverting over the long-run. Hence, these models usually produce relatively large negative term premia at the very short end. This is not the case in the models considered here.

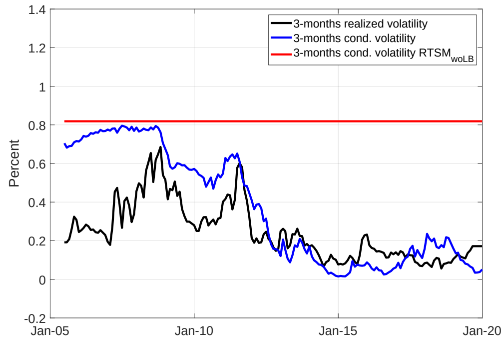
[16]This point will appear in most works that apply a shadow rate model, but the first to raise it include, e.g., Krippner (2015a), Wu and Xia (2016) and Lemke and Vladu (2016).

Figure 3: Three-months conditional volatilities of yields

(a) 1-year yield




(b) 3-year yield




Note: The chart depicts realized 3-months conditional volatilities as well as model-implied conditional volatilities based on the $RTSM_{LB}$ and $RTSM_{woLB}$ of the 1-year and 3-year yield.

On reflection, though, they seem less surprising, as in fact, both models considered here share, as an input, a considerable amount of survey information about nominal rate expectations for shorter and longer horizons. As both models fit these surveys almost equally well (see Table B.1 and Table B.3), this information allows both models to produce very similar paths of the expected short rate, which at times remains flat for a considerable time period.

In both the lower bound and in the affine model, the exclusion of this survey information impacts, in particular, on estimates of the unconditional mean and the persistence of the factor dynamics mirroring the difficulties in identifying the $\mathbb{P}$-measure in a small sample. Figure D.1 documents differences in the models in terms of their average decompositions of nominal and real yields and the inflation component. Along this dimension, it seems as if results from the lower bound models remain close to those from the survey-informed models. In particular, the lower bound model without surveys still implies stickiness of rate expectations at the effective lower bound. The main difference is that these expectations are then lower on average than in the models including interest rate surveys, implying somewhat higher term premia on average. The opposite is true for the affine models without interest rate surveys. Without this information the model is no longer able to produce paths of the expected short rate which remain at the ELB for an extended period of time. This is reflected in the average expected short rate path which immediately starts mean-reverting. After all, this leads to premia which are substantially lower and highly negative across all maturities.

Among models that include surveys, differences in forecast performance are again small (see Table B.5). Overall, including surveys leads to lower forecast errors in terms of root mean squared errors (RMSEs), in particular, for shorter maturities and at shorter forecast horizons. Both models including survey information about interest rate expectations outperform their counterparts not including this information in terms of forecasting errors. While differences are minor for the shadow rate model, RMSEs in the affine model without survey information exceed those of the other models more than two-fold. Thus, in line with the results of Kim and Orphanides (2012), it shows that surveys can help to produce more plausible in-sample forecasts in affine models.

For longer maturities, results are more mixed as surveys do not necessarily seem to improve forecast performance even for shorter forecast horizons. However, this is not a surprising result given the high uncertainty surrounding such long-term forecasts. Also, as documented by Crump, Eusepi, and Moench (2017), it shows that long-term surveys systematically overshoot long-term rate realizations. However, despite the poor performance of these surveys, we still see good reason to include them, considering this as a trade-off between a better forecasting performance and producing model-implied rate expectations close to what market participants actually believed at a certain point in time.

Marked differences in forecast performance become evident in almost all models when samples before and after reaching the ELB are compared.[17] For both lower bound models and the affine model including surveys, the forecast errors become very small for the short maturities of 6-months and 1-year, ranging between 8 and 29 bps at the 6- and 1-year forecast horizon. While the lower bound model without surveys also fares quite well in this sample, it is still outperformed by both models including surveys, even though none of these can beat the random walk, which performs equally well.[18]

Overall, the similarity of both model-implied yield decompositions means, that for plain decompositions of the yield curve, affine models may be appropriate, which for policy analysis would be a particularly useful outcome for at least two reasons: 1) these models are much easier to estimate given that observable pricing factors can be used. 2) The use of observable factors allows them to be computed at a daily frequency, thus facilitating more timely analysis. However, it is also important to note that similar decompositions of the affine and non-affine

[17]Here, the lower bound period is defined as the sub-sample starting in June 2012.

[18]Note that for inflation forecasts, the sample period plays less of a role, with forecast performances across models being roughly the same over the full and both sub-samples (see Table B.6).

model should not hide the fact that the affine model still regards rate realisations well below the ELB as likely outcomes. Further, it also still fails to replicate the stylized fact of reduced interest rate volatility at the ELB.

### 4.3 Lower Bound Implied Non-linearities and Inflation Shocks

The following subsection discusses ELB-implied structural changes in yield curve responses to shocks. While real and nominal decompositions do not necessarily have to differ by much in non-linear lower bound and affine models (see Section 4.2), the lower bound may still be of importance for how external shocks transmit along the yield curve. Given that inflation enters as observable factors, while at the same time the factor error standard deviation $\Sigma$ is assumed to be diagonal (see Footnote 8), the model allows analysis of the response of yield components to a typical shock-induced increase in inflation. Thus, inflation shocks are easily fed into the model. Although, factor dynamics are linear, yields are non-linear functions of those states. Ultimately, this is done by computing the difference between expected model-implied yields, expectations and premia conditional on state $X_t$ and $X_t^{shock}$. Thus, e.g., for yields it follows for the impulse response ($IR$) in all future periods $t+h$ conditional on $t$ that

$$IR_{t+h} = E_t^{\mathbb{P}}(y_{t+h}|X_t^{shock}) - E_t^{\mathbb{P}}(y_{t+h}|X_t) \tag{28}$$

In the following, we consider a 10-bp shock to $X_t^{\Pi}$.[19]

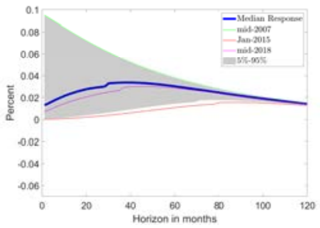
Figure 4 and Figure 5 show the impulse responses to such a shock at the 2- and 10-year maturity over the entire sample. To mark periods near to or far away from the ELB, impulse responses for June 2007 and January 2015 are plotted together with the median impulse response and the one for mid-2018, when the ECB was still approaching an exit from its unconventional measures.

At the 2-year maturity, the shock turns out to be quite persistent, fading after around 3 years, and it transmits mainly through expected inflation, while the reaction of the inflation

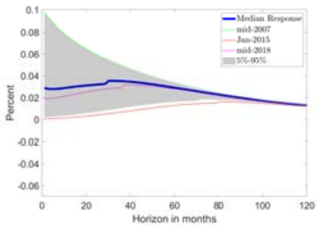
[19]Note that for the computation of the impulse responses to inflation shocks, it might be important that inflation enters the model subject to a measurement error. As inter alia discussed by Joslin, Le, and Singleton (2013), this brings about the advantage that the model has greater flexibility for fitting the cross-section of yields. However, the improved fit of yields tends to come at the cost of a worse fit of the macro factor, so that large parts of its volatility are assigned to its measurement error. On the one hand, this helps the modeller to arrive at more reliable yield decompositions. On the other hand, this may imply less reliable impulse responses of the yield curve components to macro shocks.

Figure 4: Impulse responses to a typical 10 bps increase in inflation at the 2-year maturity

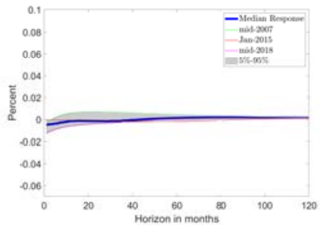
(a) Nominal yield



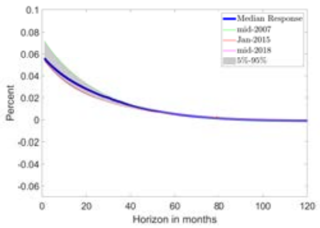

(b) Nominal expectation



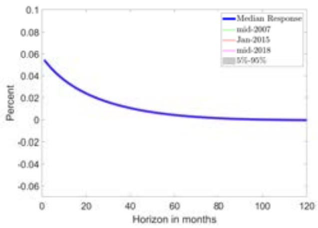

(c) Term premium



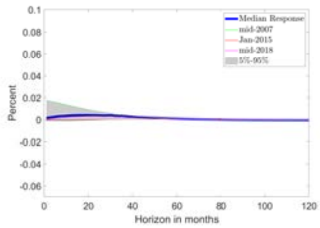

(d) Inflation compensation



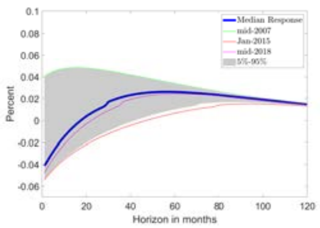

(e) Inflation expectation



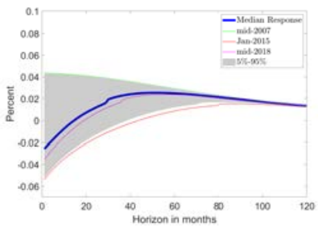

(f) Inflation risk premium



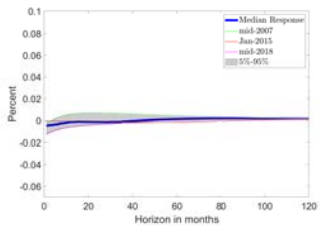

(g) Real yield




(h) Real expectation




(i) Real risk premium




Note: Panel (a)-(i) depict the impulse responses of nominal, inflation and real components of 2-year yields to a typical 10 bps increase in inflation based on the lower bound model $RTSM_{LB}$. In the panels, grey areas depict the range of responses over the sample.

risk premium is very muted (see Panels (d) to (f) in Figure 4). As implied by the linear state dynamics, the response of inflation expectations is the same over the sample. In line, with the observation of inflation risk premia converging towards zero at the lower bound, Panel (f) in Figure 4 suggests that in cases where interest rates were expected to remain at the ELB for long (e.g. in January 2015; see red line in Figure 4) the inflation risk premium at the 2-year horizon no longer reacts.

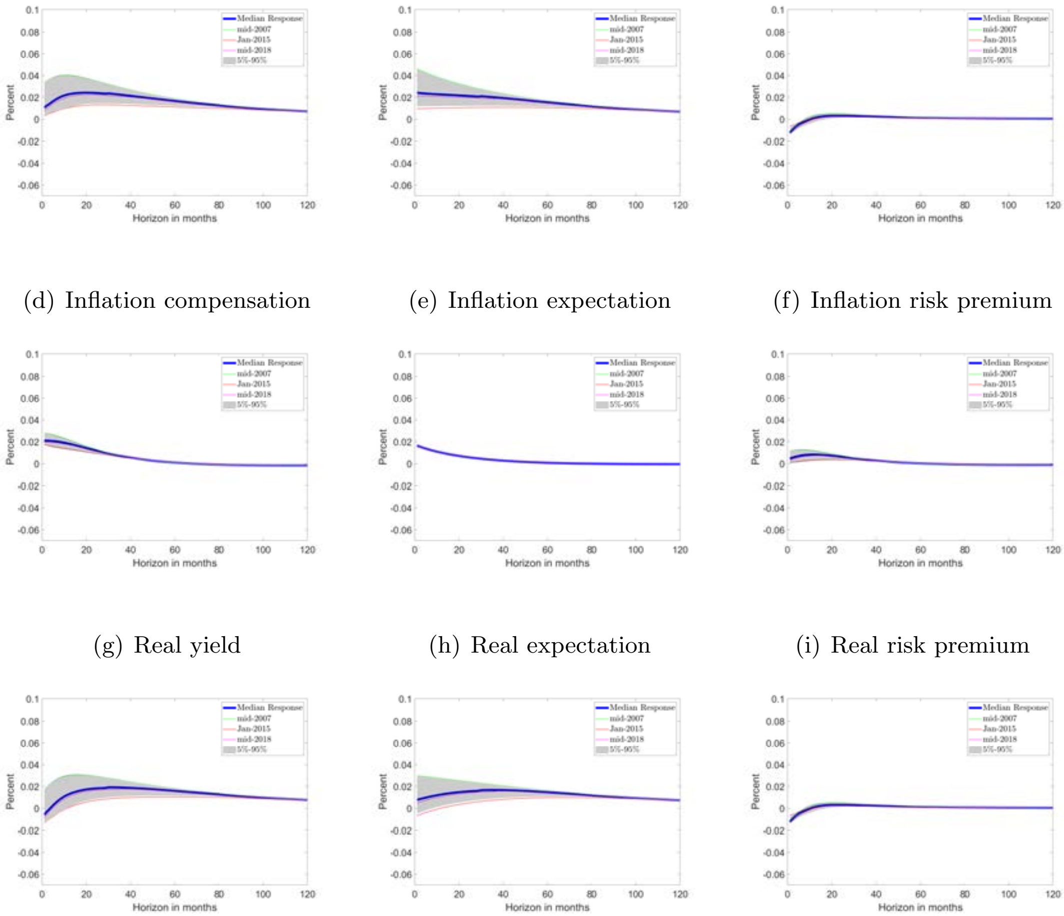
As the comparison of impulse responses over the sample reveals, the response of nominal and real yields is highly dependent on the distance to the ELB (Panels (a) to (c) and (g) to (i) of

Figure 5: Impulse responses to a typical 10 bps increase in inflation at the 10-year maturity

(a) Nominal yield (b) Nominal expectation (c) Term premium

(d) Inflation compensation (e) Inflation expectation (f) Inflation risk premium

(g) Real yield (h) Real expectation (i) Real risk premium




Note: Note: Panel (a)-(i) depict the impulse responses of nominal, inflation and real components of 10-year yields to a typical 10 bps increase in inflation based on the lower bound model $RTSM_{LB}$. In the panels, grey areas depict the range of responses over the sample.

Figure 4)). Away from the ELB, the 2-year yield reacts almost one to one to the shock due to the high persistence of the model, and it is mainly driven by a positive response of its expectations component. The closer yields are to the ELB, however, the more muted their reaction, until it is almost zero when at the ELB like in early 2015. This pattern in nominal yields eventually has important implications for real yields. While the latter respond in a muted but positive fashion to a typical increase in inflation when nominal yields are far from the ELB, their response turns quite negative once nominal yields are at the lower bound.

In principle, the muted reaction in nominal yields is well in line with the narrative of a successful implementation of forward guidance by the Eurosystem, anchoring short-rate expectations at the lower bound.[20] Against this background, these results prove to be meaningful for policy makers as they show that when short-rate expectations are successfully anchored at the lower bound, shocks to inflation c.p. can create an additional accommodative impact on the economy by lowering real rate expectations. This opens up an additional transmission channel of forward guidance beyond its direct impact on financing costs through lowering medium- to long-term yields.[21]

It is worth emphasizing that this is not an obvious result, as the ELB naturally restricts interest rates only to the downside and not to the upside. For a rather technical rational for this result, recall that a binding lower bound implies a shadow rate which lies below that lower bound, and the further out the ELB is binding, the more negative this shadow rate tends to be. Also recall that the shadow rate is nothing more than the sum of all factors, so that the latent factors will be such that smaller shocks to them will not raise the short rate above the ELB. This broadly explains why, in the model, short-term yields would not react to shocks in the inflation factor.

For completeness, Appendix F reports the same exercise conducted in the affine model. Naturally, these shocks are not conditional on date $t$, but are rather similar across the entire sample. Thus, the affine model fails to describe the non-responsiveness of short-term nominal rates potentially implied by monetary policy throughout the ELB period. This implies that the affine model also fails to produce negative responses of the real rate to a typical increase in inflation throughout this period. Given the observed stickiness of nominal short rates and given the forward guidance that was in place, these might be considered unfavorable characteristics for a term structure model if being used for structural analyses of yield curve responses to macro shocks.

These findings are in line with other papers documenting structural non-linearities induced by the ELB. Mertens and Williams (2018), in a small structural model, show that the ELB has direct implications for the distribution of both interest rate expectations and inflation, as the ELB confines the central bank to acting as a stabilizer in the presence of shocks to the

[20] Feroli, Greenlaw, Hooper, and Mishik (2017) for example show that forward guidance can break the link between macro news and yields, leaving the latter insensitive to macro shocks.

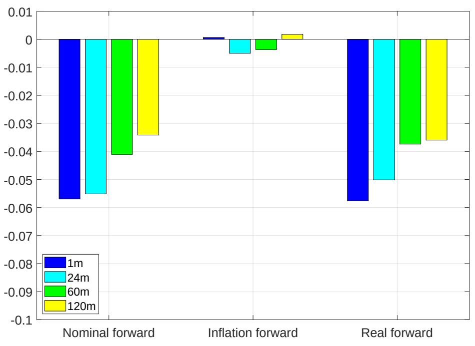
[21] The presented results are qualitatively in line with results in an earlier version of Roussellet (2020), in which the author in a similar exercise finds a lower impact of a negative inflation shock on nominal yields in times during which the zero lower bound is binding. This earlier version is available under `https://www.guillaumeroussellet.com/uploads/1/1/9/7/119769374/full_paper_atsm_2018.pdf`.

economy. With respect to the effectiveness of monetary policy, King (2019) combines a model of Vayanos and Vila (2009) with a lower bound and shows that unconventional monetary policy loses some of its power to affect yields even at longer maturities once closer to the lower bound. Finally, Geiger and Schupp (2018), in a nominal shadow rate term structure model, show that conventional monetary policy becomes less effective at the lower bound as both rate expectations and term premia react less to conventional monetary policy shocks.

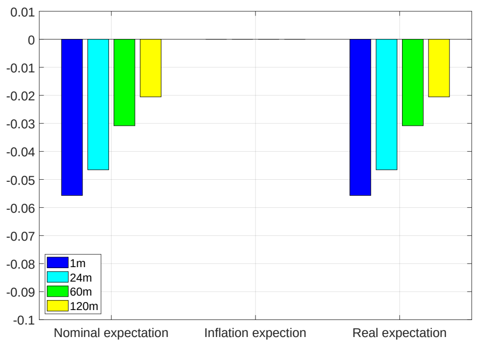
### 4.4 Quantifying the Impact of the Effective Lower Bound

Figure 6: Impact of a changing lower bound on yield components

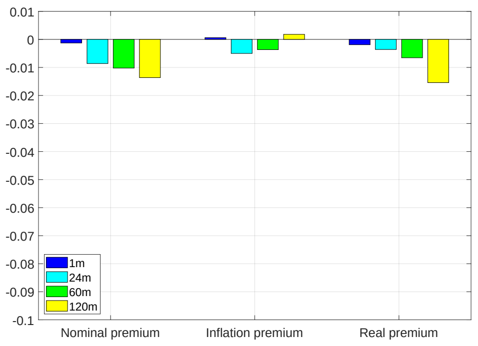
(a) Forwards




(b) Expectations




(c) Premia




Note: Panel (a) depicts the impact of a 10-bp cut in the effective lower bound on 1-month nominal, real and inflation forwards averaged over the sample at different maturities. Panel (b) depicts the impact of a 10-bp cut in the lower bound on nominal, real and inflation forward expectations averaged over the sample at different maturities. Panel (c) depicts the impact of a 10-bp cut in the lower bound on nominal, real and inflation risk premia averaged over the sample at different maturities.

The following sections discuss the isolated impact of changes in the lower bound on the different yield components. By computing model-implied yield and forward components under different calibrations of the ELB, we are able to quantify the impact that changes in the lower bound have on nominal and real yields, and potentially also on inflation components. Formally, this means that we recompute all yield components conditional on the filtered pricing factors, but differing assumptions about the ELB. For example, the impact of a 10-bp cut in the ELB on forward rates is computed as

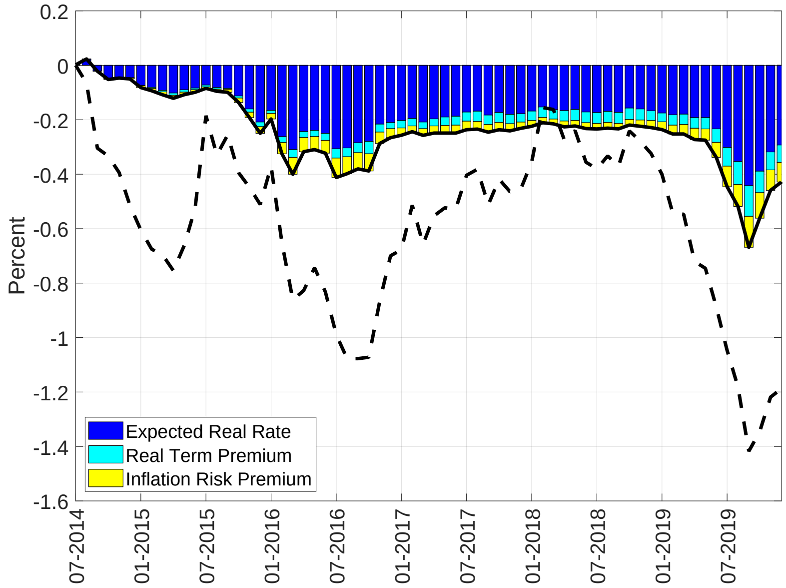
$$\Delta f_t|\Delta ELB_t = f(X_t|ELB_t - 10bps) - f(X_t|ELB_t) \tag{29}$$

Along these lines, Figure 6 depicts the average changes of forward components over the entire sample due to a 10-bp cut in the ELB at each point in the sample for different maturities.[22] The average impact of such a cut on nominal forwards is around 5 to 6 bps for the 12-month forward rate and decreases to around 2 to 3 bps at the 10-year maturity. As the results for inflation and real forwards shows, the ELB impact works mainly through real components. This is unsurprising, given that the model does not constrain the inflation process, such that non-linearities mainly propagate through nominal and real rates. Interestingly, it shows that changing the ELB mainly affects the expectations component, while the effect on premia is only around 1 bps for a 10-bp cut. Naturally, these effects are greater towards the end of the sample, when rates are close to the ELB, and somewhat smaller when rates were still some distance away from it (see Figure E.1 and E.2).

In the following, we employ our model to estimate the impact monetary policy in the euro area had on long-term yields by allowing interest rates to fall below zero. The assumption of this exercise is that had the ECB never cut rates below zero, market participants would have ruled out negative rates for good, effectively truncating the rate distribution at zero. Technically, the isolated effect of lowering the ELB is computed by keeping a zero lower bound over the entire sample and compare resulting yields and yield components to ones obtained by keeping latent factors and inflation constant as of the introduction of negative rates, but feeding the model with the actual lower bound. Thus, we can obtain the decrease in yields that would have been observed under constant macro conditions, thus only induced by the decreasing lower bound.

[22] In particular, for each point in time the ELB is lowered by 10 bps from its prevailing level as it is observed by the model at this point in time.

Figure 7: Cumulative impact of lowering the lower bound on the 10-year yield since mid-2014




Note: The chart depicts the isolated cumulative change of the 10-year yield induced only by changes in the effective lower bound since mid-2014. The dashed black line depicts the actual change in the 10-year yield.

As can be seen in Figure 7, the decreasing lower bound added as much as around 40 bps to the overall cumulative change in the 10-year yield since mid-2014.[23] In particular, in line with the results presented above, this contribution was almost entirely transmitted through lower real rate expectations. This strengthens the conclusion that lowering the lower bound below zero has been an effective tool for injecting real stimulus into the economy.

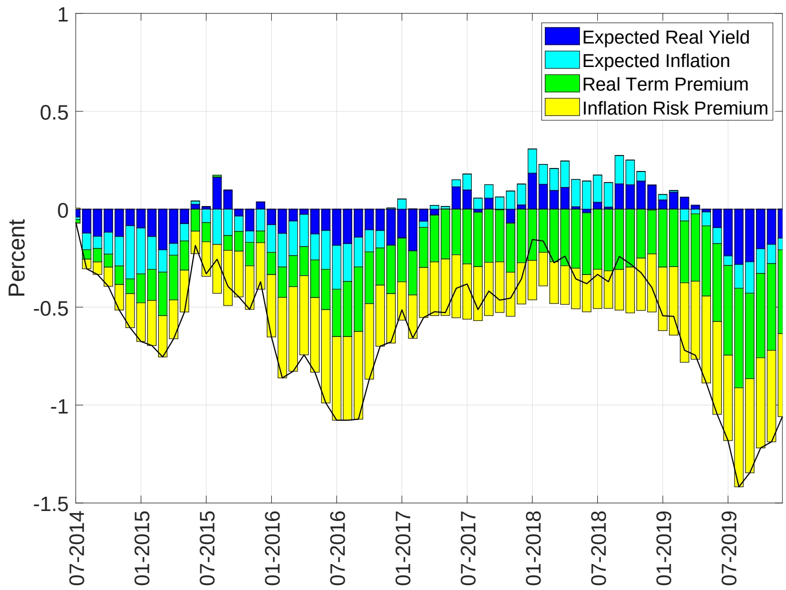
## 4.5 The Decline in Long-term Yields in the Context of the Eurosystem's Unconventional Measures

In the following, the proposed model is applied to decompose the change in nominal long-term rates between mid-2014 and mid-2016. This decline is often considered to have been initiated in anticipation of the Eurosystem's unconventional monetary policy measures, in particular its large-scale asset purchases. Commonly, such programmes are considered to affect yields mainly

[23] A similar counterfactual scenario has been computed by Rostagno, Altavilla, Carboni, Lemke, Motto, Saint-Guilhem, and Yiangou (2019) who derive the impact of the ECB's negative interest rate policy from simulations based on a 3-month Euribor options-implied distribution. Their results suggest a slightly bigger impact of negative rates, but one that is broadly in the same ballpark as those presented here.

through two channels: 1) the duration extraction or portfolio rebalancing channel affecting risk premia (see Vayanos and Vila (2009)) and 2) the rate signalling channel affecting rate expectations (see Bauer and Rudebusch (2014)). Indeed, the results support the view that monetary policy had an impact through these channels. In particular, this view finds support in the finding that the decline in nominal rate expectations and premia was to a good extent driven by real rate expectations and real risk premia (see Figure 8).

Figure 8: Decomposition of cumulative change in the nominal 10-year yield




Note: The chart depicts the decomposition of the cumulative changes in the nominal 10-year yield between June 2014 and December 2019.

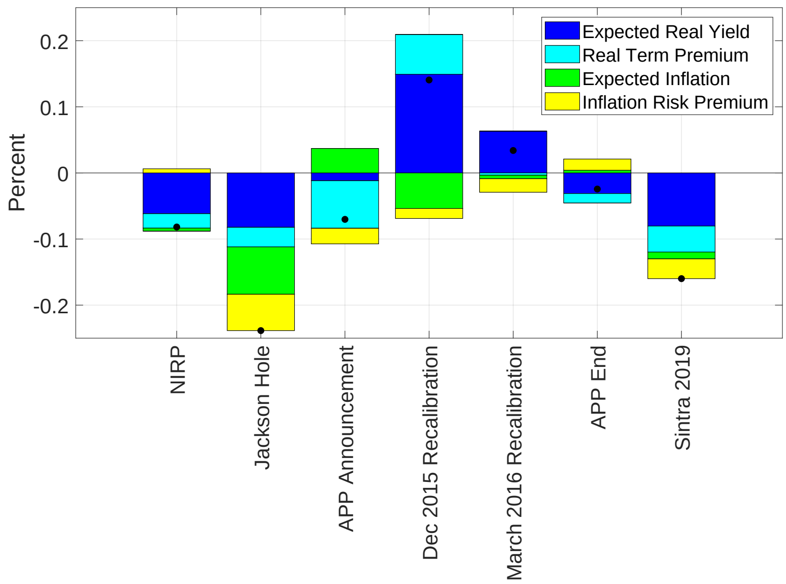
At the same time, however, model results also imply that the decline in yields also reflects to a large extent a decline in inflation expectations and the inflation risk premium (see Figure 8). In fact, almost half of that decline of around 100 bps is accounted for by decreasing inflation components, which implies that the observed change in nominal yields in 2014 or 2016 was also the result of increasing expectations about low future inflation.[24]

This may seem puzzling given the plethora of monetary policy measures and the accompa-

[24]Note that since around 2012 implied levels of the IRP are found to be negative across all maturities (see e.g. Camba-Mendez and Werner (2017) or García and Werner (2012) for the euro area and Carriero et al. (2018) and Christensen and Spiegel (2019) for the UK and Japan, respectively). As can be shown, such negative IRPs would in general be expected in a situation in which investors were anticipating low future growth paired with low inflation (see Appendix G).

nying decline in real rate expectations and premia mentioned above. One might be tempted to conclude that the unprecedented measures taken by the ECB – while indeed lowering yields – might have led the market to believe that the economic outlook may be worse than previously expected. In addition, there might even have been a lack of trust in the abilities of monetary policy to reverse the negative inflation trend. In fact, a similar line of argument has been advocated for Japan, for which Christensen and Spiegel (2019) come to the conclusion that the introduction of negative interest rates in January 2016 did indeed have the perverse effect of lowering inflation expectations, contrary to its original intention. Overall, such a pattern would be in line with the interpretation that many of the measures were perceived as negative information shocks rather than accommodative monetary policy shocks as defined by Jarociński and Karadi (ming).[25]

Figure 9: Changes in the 10-year yield around selected monetary policy events




Note: The chart depicts decomposed changes in the 10-year yield around selected monetary policy events. The events comprise the first introduction of negative interest rate policy (NIRP), Mario Draghi's Jackson Hole speech in 2014, in which preparations for asset purchases were first mentioned, the official announcement of the ECB's asset purchase programme (APP), its recalibrations in December 2015 and March 2016, the first announcement of its end by end-2018, and Mario Draghi's speech in Sintra in 2019.

One caveat of the above analysis, of course, is that it is not based on a structural analysis,

[25] Alternatively, observations are also in line with results by Vaccaro-Grange (2019), who finds that the ECB's unconventional monetary policy measures have had a negative impact on inflation between 2014 and 2016 via the credit cost channels as they significantly lowered financing costs of firms.

meaning it remains silent on the exact drivers of the observed change in yields. The analysis takes a closer look at changes in the 10-year yield around important monetary policy events in the euro area in an attempt to close in on the pure policy impact since 2014. Figure 9 depicts the decomposed month-on-month changes around selected policy decisions and announcements. The events comprise the first introduction of negative interest rate policy (NIRP), Mario Draghi's Jackson Hole speech in 2014, in which preparations for asset purchases were first mentioned, the official announcement of APP, its recalibrations in December 2015 and March 2016, the first announcement of its end by end-2018, and Mario Draghi's Sintra speech in which he held out the prospect for additional monetary stimulus. While a marked decline in expected real yields and the real risk premium was observed around the majority of these events, it is striking that a marked increase in inflation expectations occurred only around the APP announcement in January 2015. By contrast, in the month of Mario Draghi's Jackson Hole speech in 2014 as well as around the decisions of the Governing Council in December 2015 – widely considered a disappointment – inflation expectations decreased considerably. While these monthly changes again cannot be considered structural responses to monetary policy, they still highlight that monetary policy over the last years has struggled to sustainably create more optimism about long-term inflation expectations with its decisions. In the end, a stronger increase in inflation expectations was only observed late in 2016, well into the ECB's APP and coinciding with a general improvement in the economic outlook on a global level. While not offering final conclusions on the exact impact monetary policy had on long-term yields in the sample considered, the exercise should emphasize the importance of not only looking into nominal yield decomposition, when analyzing the effectiveness of the policy tools used.

# 5 Concluding Remarks

I propose a joint real-nominal model for the euro area which incorporates a time-varying lower bound for nominal yields as a new and unique feature. Overall, the model is able to produce a satisfying fit of both nominal yields and inflation-linked swap rates. At the same time, it fits survey information about interest rate and inflation expectations quite well, which indicates a plausible decomposition of all yield components despite my small sample, which was limited in size due to constraints on the availability of inflation-linked swap rates.

As is shown by a shock analysis within the proposed model, shock responses of both nominal and real yields are affected by the degree to which the ELB is binding, underlining its importance for structural analysis of the economy. In addition, the importance of the ELB for monetary policy makers is highlighted by further analyses showing that the ELB itself may be a tool for monetary policy to lower real rate expectations and thus induce monetary stimulus.

At the same time, comparing results from the lower bound model with those from an affine version of that model suggests that the incorporation of a lower bound does not necessarily make a substantial difference in terms of the decomposition of yields or inflation components if both models are informed by survey expectations. Nevertheless, the lower bound model is better at replicating observed second moments of yields once they approach the lower bound, as affine models per assumption imply constant conditional volatility of yields.

Based on the proposed model, the decline in long-term nominal yields since mid-2014 is decomposed into real and inflation components. On the one hand, the results support the conclusion that, to some extent, the decline may indeed have been driven by monetary policy, in particular its large-scale asset purchases, which may be the driver of the implied decline in real rate expectations and the real risk premium. On the other hand, according to the model-implied decomposition, the decline in nominal yields was to a large extent also driven by falling inflation expectations and inflation risk premia. This lends some support to the narrative that the Eurosystem's unprecedented unconventional measures might have worsened the perceived outlook for inflation through negative information effects, following the argument of Christensen and Spiegel (2019) in the case of Japan. Indeed, monthly changes in yield components around important monetary policy events show that neither inflation expectations nor the inflation risk premium increased in most months in which those measures were decided.

Some caveats do remain for the analyses presented in this paper. While my model was able to produce persistent interest rate and inflation expectations, yet allowing for some volatility, including at long-term horizons, the fundamental assumption of stationarity does not allow any conclusions about whether or not the unconditional mean of interest rates or inflation has changed. Hence, while long-term rate and inflation expectations may temporarily decrease, they will always return to their constant unconditional mean. Hence, no conclusions can be drawn about whether or not the real natural rate has decreased permanently. The same holds true for any conclusions about a possible permanent de-anchoring of inflation expectations.

The analysis could thus be extended upon in the future by introducing greater flexibility in this regard as suggested by, e.g., Bauer and Rudebusch (2019) or Brand, Goy, and Lemke

(2020), who allow for a unit root in their expectations components. Another aspect that could be addressed in future work, is the possibility of a more structural modelling of the inflation process by inter alia adding more macroeconomic structure to the model. This could reduce the reliance on survey information and increase the weight of market data.

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# Appendix

## A Appendix - Nominal Pricing Recursions

In the following nominal bond pricing recursions are derived following Ang et al. (2008). Nominal prices $P_{n+1,t}$ are related to real prices $P^*_{n+1,t}$ via the price deflator $Q_t$:

$$P_{n+1,t} = P^*_{n+1,t}Q_t = E_t[m^*_{t+1}\frac{Q_t}{Q_{t+1}}p^{*n}_{t+1}Q_t] = E_t[m_{t+1}p^n_{t+1}], \tag{A.1}$$

Further, for the nominal pricing kernel it holds that

$$m_{t+1} = m^*_{t+1}\frac{Q_t}{Q_{t+1}} = m^*_{t+1}exp(-\pi_{t+1}) = exp(-r^*_t - \pi_{t+1} - 0.5\lambda'_t\lambda_t - \lambda'_t\epsilon_{t+1}). \tag{A.2}$$

Further, nominal prices are assumed to be exponentially affine functions of the factors $X_t$:

$$p^n_t = exp(A_n + B_nX_t) \tag{A.3}$$

Thus,

$$\begin{aligned} P_{n+1,t} =& exp(-i^* - \pi_{t+1} - 0.5\lambda'_t\lambda_t - \lambda'_t\epsilon_{t+1})exp(A_n + B'_nX_{t+1}) && \text{(A.4)} \\ =& exp(-i^* - \rho_0^{\pi,\mathbb{P}} - \rho_1^{\pi,\mathbb{P}}X_t - \Sigma^\pi\epsilon_{t+1} - 0.5\lambda'_t\lambda_t - \lambda'_t\epsilon_{t+1}\dots \\ & + A_n + B'_n(\rho_0^{\mathbb{P}} + \rho_1^{\mathbb{P}}X_t + \Sigma\epsilon_{t+1})) && \text{(A.5)} \\ =& exp(-i^* - \rho_0^{\pi,\mathbb{P}} - \rho_1^{\pi,\mathbb{P}}X_t - 0.5\lambda'_t\lambda_t + A_n + B'_n\rho_0^{\mathbb{P}} + B'_n\rho_1^{\mathbb{P}}X_t)\dots \\ & E_t[exp(\epsilon_{t+1}(B'_n\Sigma - \lambda'_t - \Sigma^\pi))] && \text{(A.6)} \end{aligned}$$

To further simplify A.6, note that $E(exp(b\epsilon)) = exp(0.5bIb')$, so that

$$\begin{aligned} P_{n+1,t} =& exp(-i^* - \rho_0^{\pi,\mathbb{P}} - \rho_1^{\pi,\mathbb{P}}X_t - 0.5\lambda'_t\lambda_t + A_n + B'_n\rho_0^{\mathbb{P}} + B'_n\rho_1^{\mathbb{P}}X_t)\dots \\ & E_t[exp(0.5B'_n\Sigma'\Sigma B'_n + 0.5\lambda'\lambda + 0.5\Sigma^{\pi'}\Sigma^\pi - B'_n\Sigma\lambda'_t - B'_n\Sigma\Sigma^\pi + \Sigma^\pi\lambda'_t)] && \text{(A.7)} \\ & && \text{(A.8)} \end{aligned}$$

Substituting for $i^*, \pi_{t+1}$ and $\lambda_t$

$$
\begin{aligned}
=&exp(-\delta_0 - \delta_1 X_t - \rho_0^{\pi,\mathbb{P}} - \rho_1^{\pi,\mathbb{P}} X_t + A_n + B_n'(\rho_0^{\mathbb{P}} - \Sigma\lambda_0) + 0.5B_n'\Sigma'\Sigma B_n \ldots \\
&+ 0.5\Sigma^{\pi'}\Sigma^{\pi} - B_n'\Sigma\Sigma^{\pi} - B_n'\Sigma'\lambda_0 - [\delta_1' - \rho_1^{\pi,\mathbb{P}} + B_n'(\rho_1^{\mathbb{P}} - \Sigma\lambda_1) + \Sigma^{\pi,\mathbb{P}}\lambda_1]X_t)
\end{aligned} \tag{A.9}
$$

From A.3 for an n-period bond it then holds that

$$
\begin{aligned}
A_{n+1} =& -\delta_0 - \rho_0^{\pi,\mathbb{P}} + A_n + B_n'(\rho_0^{\mathbb{P}} - \Sigma'\lambda_0) + 0.5B_n'\Sigma'\Sigma B_n \ldots \\
&+ \Sigma^{\pi}\lambda_0 + 0.5\Sigma^{\pi'}\Sigma^{\pi} - B_n'\Sigma\Sigma^{\pi}
\end{aligned} \tag{A.10}
$$

$$
B_{n+1} = -\delta_1' - \rho_1^{\pi,\mathbb{P}} + B_n'(\rho_1^{\mathbb{P}} - \Sigma'\lambda_1) + \Sigma^{\pi}\lambda_1, \tag{A.11}
$$

and

$$
A_1 = -\delta_0 - \rho_0^{\pi,\mathbb{P}} + \Sigma^{\pi}\lambda_0 + 0.5\Sigma^{\pi'}\Sigma^{\pi} \tag{A.12}
$$

$$
B_1 = -\delta_1' - \rho_1^{\pi,\mathbb{P}} + \Sigma^{\pi}\lambda_1. \tag{A.13}
$$

Continuously compounded interest rates then follow

$$
\begin{aligned}
i_{n,t} =& -\frac{1}{n}log(P_{n,t}) \\
=& -\frac{1}{n}(-A_n - B_n X_t) \\
=&a_n + b_n' X_t
\end{aligned} \tag{A.14}
$$

with $a_n = -A_n/n$ and $b_n = -B_n/n$.

# B Appendix - Model performance

Table B.1: In-sample model fit of yields and survey interest rate forecasts of model $RTSM_{LB}$

| Maturity in months | 1 | 3 | 6 | 12 | 24 | 36 | 60 | 84 | 120 | avg |
|---|---|---|---|---|---|---|---|---|---|---|
| Yields (MAE) | | | | | | | | | | |
| Total sample: | 5 | 4 | 4 | 4 | 3 | 4 | 3 | 3 | 5 | 4 |
| Pre-ELB sample: | 9 | 5 | 4 | 5 | 5 | 5 | 3 | 4 | 6 | 5 |
| ELB sample: | 1 | 3 | 3 | 3 | 2 | 2 | 3 | 2 | 4 | 3 |
| Expected 3M-rate in x months | 3 | 6 | 9 | 12 | 15 | 18 | 21 | $72-120$ | | |
| Interest rate surveys (MAE) | | | | | | | | | | |
| Total sample: | 8 | 11 | 11 | 12 | 13 | 13 | 16 | 13 | | |
| Pre-ELB sample: | 12 | 15 | 15 | 17 | 18 | 18 | 24 | $--$ | | |
| ELB sample: | 5 | 6 | 7 | 8 | 8 | 8 | 9 | 13 | | |

Note: This table shows the mean absolute errors (MAE) of model-implied yields and short-rate expectations compared to observed yields and survey forecasts for selected sample periods in basis points obtained based on the model $RTSM_{LB}$. The total sample covers the period June 2005 to December 2019, while the pre-ELB sample covers the period June 2005 to June 2012, and the ELB sample the period July 2012 to December 2019.

Table B.2: In-sample model fit of inflation-linked swap rates and survey inflation forecasts of model $RTSM_{LB}$

| Maturity in months | 12 | 24 | 36 | 48 | 60 | 84 | 108 | 120 | avg |
|---|---|---|---|---|---|---|---|---|---|
| Yields (MAE) | | | | | | | | | |
| Total sample: | 4 | 3 | 2 | 2 | 2 | 2 | 2 | 3 | 2 |
| Pre-ELB sample: | 4 | 3 | 2 | 3 | 3 | 2 | 3 | 3 | 3 |
| ELB sample: | 3 | 2 | 2 | 1 | 1 | 1 | 2 | 2 | 2 |
| Expected y-o-y inflation in x years | 1 | 2 | 5 | $72-120$ | | | | | |
| Inflation expectations surveys (MAE) | | | | | | | | | |
| Total sample: | 31 | 29 | 5 | 7 | | | | | |
| Pre-ELB sample: | 23 | 17 | 3 | 7 | | | | | |
| ELB sample: | 41 | 42 | 7 | 6 | | | | | |

Note: This table shows the mean absolute errors (MAE) of model-implied inflation expectations under the risk-neutral and historical probability measure compared to observed inflation-linked swap rates and survey inflation forecasts for selected sample periods in basis points obtained based on the model $RTSM_{LB}$. The total sample covers the period June 2005 to December 2019, while the pre-ELB sample covers the period June 2005 to June 2012, and the ELB sample the period July 2012 to December 2019.

Table B.3: In-sample model fit of yields and survey interest rate forecasts of model $RTSM_{woLB}$

| Maturity in months | 1 | 3 | 6 | 12 | 24 | 36 | 60 | 84 | 120 | avg |
|---|---|---|---|---|---|---|---|---|---|---|
| Yields (MAE) | | | | | | | | | | |
| Total sample: | 6 | 3 | 2 | 5 | 6 | 5 | 4 | 4 | 7 | 5 |
| Pre-ELB sample: | 9 | 4 | 4 | 9 | 7 | 5 | 5 | 6 | 8 | 7 |
| ELB sample: | 3 | 2 | 1 | 2 | 4 | 4 | 3 | 3 | 6 | 3 |
| Expected 3M-rate in x months | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 72 – 120 | | |
| Interest rate surveys (MAE) | | | | | | | | | | |
| Total sample: | 7 | 11 | 13 | 14 | 14 | 15 | 17 | 18 | | |
| Pre-ELB sample: | 9 | 13 | 16 | 17 | 18 | 21 | 29 | −− | | |
| ELB sample: | 5 | 9 | 10 | 11 | 11 | 9 | 7 | 18 | | |

Note: This table shows the mean absolute errors (MAE) of model-implied yields and short-rate expectations compared to observed yields and survey forecasts for selected sample periods in basis points obtained based on the model $RTSM_{woLB}$. The total sample covers the period June 2005 to December 2019, while the pre-ELB sample covers the period June 2005 to June 2012, and the ELB sample the period July 2012 to December 2019.

Table B.4: In-sample model fit of inflation-linked swap rates and survey inflation forecasts of model $RTSM_{woLB}$

| Maturity in months | 12 | 24 | 36 | 48 | 60 | 84 | 108 | 120 | avg |
|---|---|---|---|---|---|---|---|---|---|
| Yields (MAE) | | | | | | | | | |
| Total sample: | 4 | 2 | 3 | 3 | 3 | 2 | 2 | 3 | 3 |
| Pre-ELB sample: | 4 | 3 | 3 | 2 | 3 | 2 | 2 | 3 | 3 |
| ELB sample: | 4 | 2 | 3 | 3 | 3 | 2 | 2 | 3 | 3 |
| Expected y-o-y inflation in x years | 1 | 2 | 5 | $72-120$ | | | | | |
| Inflation expectations surveys (MAE) | | | | | | | | | |
| Total sample: | 29 | 31 | 8 | 10 | | | | | |
| Pre-ELB sample: | 20 | 19 | 7 | 10 | | | | | |
| ELB sample: | 48 | 43 | 10 | 10 | | | | | |

Note: This table shows the mean absolute errors (MAE) of model-implied inflation expectations under the risk-neutral and historical probability measure compared to observed inflation-linked swap rates and survey inflation forecasts for selected sample periods in basis points obtained based on the model $RTSM_{woLB}$. The total sample covers the period June 2005 to December 2019, while the pre-ELB sample covers the period June 2005 to June 2012, and the ELB sample the period July 2012 to December 2019.

Table B.5: Yield forecasts

| Forecast horizon | 6-months | | | | 12-months | | | | 24-months | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Yield maturity | 6M | 1Y | 5Y | 10Y | 6M | 1Y | 5Y | 10Y | 6M | 1Y | 5Y | 10Y |
| Full Sample: | | | | | | | | | | | | |
| $RTSM_{LB}$: | 0.58 | 0.60 | 0.64 | 1.04 | 0.94 | 0.93 | 0.81 | 1.11 | 1.51 | 1.46 | 1.12 | 1.23 |
| $RTSM_{LB}^{noIRSurveys}$: | 0.63 | 0.68 | 1.08 | 1.36 | 0.98 | 1.00 | 1.25 | 1.46 | 1.53 | 1.54 | 1.60 | 1.69 |
| $RTSM_{woLB}$: | 0.57 | 0.61 | 1.22 | 1.68 | 0.90 | 0.87 | 1.20 | 1.61 | 1.39 | 1.26 | 1.09 | 1.40 |
| $RTSM_{woLB}^{noIRSurveys}$: | 1.20 | 1.19 | 0.97 | 1.63 | 1.71 | 1.60 | 1.03 | 1.61 | 2.64 | 2.34 | 1.09 | 1.43 |
| $RandomWalk$: | 0.63 | 0.67 | 0.64 | 0.49 | 0.99 | 0.96 | 0.74 | 0.74 | 1.47 | 1.43 | 1.11 | 0.95 |
| pre ELB-period: | | | | | | | | | | | | |
| $RTSM_{LB}$: | 0.86 | 0.88 | 0.75 | 0.83 | 1.38 | 1.35 | 0.90 | 0.85 | 2.25 | 2.14 | 1.20 | 0.87 |
| $RTSM_{LB}^{noIRSurveys}$: | 0.92 | 0.97 | 0.80 | 0.62 | 1.42 | 1.41 | 1.00 | 0.71 | 2.26 | 2.18 | 1.46 | 0.99 |
| $RTSM_{woLB}$: | 0.84 | 0.90 | 1.76 | 2.41 | 1.32 | 1.27 | 1.73 | 2.35 | 2.08 | 1.84 | 1.49 | 2.08 |
| $RTSM_{woLB}^{noIRSurveys}$: | 1.60 | 1.54 | 1.16 | 2.33 | 2.37 | 2.15 | 1.25 | 2.34 | 3.79 | 3.26 | 1.26 | 2.17 |
| $RandomWalk$: | 0.63 | 0.64 | 0.55 | 0.49 | 0.99 | 0.96 | 0.74 | 0.66 | 1.47 | 1.43 | 1.11 | 0.95 |
| ELB-period: | | | | | | | | | | | | |
| $RTSM_{LB}$: | 0.08 | 0.10 | 0.49 | 1.20 | 0.14 | 0.17 | 0.66 | 1.28 | 0.29 | 0.35 | 0.91 | 1.50 |
| $RTSM_{LB}^{noIRSurveys}$: | 0.09 | 0.20 | 1.28 | 1.78 | 0.17 | 0.31 | 1.40 | 1.89 | 0.41 | 0.59 | 1.67 | 2.15 |
| $RTSM_{woLB}$: | 0.08 | 0.10 | 0.34 | 0.51 | 0.14 | 0.17 | 0.47 | 0.55 | 0.30 | 0.36 | 0.77 | 0.72 |
| $RTSM_{woLB}^{noIRSurveys}$: | 0.64 | 0.71 | 0.79 | 0.55 | 0.72 | 0.79 | 0.84 | 0.55 | 1.03 | 1.07 | 1.03 | 0.62 |
| $RandomWalk$: | 0.08 | 0.09 | 0.27 | 0.37 | 0.13 | 0.14 | 0.40 | 0.54 | 0.23 | 0.23 | 0.52 | 0.67 |

Note: This table shows the root mean squared errors (RMSE) of in-sample model-implied yield forecasts for 6-month, 1-year, 5-year and 10-year yields. Forecasts are computed based on a lower bound model with and without surveys ($RTSM_{LB}$ and $RTSM_{LB}^{noIRSurveys}$) and an affine model with and without surveys ($RTSM_{woLB}$ and $RTSM_{woLB}^{noIRSurveys}$). The total sample covers the period June 2005 to December 2019 while the pre-ELB sample covers the period June 2005 to June 2012 and the ELB sample the period July 2012 to December 2019.

Table B.6: Inflation forecasts

| Sample | Full Sample | | | pre-ELB period | | | ELB period | | |
|---|---|---|---|---|---|---|---|---|---|
| Forecast horizon | 6M | 1Y | 2Y | 6M | 1Y | 2Y | 6M | 1Y | 2Y |
| $RTSM_{LB}$: | 0.76 | 0.86 | 1.05 | 0.89 | 1.00 | 1.13 | 0.63 | 0.78 | 1.04 |
| $RTSM_{LB}^{noIRSurveys}$: | 0.80 | 0.89 | 1.03 | 0.91 | 0.99 | 1.09 | 0.69 | 0.84 | 1.05 |
| $RTSM_{woLB}$: | 0.75 | 0.85 | 1.05 | 0.88 | 0.99 | 1.14 | 0.61 | 0.76 | 1.04 |
| $RTSM_{woLB}^{noIRSurveys}$: | 0.91 | 1.09 | 1.30 | 1.05 | 1.31 | 1.52 | 0.78 | 0.95 | 1.17 |
| $Surveys$: | $--$ | 0.89 | 1.05 | $--$ | 1.06 | 1.16 | $--$ | 0.75 | 1.04 |
| $RandomWalk$: | 0.78 | 1.21 | 1.49 | 0.99 | 1.57 | 1.71 | 0.55 | 0.85 | 1.31 |

Note: This table shows the root mean squared errors (RMSE) of in-sample model-implied and survey year-on-year inflation forecasts for 6 month, 1 year and 2 years ahead. Model-implied forecasts are computed based on a lower bound model with and without surveys ($RTSM_{LB}$ and $RTSM_{LB}^{noIRSurveys}$) and an affine model with and without surveys ($RTSM_{woLB}$ and $RTSM_{woLB}^{noIRSurveys}$). The total sample covers the period June 2005 to December 2019 while the pre-ELB sample covers the period June 2005 to June 2012 and the ELB sample the period July 2012 to December 2019.

# C Appendix - Parameter Estimates

Table C.1: **Parameter estimates**

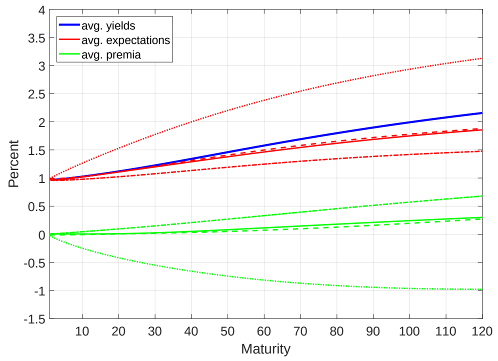
| | $RTSM_{LB}$ | | | | $RTSM_{woLB}$ | | | |
|---|---|---|---|---|---|---|---|---|
| $\rho_0^{\mathbb{Q}}$ | -0.002 | 0 | 0 | 0.137 | 0.015 | 0 | 0 | 0.072 |
| | (0.011) | (-) | (-) | (0.023) | (0.007) | (-) | (-) | (0.030) |
| $\rho_1^{\mathbb{Q}}$ | 0.999 | 0 | 0 | 0 | 0.999 | 0 | 0 | 0 |
| | (–) | (–) | (–) | (–) | (–) | (–) | (–) | (–) |
| | 0 | 0.992 | 0 | 0 | 0 | 0.998 | 0 | 0 |
| | (–) | (0.002) | (–) | (–) | (–) | (0.000) | (–) | (–) |
| | 0 | 0 | 0.859 | 0 | 0 | 0 | 0.949 | 0 |
| | (–) | (–) | (0.026) | (–) | (–) | (–) | (0.010) | (–) |
| | -0.015 | 0.275 | -0.043 | 0.962 | -0.014 | 0.103 | -0.063 | 0.988 |
| | (0.005) | (0.066) | (0.037) | (0.006) | (0.001) | (0.069) | (0.042) | (0.003) |
| $\lambda_0$ | -122.171 | -731.478 | -29.833 | 43.388 | -211.073 | -119.720 | 59.519 | 78.184 |
| | (179.258) | (372.541) | (-29.833) | (87.033) | (136.670) | (159.838) | (322.842) | (90.220) |
| $\lambda_1$ | -28.016 | 48.661 | -5.240 | 88.012 | -6.311 | 192.222 | 62.068 | 119.449 |
| | (42.508) | (411.087) | (150.363) | (69.841) | (53.822) | (52.676) | (392.874) | (48.5413) |
| | 236.948 | -1343.800 | 902.023 | 100.308 | 129.493 | -700.193 | 800.670 | -111.600 |
| | (108.872) | (530.727) | (312.138) | (128.362) | (70.894) | (417.486) | (429.347) | (70.557) |
| | -139.261 | -384.688 | -19.980 | -42.946 | -210.516 | -319.586 | -566.944 | 123.171 |
| | (60.077) | (444.814) | (208.762) | (70.969) | (73.472) | (456.066) | (596.253) | (79.353) |
| | -33.206 | -493.048 | -177.157 | -88.005 | -27.032 | -86.377 | -152.416 | -43.853.345 |
| | (14.493) | (150.363) | (137.567) | (29.316) | (9.912) | (79.744) | (38.476) | (18.668) |
| $\Sigma$ | 0.484 | 0 | 0 | 0 | 0.240 | 0 | 0 | 0 |
| | (0.095) | (–) | (–) | (–) | (0.047) | (–) | (–) | (–) |
| | 0 | 0.042 | 0 | 0 | 0 | 0.041 | 0 | 0 |
| | (–) | (0.007) | (–) | (–) | (–) | (0.006) | (–) | (–) |
| | 0 | 0 | 0.370 | 0 | 0 | 0 | 0.132 | 0 |
| | (–) | (–) | (0.110) | (–) | (–) | (–) | (0.014) | (–) |
| | 0 | 0 | 0 | 0.273 | 0 | 0 | 0 | 0.273 |
| | (–) | (–) | (–) | (0.044) | (–) | (–) | (–) | (0.051) |
| $\delta_0$ | 0 | | | | 0 | | | |
| | (–) | | | | (–) | | | |
| $\delta_1$ | 1 | 1 | 1 | 0.461 | 1 | 1 | 1 | -0.173 |
| | (–) | (–) | (–) | (0.164) | (–) | (–) | (–) | (0.107) |

Note: The table depicts parameter estimates for both the joint real-nominal model incorporating a lower bound ($RTSM_{LB}$) and the joint real-nominal model not including the lower bound ($RTSM_{woLB}$). Asymptotic quasi-maximum standard errors in parentheses.

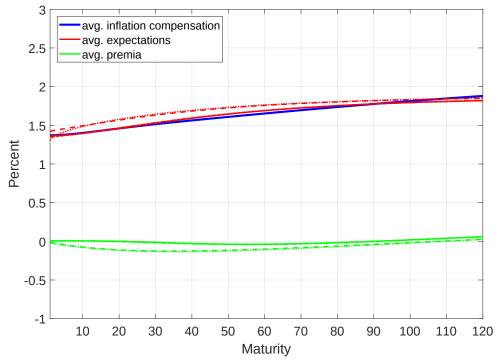
# D Appendix - Decomposing the term structure with and without surveys

Figure D.1: **Average decomposition of yield components**

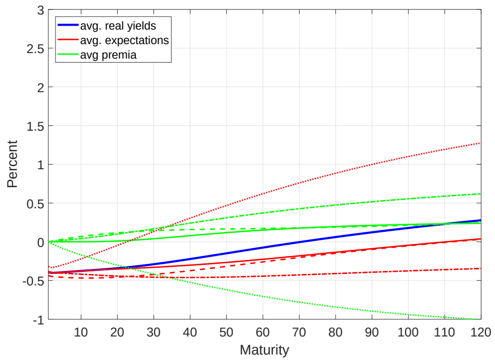
(a) Nominal yields




(b) Inflation-linked swap rates




(c) Real yields

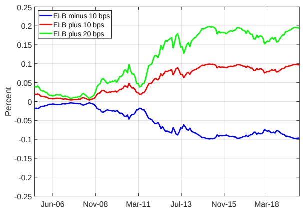



Note: Panel (a) depicts the average model-implied decomposition of the term structure of nominal yields. Panel (b) depicts the average model-implied decomposition of the term structure of inflation-linked swap rates. Panel (c) depicts the average model-implied decomposition of the term structure of real yields. In the panels, solid lines are based on the lower bound model ($RTSM_{LB}$), while dashed lines depict results from the affine model ($RTSM_{woLB}$), and dotted lines from the affine model without surveys, ($RTSM_{woLB}^{noIRSurveys}$) and dashed dotted lines from the lower bound model without surveys ($RTSM_{LB}^{noIRSurveys}$).

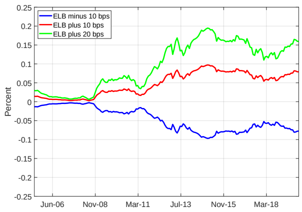
# E Appendix - Impact of changes in the effective lower bound on yield components

Figure E.1: **Impact of changes in the ELB on 2-year forward components**

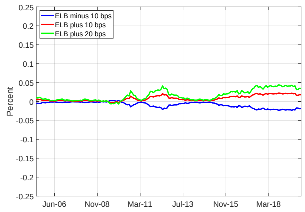
(a) Nominal yield



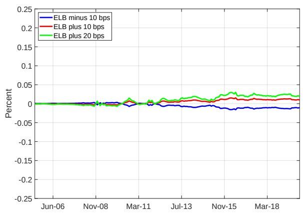

(b) Nominal expectation



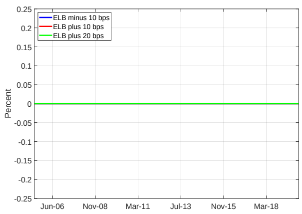

(c) Term premium



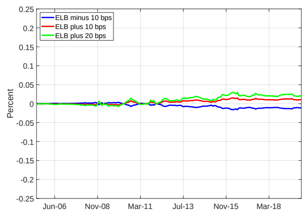

(d) Inflation compensation



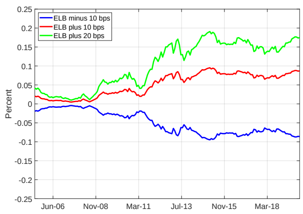

(e) Inflation expectation



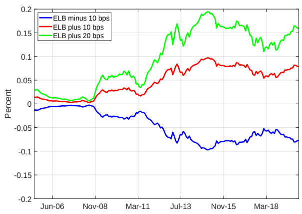

(f) Inflation risk premium



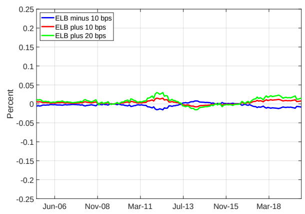

(g) Real yield




(h) Real expectation




(i) Real risk premium



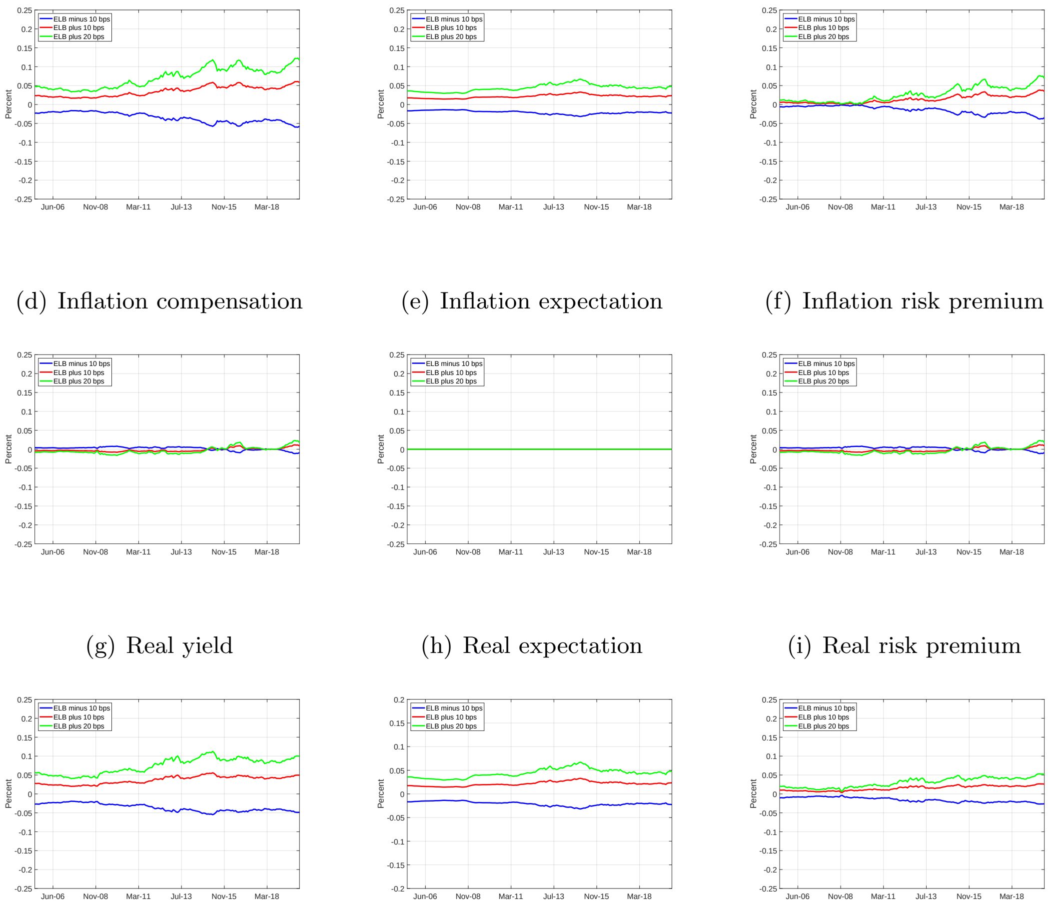

Note: Panels depict the impact of a -10, 10 and 20-bp change in the effective lower bound on 2-year forward components. Impacts are obtained by first computing counterfactual components based on the originally filtered states and estimated parameters but with a changed ELB. Subsequently, the differences between these counterfactuals and actual model-implied components are computed.

Figure E.2: **Impact of changes in the ELB on 10-year forward components**

(a) Nominal yield (b) Nominal expectation (c) Term premium

(d) Inflation compensation (e) Inflation expectation (f) Inflation risk premium

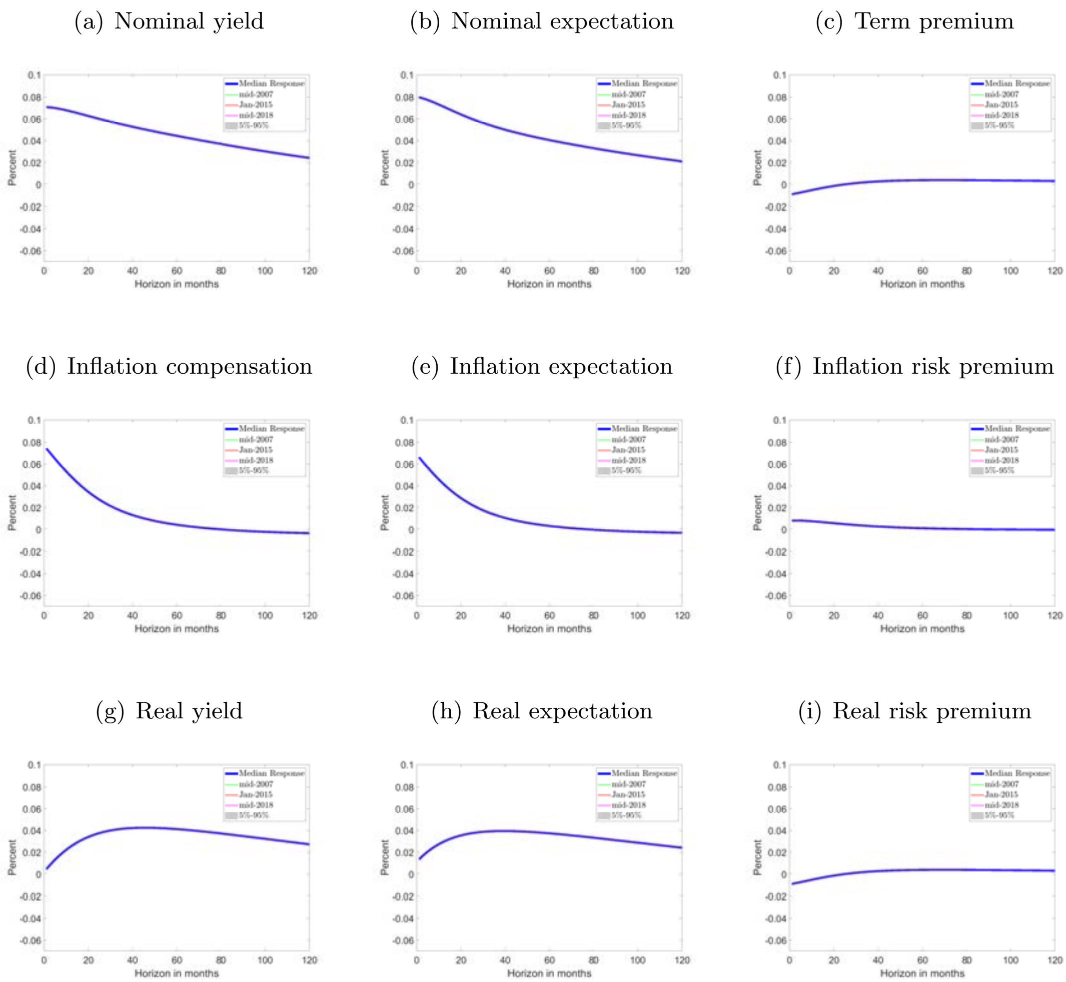
(g) Real yield (h) Real expectation (i) Real risk premium




Note: Panels depict the impact of a -10, 10 and 20-bp change in the effective lower bound on 2-year forward components. Impacts are obtained by first computing counterfactual components computed based on the originally filtered states and estimated parameters but with a changed ELB. Subsequently, the differences between these counterfactuals and actual model-implied components are computed.

# F Appendix - Inflation shocks in the affine model

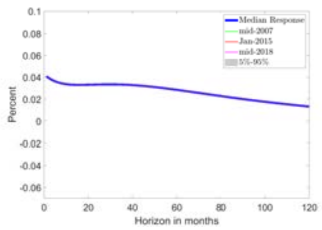
Figure F.1: **Impulse responses to a typical 10 bps increase in inflation at the 2-year maturity**



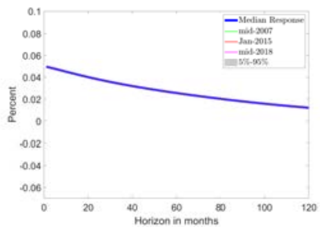

Note: Note: Panel (a)-(i) depict the impulse responses of nominal, inflation and real components of 10-year yields to a 10 bps increase in inflation based on the affine model $RTSM_{woLB}$. In the panels, grey areas depict the range of responses over the sample.

Figure F.2: **Impulse responses to a typical 10 bps increase in inflation at the 10-year maturity**

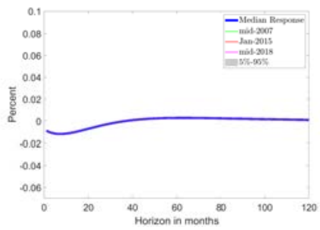
(a) Nominal yield



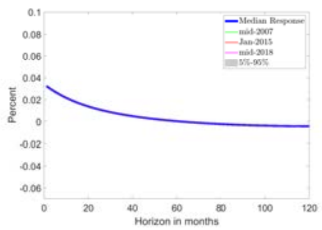

(b) Nominal expectation



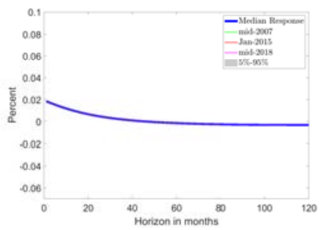

(c) Term premium



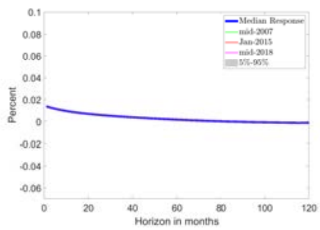

(d) Inflation compensation



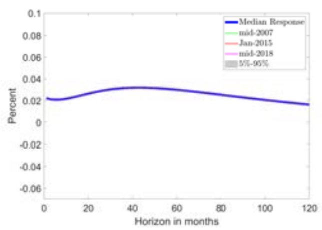

(e) Inflation expectation



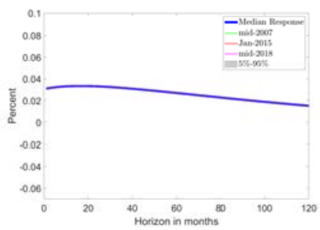

(f) Inflation risk premium



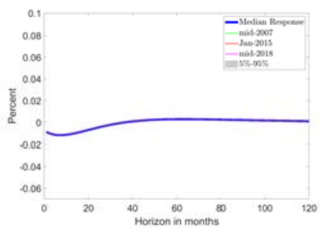

(g) Real yield




(h) Real expectation




(i) Real risk premium




Note: Note: Panel (a)-(i) depict the impulse responses of nominal, inflation and real components of 10-year yields to a typical 10 bps increase in inflation based on the affine bound model $RTSM_{woLB}$. In the panels, grey areas depict the range of responses over the sample.

# G Appendix - Negative inflation risk premia

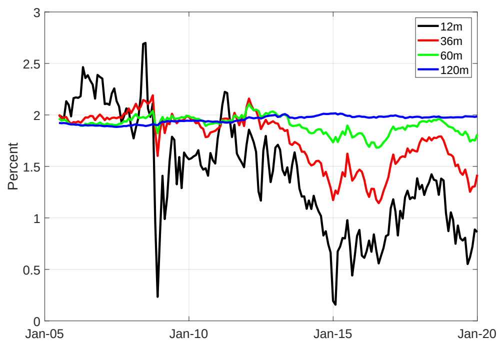
The appendix focuses on the model-implied inflation risk premia and in particular on the finding that these have been negative since around 2011. In general, market-based inflation compensation is the sum of genuine inflation expectations and the inflation risk premia demanded by investors. In the model, inflation compensation is defined as the $\mathbb{Q}$-expectation about the inflation factor $\Pi$, while genuine inflation expectations are obtained under the historical probability measure $\mathbb{P}$. It then holds, that

$$E_t^{\mathbb{Q}}(\Pi_{t+h}) = E_t^{\mathbb{P}}(\Pi_{t+h}) + IRP_t, \tag{G.1}$$

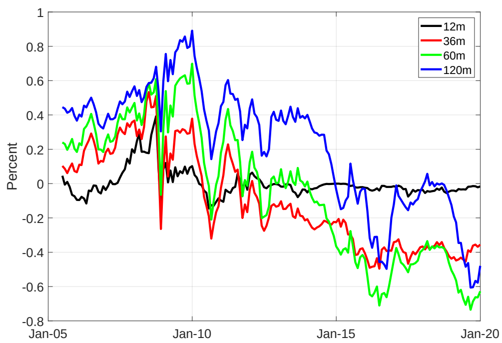
with the inflation risk premium ($IRP$) obtained as described by Equation 25. The model identifies the dynamics of the inflation factor under the $\mathbb{Q}$- and $\mathbb{P}$-measure and thus, the inflation risk premium, on the back of the information included on survey inflation expectations and ILS rates. The model implies an unconditional mean of inflation of around 1.9% (see Table 1), which seems to be in line with the Eurosystem's declared intention of keeping inflation below, but close to, 2% over the medium term.

Figure G.1: Inflation expectations and inflation risk premia

(a) Expectations




(b) Premia




Note: Panel (a) depicts the model-implied expectations about the 1-month inflation 1 year, 3 years, 5 years and 10 years ahead. Panel (b) depicts the normalized model-implied 1-month inflation risk premia 1 year, 3 years, 5 years and 10 years ahead.

Our model generates negative inflation risk premia, in particular at short- and medium-term maturities, since roughly the beginning of 2013, confirming the results by Camba-Mendez and Werner (2017). While negative inflation risk premia were also observed temporarily in the course of the financial crisis, this phenomenon is far more persistent over the second half of the sample (see Figure G.1). Since they turned negative in around early 2013, they followed a remarkable downward trend down to levels of below -0.6% in fall 2016 and late 2019. Economically, this may be interpreted as investors demanding a positive inflation risk premium, insuring against a higher-than-expected inflation outcome prior to 2013. Since 2013, they have since been willing to accept negative inflation risk premia, which may reflect some concerns about lower-than-expected inflation outcomes.

While negative risk premia are neither a new nor abnormal phenomenon, they often raise eyebrows when mentioned as their economic interpretation is not straightforward. It is easiest to think about negative premia as an insurance premium. If a given asset is promising safe returns in adverse states of the world, any risk-averse investor may be willing to pay more than the expected return to alleviate his situation, were this adverse state to materialize. Even though this is a generally applicable explanation for negative rates, it is still worth investigating, how risk premia and their signs are determined for any given asset.

Readers may recall, that any asset pricing model including those discussed in this paper build on a fundamental pricing equation:[26]

$$P_{n,t} = E_t[M_{t+1}, X_{n,t+1}], \tag{G.2}$$

where $M_{t+1}$ is the stochastic discount factor and $X_{n,t+1}$ the asset's payoff in $t+1$.

One-period gross holding returns are further defined as

$$1 + R_{n,t+1} = \frac{X_{n,t+1}}{P_{n,t}}, \tag{G.3}$$

so that Equation G.2 can be expressed as

$$1 = E_t[M_{t+1}(1 + R_{1,t+1})] \tag{G.4}$$

[26]The following derivations follow Geiger (2011).

As both $M_{t+1}$ and the one-period return $R_{n,t+1}$ are considered random variables, it holds that

$$E_t[M_{t+1}(1+R_{n,t+1})] = E_t(M_{t+1})E_t(1+R_{n,t+1}) + cov_t(M_{t+1}, R_{n,t+1}) \tag{G.5}$$

Substituting G.5 into G.4 yields

$$1 + E_t(R_{n,t+1}) = \frac{1 - cov_t(R_{n,t+1}, M_{t+1})}{E_t(M_{t+1})} = \frac{1}{M_{t+1}} - \frac{cov_t(R_{n,t+1}, M_{t+1})}{E_t(M_{t+1})} \tag{G.6}$$

If the covariance between the one-period return and the stochastic discount factor is zero, G.6 collapses to

$$1 + E_t(R_{n,t+1}) = \frac{1}{E_t(M_{t+1})} = 1 + R^f_{n,t+1} \tag{G.7}$$

Hence, when the covariance term is zero, the return is considered to be risk-free.[27] The risk premium is positive only if $cov_t(R_{n,t+1}, M_{t+1}) < 0$ and vice versa.

At this point, we know the conditions for the risk premium to be zero, positive or negative. To further gain some economic intuition of what these conditions imply, it is helpful to consider a simple two-period optimization problem some investor might be facing.

Let us assume that the investor wants to maximize her utility through the current and next period's consumption subject to some budget constraint.

$$\begin{aligned} &\max_{C_t, C_{t+1}} U(C_t, C_{t+1}) \\ &s.t.\ \ C_t = e_t - P_t \\ &and\ \ C_{t+1} = e_{t+1} + P_t(1 + R_{t+1}) \end{aligned} \tag{G.8}$$

[27]To see that this holds in the model presented in the main text, recall that we assume the real pricing kernel to equal $m^*_{t+1} = exp(-si^*_1 - 0.5\lambda_t{\prime}\lambda_t - \lambda_t\epsilon_t)$. Recall that if $x$ is a normally distributed random variable, $Y = e^x$ is log-normally distributed with $E(Y) = exp(E(x) + 0.5var(x))$. Thus $E(m^*_{t+1}) = exp(-i_{1,t})$, where $i_{1,t}$ is the one period risk-free rate.

where $C$ denotes consumption, $e$ endowments and $R$ the return of assets held, which are denoted by $P$. Further, we assume additive intertemporal utility

$$U(C_t, C_{t+1}) = u(C_t) + \beta E_t[u(C_{t+1})] \tag{G.9}$$

From the first-order conditions (FOCs) it then follows

$$P_t = E_t \left[ \beta \frac{u\prime(C_{t+1})}{u\prime(C_t)} X_{t+1} \right]. \tag{G.10}$$

From Equation G.10 and Equation G.2 (the no-arbitrage pricing formula), it then follows that

$$E_t(M_{t+1}) = E_t \left[ \beta \frac{u\prime(C_{t+1})}{u\prime(C_t)} \right], \tag{G.11}$$

saying that the SDF in this set-up equals the marginal rate of substitution multiplied by the investor's subjective discount factor. Dividing Equation $X$ by the price $P_t$ then yields

$$1 = E_t \left[ \beta \frac{u\prime(C_{t+1})}{u\prime(C_t)} \frac{X_{t+1}}{P_t} \right], \tag{G.12}$$

with $X_{t+1}/P_t = 1 + R_{t+1}$ as the gross return of the asset held, such that it holds that

$$1 = E_t \left[ \beta \frac{u\prime(C_{t+1})}{u\prime(C_t)} R_{t+1} \right]. \tag{G.13}$$

Note that for any two random variables $x$ and $y$ it holds that $E_t(xy) = E_t(x)E_t(y) + cov_t(x, y)$. Therefore, Equation G.13 can be rewritten as

$$E_t(1 + R_{t+1}) = \frac{1 - cov_t \left( R_{t+1} \beta \frac{u\prime(C_{t+1})}{u\prime(C_t)} \right)}{E_t \left( \beta \frac{u\prime(C_{t+1})}{u\prime(C_t)} \right)}. \tag{G.14}$$

Equation G.14 illustrates that the risk premium and its sign crucially depend on the covariance between the asset's gross return and the stochastic future consumption. Assume a situation, in which the marginal rate of substitution is high, i.e. expected marginal utility in the next period is higher than marginal utility today, which means that the investor would prefer some more consumption in the next period over consumption today. If the asset in such situations typically yields a lower return, such that the covariance term in G.14 is negative, the investor overall demands a higher gross return. Note that while the covariance between asset returns and consumption growth determines the sign of the asset-specific risk premium, volatility in returns and consumption also plays a role for its size, as it holds that

$$cov_t\left((1+R_{t+1})\beta\frac{u\prime(C_{t+1})}{u\prime(C_t)}\right) = corr\left((1+R_{t+1})\beta\frac{u\prime(C_{t+1})}{u\prime(C_t)}\right)\sqrt{var(1+R_{t+1})}\sqrt{var(M_{t+1})} \tag{G.15}$$

To derive the gross return for a risk-free asset, all that needs to be done is to set the covariance term in Equation G.14 to zero and assume that the risk-free asset's return is known with certainty, yielding

$$E_t(1+R_t^f) = \frac{1}{E_t\left(\beta\frac{u\prime(C_{t+1})}{u\prime(C_t)}\right)} = \frac{1}{E(M_{t+1})}. \tag{G.16}$$

**Pricing nominal assets**

The above is easily translated into nominal space. Let $CPI_t$ be the price index, then a nominal bond costs in nominal terms $P_{i,t}^{\$}$ and in units of goods $\frac{P_{i,t}^{\$}}{CPI_{,t}}$; it pays \$1 or equivalently $\frac{\$1}{CPI_t}$ in units of goods. An investor is faced with a maximization problem according to Equation G.17 with a modified budget constraint

$$\max_{C_t, C_{t+1}} U(C_t, C_{t+1}) \tag{G.17}$$
$$s.t. \;\; C_t = e_t - \frac{P_t^{\$}}{CPI_t}$$
$$and \;\; C_{t+1} = e_{t+1} + \frac{P_t^{\$}(1+R_{t+1}^{\$})}{CPI_{t+1}}$$

The FOCs then yield

$$\frac{1}{1+R_{n,t+1}^{\$}} = E_t\left[M_{t+1}, \frac{CPI_t}{CPI_{t+1}}\right] \tag{G.18}$$

so that it now holds that

$$E_t(1+R_{t+1}^{\$}) = \frac{1 - cov_t\left(\frac{CPI_t}{CPI_{t+1}}, \beta\frac{u\prime(C_{t+1})}{u\prime(C_t)}\right)}{E_t\left(\beta\frac{u\prime(C_{t+1})}{u\prime(C_t)}\right)}. \tag{G.19}$$

Hence, nominal gross returns now comprise a risk premium which depends on inflation and is non-zero if and only if the covariance between the stochastic discount factor and inflation is non-zero. Importantly, it is positive if and only if this covariance is negative; otherwise it's negative. The covariance between the SDF and inflation is positive if inflation is expected to be low, whenever expected future marginal utility is expected to be higher compared to current marginal utility. This is the case when future consumption is expected to be lower. Thus, we would expect to see negative inflation risk premia if investors were to expect states in which low consumption growth comes with low inflation. Note that this does not require investors to expect deflation. In fact, given G.19 no conclusions about deflation expectations can be drawn from the sign of the inflation risk premium.

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