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Gábor Fukker, Christoffer Kok

On the optimal control of interbank contagion in the euro area banking system

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Abstract

In this paper we present a methodology of model-based calibration of additional capital needed in an interconnected financial system to minimize potential contagion losses. Building on ideas from combinatorial optimization tailored to controlling contagion in case of complete information about an interbank network, we augment the model with three plausible types of fire sale mechanisms. We then demonstrate the power of the methodology on the euro area banking system based on a network of 373 banks. On the basis of an exogenous shock leading to defaults of some banks in the network, we find that the contagion losses and the policy authority's ability to control them depend on the assumed fire sale mechanism and the fiscal budget constraint that may or may not restrain the policy authorities from infusing money to halt the contagion. The modelling framework could be used both as a crisis management tool to help inform decisions on capital/liquidity infusions in the context of resolutions and precautionary recapitalisations or as a crisis prevention tool to help calibrate capital buffer requirements to address systemic risks due to interconnectedness.

Keywords: Interbank networks, contagion, fire sales, stress testing, macroprudential policy, optimal control

JEL: C61, D85, G01, G18, G21, G28, L14

Non-technical summary

In the aftermath of the global financial crisis, it has become clear that a main source of systemic risk is the high level of interconnectedness of the financial system. Therefore, following the early and rather theoretical works in the economic literature, regulators started to incorporate network analytical tools and models of financial system interconnections into their regular analytical and policy framework.

In order to cater for the risks related to the interconnected nature of the financial system, regulators and prudential authorities introduced new, mainly capital-based instruments primarily focused on the most systemic institutions. Although in theory the aim of the introduced measures is exactly to internalize the losses generated by financial institutions, the calibration of some of these measures is still based on simplistic or rules of thumb approaches without bringing complex network models to the policy space.

Our work aims at providing an approach that not just quantifies potential contagion losses but also serves as a tool that is able to calibrate the optimal response of the policymaker either as an ex-ante or ex-post application of the model. In other words, we aim to fill the gap between contagion models and optimal policy responses in a rigorously proven manner.

We focus on the most classical example of direct interbank contagion when bank balance sheets are directly connected via interbank loans. Exogenously defaulting banks are not able to pay back their outstanding obligations which may lead to default cascades. In addition, institutions in need of liquidity may decide to sell a fraction of their illiquid assets at fire sale prices, resulting in further losses. At the same time, banks may decide to withdraw their short-term assets from other banks which adds an additional liquidity shock to the system. All actions happen simultaneously in the system due to the payment equilibrium approach used. An additional amount of initial captital and/or liquidity (infusion) is able to minimise or even eliminate contagion losses. To solve the optimization problem, we use combinatorial optimization techniques to find the subset of banks which are to be saved by this infusion.

The modelling framework is applied to a network of close to 400 euro area banks. We run our simulations for several different types of fire sale assumptions as well as with and without an upper bound on the amount of infusion that the policy maker is able to provide.

Our main findings show that if the initial shock and the assumed fire sale mechanism are severe enough, the benefit from preventing banks from defaulting is higher than not intervening at all. When an upper bound on the infusion amount is assumed (e.g. due to a public budget constraint), gains are much lower which shows a non-linear property of the optimal infusion. This phenomenon is due to the fact that in case of a budget constraint, some banks may be excluded from the possible set of infused banks. We also show that the overall amount of contagion losses, and as a corollary the net benefit of infusion, crucially depends on the assumed fire sale mechanism.

The framework provides a rigorous approach to calculate systemic contagion losses in the context of tail risk events and how to minimise such losses. The modelling framework could be applied to calibrate ex-post infusion measures in the context of bank resolutions, such as a precautionary recapitalisation or provision of emergency liquidity assistance to ensure the resolution entity is able to honour its short-term payment obligations (i.e. liquidity in resolution). The framework could also be used to inform calibration of ex-ante capital buffer requirements, such as G-SII/O-SII buffers or Systemic Risk Buffers. In this context, we furthermore compare the model-derived optimal capital support to defaulting banks with standard network and regulatory-based measures of systemicness typically applied in the calibration of such buffers. In line with other studies, we find that such simple network metrics may not be sufficient to identify banks with the most systemic footprint and that more complex methods, such as the one presented here, can add value for policy assessments.

1 Introduction

While interconnections between financial institutions might have a stabilization property due to more diverse transferring of risk, it has been proven that this may only be the case up to a certain magnitude of shocks hitting the financial system (Acemoglu, Ozdaglar, and Tahbaz-Salehi 2015b; Roncoroni et al. 2019). The global financial crisis (GFC) was a clear illustration of a shock of sufficient magnitude to trigger contagion across the system requiring massive and costly taxpayer-financed policy interventions. Also the more recent market turmoil in February-March 2020 illustrated the potential for pernicious fire sales amplifying losses at the system level, again necessitating reactive policy interventions.

Thus financial interconnectedness has been recognized as one key source of systemic risk which in the aftermath of the GFC regulators have sought to mitigate by imposing higher capital requirements, especially aimed at systemically important financial institutions (BCBS 2012; BCBS 2013)[1].

Against this background, in this paper we present a rigorous modelling framework that allows for deriving the optimal response in the face of potential contagion losses to a material adverse shock to the banking system. The framework can be applied to help inform policy decisions aiming at optimally controlling (i.e. minimising) interbank contagion, while taking into account different fire sale mechanisms and limitations on the use of public resources.

Contagion models can be classified along several dimensions like the type of recovery rate, the presence of uncertainty and the point at which losses are triggered. Focusing on the recovery rate being a key feature since contagion models exist, approaches can assume an exogenous recovery rate or endogenous recovery rate. The latter one is widely known by the name of the authors (Eisenberg and Noe 2001) where equilibrium payments are proportional to the asset-to-debt ratio of financial institutions. This method has been further developed by Rogers and Veraart (2013) with the inclusion of bankruptcy costs. However, there are prominent examples for the application of an exogenous recovery rate as well, see Cont, Moussa, and Santos (2010) for a model applied to Brazil or Bardoscia et al. (2019) who focus on the contagion channel of credit valuation adjustments in a forward looking manner applied to the UK banking system. A more recent example for an application on European data is Covi, Gorpe, and Kok (2021) who use large

[1]Other policy responses in the markets for derivatives and securities financing transactions introduced mandatory clearing and more stringent collateral requirements.

exposure data of the euro area banking system.

In this work, we use the endogenous loss given default (LGD) approach inspired by Minca and Sulem (2014). Their key idea is that since payments are proportional to the asset-to-debt ratio of banks (Eisenberg and Noe 2001; Rogers and Veraart 2013), the asset-to-debt ratio can be iterated by a mapping to find an equilibrium value. By changing the value of cash infusion, this asset-to-debt ratio increases to a target value and the optimal value of additional cash needed to minimize interbank losses can be derived. Amini, Minca, and Sulem (2017) show a similar procedure for the exogenous recovery rate case.

Our application contributes to the two initial papers of Minca and Sulem (2014) and Amini, Minca, and Sulem (2017) on the optimal control problem in several aspects. First, we extend the model for three different types of fire sale mechanisms widely used in the literature and provide simple analytical solutions for the optimal amount of infusions. The way and degree to which assets are fire sold are notoriously difficult to pin down. In this light, we adopt a suite of model approach to ensure that our findings are robust to specific fire sale model assumptions. Second, we apply the methodology to a granular database of 373 euro area banks based on confidential ECB supervisory reporting. Third, we impose a realistic stress scenario on the network based on an official ECB stress test application. Fourth, we analyse situations when it is not worth to save a bank, also considering situations where the policy authorities have limited public funds available. Finally, we elaborate on the possible policy applications of the methodology.

More specifically, to illustrate how the model works we obtain an exogenous shock to the banking system from the capital shortfalls derived in the ECB's macroprudential perspective of the 2018 EBA stress test (Budnik 2019; Budnik et al. 2019). Banks having a positive capital shortfall, meaning its capital level falls below the regulatory minimum (defined by an assumed threshold level) due to the stress imposed, might have difficulties in paying back their obligations to other entities. At the same time, fire sale losses may turn up at banks under distress or at banks who fund themselves from banks in default when they are forced to sell an amount of their illiquid assets at discounted prices. Particularly, we implement three types of fire sale mechanisms well-known in the literature, namely: banks sell illiquid assets to cover the liquidity shortfall from interbank losses, short-term funding is withdrawn from defaulted banks and they need to sell some of their illiquid assets, or the short-term funding is withdrawn by the defaulted banks and fire sales occur also at banks who have not defaulted yet.[2] Actions happen simultaneously and the final equilibrium can be interpreted as an instantaneous equilibrium.

In the next step, the policymaker (a central bank or the government) is then assumed to calibrate an additional amount of liquid assets or capital to be injected for a bank to minimize contagion losses both from direct exposures and from fire sale losses. Liquidity and solvency are in practice different, but equivalent in our modelling framework: both capital buffers and liquidity reflect absorption capacity and once any of them is exhausted, the bank defaults. Practically, the policymaker's action can materialize as ex-post actions in the form of an emergency liquidity assistance provided by a national central bank or as a government recapitalization (e.g. precautionary recapitalization). To prevent such contagion losses, capital requirements or liquidity requirements can also be used as ex-ante measures. We demonstrate that it is possible to achieve no losses at all even if we do not save all banks that have been initially defaulted. That is, letting an institution go bankrupt does not necessarily have a systemic risk aspect if its default is not generating further significant losses to other banks. As such, the framework can also prove useful in "public interest" assessments in the context of bank resolution decisions. Throughout the paper we will refer to the additional amount of liquidity or capital needed as *infusion*. Our results also point to important non-linear contagion effects when the policymaker faces binding budget constraints: when we are not able to save a highly contagious bank due to the presence of a budget constraint, a contagion mechanism will still deliver significant amount of losses, hence the benefit from an infusion is not linear in the budget constraint. In such a scenario, also the specific fire sale assumption is shown to have important implications for the infusion results since infusions are a function of existing fire sale losses in the system[3].

We assume the interbank network to be static and not responding to changing market conditions. However, in principle it is possible to extend the model with a block that endogenously adjusts the changes in the given interbank network, like Hałaj and Kok (2015). An empirically estimated application of an endogenous network formation block would mean a substantial step forward in the systemic risk literature.

The rest of the paper is structured as follows. In section 2, we introduce

[2]In reality, fire sales may be driven simultaneously by all three elements and a mechanism combining these components may be more realistic. However, in order to better disentangle the different assumptions and to avoid pre-judging the exact mechanism at play we study here each element in turn.

[3]In the absence of a budget constraint, fire sale losses are not present.

the balance sheet interconnectedness through direct exposures and the fire sale mechanisms of illiquid assets. In section 3, the optimisation procedure for determining additional liquidity is described. We analyze our model results in section 4. Section 5 discusses the possible policy applications. Section 6 summarizes and concludes our findings.

2 Endogenous recovery rate and contagion

This section introduces the modelling approach. Banks' recovery rates are endogenously determined (Eisenberg and Noe 2001).

We have a network of banks with direct exposures like unsecured loans among each other. An initial shock induces the failure of some banks which leads to a cascade of failures. The policymaker may decide either to infuse capital in some of the surviving banks after the initial shock (ex-post measure) to limit contagion effects or expect banks to hold additional capital (ex-ante measure) to withhold contagion risk. The latter is not suitable once the crisis hits but has to be built up in advance to improve the loss absorption capacity.

Let $\mathcal{N} = \{1, \ldots, n\}$ be the set of banks in the network. The bank has capital c_i and the threshold level of default is $c_{i,th}$. Thus the bank is in solvency default if $c_i < c_{i,th}$. Matrix $\mathbf{L}$ represents bilateral exposures: $l_{i,j}$ is the debt of bank i to j. Like in the usual way (see figure 1 for a graphical representation), $\sum_i l_{i,j}$ denotes interbank assets of bank j, $\sum_i l_{j,i}$ are the interbank liabilities of bank j. We distinguish short-term liabilities of the bank s_i that are a subset of total liabilities $l_i = \sum_j l_{i,j}$. γ_i is the loss absorption or liquidity buffer of the bank. We assume that banks are able to convert their high quality liquid assets (HQLA) to cash by pledging them as collateral to a central bank. y_i are illiquid (non-HQLA) financial assets. These assets are subject to fire sale at a bank-dependent average price p_i. Let $\pi_{i,j}$ be the proportion of debt l_i toward bank j: $l_{i,j} = \pi_{i,j} l_i$. Similarly, $l_{j,i} = \pi_{j,i} l_j$.

The initial asset-to-debt ratio of the bank is $r_i = \frac{\gamma_i + \sum_j \pi_{j,i} l_j}{l_i}$. We note that we did not take into account non-interbank loans and deposits, but we show in appendix B that this setup is equivalent to a simplified iteration procedure of Eisenberg and Noe (2001) introduced by Rogers and Veraart (2013).

We make two further assumptions similarly to Hałaj and Kok (2013). First, banks can use their liquid assets for interbank payments only up to the amount of excess capital. Second, this excess capital base is adjusted with interbank assets and liabilities. These assumptions lead to the definition of liquid assets like $\gamma_i = \min(c_i - c_{i,th}, HQLA) + \sum_j l_{i,j} - \sum_j l_{j,i}$. With this definition of γ_i, our model will treat liquidity and solvency identically. In Section 5, we elaborate on how this can be treated in policymaking. We say that a bank is in fundamental default (initial default) if it is defaulted by the exogenous shock to the system: $\gamma_i < 0$ or $c_i - c_{i,th} < 0$. The two assumptions above lead to the following corollary linking the possible default thresholds.

Corollary 1. A bank is in fundamental default if $c_i - c_{i,th} < 0$ if and only if $r_i < 1$.

For the simple proof, see appendix A. Note that with the definition of γ_i, we made it possible to track liquidity default as well besides solvency default (see figure 1). However, the current EBA stress test methodology focuses on solvency default and we follow this approach when we take exogenous defaults in the system.

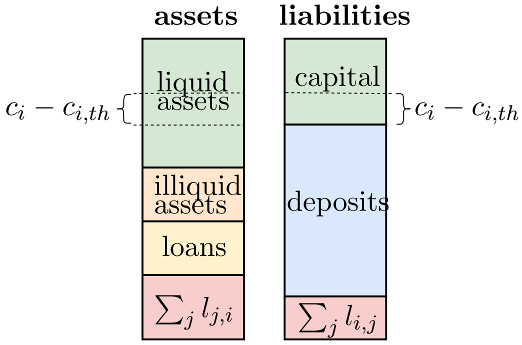

Figure 1: Balance sheet of banks and the use of liquid assets given an exogenous shock to capital.

2.1 Direct exposures

We assume that the policymaker is able to infuse capital into the banks in response to a shock. We call this an ex-post measure. Under infusion of equity ξ_i in bank i, liquid assets of bank i becomes $\gamma_i + \xi_i$ or we can equivalently say that capital becomes $c_i + \xi_i$. We denote the vector of asset-to-debt ratios by x. Then the equilibrium asset-to-debt ratio of bank i is given by

$$\Phi(x, \xi)_i = \frac{\left(\gamma_i + \xi_i + \sum_j \pi_{j,i} l_j \cdot \min\{x_j, 1\}\right)}{l_i}, \qquad (1)$$

where the minimum operation denotes that if a bank defaults, it can pay only proportionally to its available liquid assets. When it does not default, it can pay its obligations in total. Similarly to corollary 1 a bank is in default if its asset-to-debt ratio is below 1, therefore the default threshold is $\Phi < 1$.

The policymaker is also able to expect banks to hold a higher amount of capital before a shock hits. We call it ex-ante application and can be based on stress testing results. Consequently, higher capital buffers can alleviate the need for ex-post infusions.

Corollary 2. The mapping in equation (1) is equivalent to a mapping of the payment vector in an endogenous recovery rate (Rogers and Veraart 2013) model.

See the proof in appendix B. The next lemma shows that an iteration of Φ converges to a fixed point.

Lemma 1. $\Phi(x, \xi)$ *is monotone and bounded in* x *for fixed* ξ*, and there exists a largest fixed point which can be obtained via a fixed point iteration.*

Proof. $x \mapsto \Phi(x, \xi)$ is clearly a non-decreasing function of x and bounded from below and above. Let $q_0^i = \frac{(\gamma_i + \xi_i + \sum_j \pi_{j,i} l_j)}{l_i}$ as the upper bound of Φ and $q^{n+1} = \Phi(q^n, \xi)$. Since $q^{n+1} \leq q^n$, $\lim_{n\to\infty} q_n = q^*$ exists and is the largest fixed point. □

After the iteration, we denote the resulting equilibrium asset-to-debt ratio of bank i by $R_i(\xi)$. Now the loss suffered by bank i can be written as the difference between original interbank obligations (l_j) of its counterparties and payments in equilibrium which are proportional to the asset-to-debt ratios of the counterparties:

$$L^{IB,i}(\xi) = \sum_j \pi_{j,i} l_j - \sum_j \pi_{j,i} l_j \min\{R_j(\xi), 1\}.$$

Summing this over all banks i, the loss in the financial system due to interbank exposures is given by

$$
\begin{aligned}
L^{IB}(\xi) &= \sum_i L^{IB,i}(\xi) \\
&= \sum_i \left(\sum_j \pi_{j,i} l_j - \sum_j \pi_{j,i} l_j \min\{R_j(\xi), 1\} \right) \\
&= \sum_i \sum_j \pi_{j,i} l_j (1 - \min\{R_j(\xi), 1\}) \\
&= \sum_i \sum_j \pi_{j,i} l_j \left(R_j(\xi) - 1\right)^- . \qquad (2)
\end{aligned}
$$

2.2 Fire sale mechanisms

Losses from direct interbank losses in the unsecured interbank market are typically small (Glasserman and Young 2015; Bardoscia et al. 2019). For theoretical results on overall losses and their comparison in different models of contagion, see Visentin, Battiston, and D'Errico (2016) where Eisenberg–Noe type models are shown to produce lower losses than other models incorporating uncertainty.[4] At the same time, experience from the global financial crisis shows that interbank markets might dry up and rolling over the outstanding debt might become difficult for financial institutions either for defaulted or non-defaulted banks. In this case, banks would need to liquidate a fraction of their securities portfolio which would lead to a spiral effect of fire sales (Brunnermeier 2009). It is also possible that as banks receive back less of their interbank assets when a shock hits the system, they need to sell some of their illiquid assets to meet their obligations.

For practical application, we introduce three separate channels of fire sales to the framework which are applied in the literature. First, we assume that banks having a liquidity shortfall from interbank losses start selling their illiquid assets at predetermined prices. Second, short-term funding is withdrawn from all defaulted banks. Third, short-term funding is withdrawn by the defaulted banks. All assumptions are plausible and might appear in reality as a mix, but here we assess them separately to keep calculations tractable. This modelling approach does not account for the similar holdings of assets of banks. For this reason, fire sale losses might be underestimated. In a more detailed approach, fire sale prices could also depend on

[4]However, Bardoscia et al. (2019) incorporates uncertainty and reports low levels of potential losses. On the other hand, recent market turmoils have shown the importance of derivative markets which are not covered in this framework.

the similarity of overlapping portfolios subject to fire sales, see e.g Roncoroni et al. (2019) and Aldasoro, Hüser, and Kok (2020) for recent applications of portfolio overlaps.

(A) The bank covers its liquidity shortfall from interbank losses by selling illiquid assets (Hałaj and Kok 2013). From equation (2), the bank with liquid assets γ_i has to cover a liquidity shortfall of $\left(\gamma_i - \sum_j \pi_{ji} l_j (x_j - 1)^-\right)^-$, which has positive value if losses are higher than the liquid assets of the bank. The bank cannot sell more than its illiquid assets y_i. Since these assets have average value $p_i < 1$ on the market, the bank has to sell $\frac{\left(\gamma_i - \sum_j \pi_{ji} l_j (x_j - 1)^-\right)^-}{p_i}$ amount of the illiquid assets. Finally, after selling the bank suffers a loss due to a lower market price of the illiquid assets. The loss is quantified by the multiplying factor $(1 - p_i)$.

Therefore the fire sale loss of bank i is given by

$$\delta_i^A(x_{j,j\neq i}) = \min\left\{\frac{\left(\gamma_i - \sum_j \pi_{ji} l_j (x_j - 1)^-\right)^-}{p_i}, y_i\right\}(1 - p_i).$$

(B) Short-term funding is withdrawn from defaulted banks (Minca and Sulem 2014). This process further decreases the liquidity position of defaulted banks and endogenous LGDs become lower. If bank i is defaulted ($x_i < 1$), its counterparties decide to withdraw their funding s_i. In this case, a liquidity shortfall of $(\gamma_i - s_i)^-$ has to be covered. The rest of the loss function is similarly to (A) given by

$$\delta_i^B(x_i) = \min\left\{\frac{(\gamma_i - \mathbf{1}(x_i < 1)s_i)^-}{p_i}, y_i\right\}(1 - p_i).$$

(C) Short-term funding is withdrawn by defaulted banks (Covi, Gorpe, and Kok 2021). Defaulted banks are not able to further fund other banks. By withdrawing their funding from other banks, they spread distress towards institutions that are not necessarily defaulted yet. The fire sale loss of bank i is given by

$$\delta_i^C(x_{j,j\neq i}) = \min\left\{\frac{\left(\gamma_i - \sum_j \pi_{ji} s_j \mathbf{1}(x_j < 1)\right)^-}{p_i}, y_i\right\}(1 - p_i).$$

We note that in all three cases the loss from fire sales depends only on the asset-to-debt ratio of *defaulting* banks. If a bank is not defaulted, it has no impact on the losses of others at all. Having defined these fire sale losses, the new asset-to-debt ratios of banks become

$$\Phi(x,\xi)_i = \frac{\left(\gamma_i + \xi_i - \delta_i(x) + \sum_j \pi_{j,i} l_j \cdot \min\{x_j, 1\}\right)}{l_i}, \tag{3}$$

for which Lemma 1 also holds as δ is bounded.

The overall fire sales loss is then defined by

$$\Delta(\xi) = \sum_i \delta_i(\gamma_i + \xi_i). \tag{4}$$

The presence of fire sales increases even the amount of interbank losses in equation (2), see empirical results in section 4.

The total losses in the system is then obtained as the sum of interbank and fire sale losses:

$$L(\xi) = L^{IB}(\xi) + \Delta(\xi). \tag{5}$$

Optimisation results hold in the next section for a wide class of fire sale loss functions. The only property used in further proofs is that the fire sale loss of a bank depends only on the asset-to-debt ratio of defaulted banks. Fire sale losses appear on the asset side of banks.

In our application, we apply fixed discounts on all prices. In this case, portfolio overlaps do not play a role. On the contrary, when prices are determined endogenously based on the reactions of individual banks deciding to sell some of their securities, one needs to update prices based on the amounts sold using a price impact function which is increasing in the amounts sold (Cont and Schaanning 2019). This, in our example, would need us to keep track of portfolio matrices and price changes at given aggregation level within the iteration.

3 Optimisation

The goal of this section is to provide both economic intuition and a mathematical framework to the problem of optimal control in the interbank network. The policymakers' goal is to minimize the contagion losses described above that may arise in the stress situation. In this step, we assume that the initial stress is completely known and banks' balance sheets are also monitored in real time by the policymaker. Though this is not necessarily the case in normal times[5], in a crisis situation regulators might be able to request special reports of banks on their solvency and liquidity positions and interbank exposures from which contagion channels can be reconstructed.

It is clear that contagion losses could be minimized if all banks would be given an infinite amount of liquidity, but we show that this solution is far from optimal. What is laid down in the following formulae can be summarized as follows. Once a shock hits, the policymaker decides how much money it can give to which banks. The policymaker has to calibrate her response taking into account various considerations. First of all, the set of possible solutions to the policymaker's optimization problem is introduced by her budget constraint (e.g. the fiscal envelope or the amount of liquidity that can be offered against eligible collateral). Second, the policymaker does not infuse any amount into a bank which is not initially defaulted. This is completely intuitive from an economic perspective: only banks which are vulnerable need to be supported. Although this might be subject to criticism from a mathematical perspective since other banks, which are assumed to be healthy before contagion takes place, may also amplify the contagion effect. Third, if given a constraint on the amount of total infusion, we are not able to save a bank, then we do not infuse any amount in that bank. This is the intuitive meaning of Assumption 1 below which in the end discretizes the set of possible solutions to explore; there will be a subset of banks which are saved while the remaining ones are not. As a consequence, the optimisation is carried out by exploring all possible subsets of initially defaulting banks, calculating the losses in the presence of infusions and choosing the subset which yields the lowest losses while ensuring that the amount of total infusion is below the budgetary threshold (see Proposition 1 below). Meanwhile, infusions are calculated in such a way that banks are pushed back to the region of solvency/liquidity, whichever is breached without infusion.

Another main feature of the model is that despite being an equilibrium payment algorithm, it is still possible to introduce an ordering of events coming from the fact that all losses depend only on the asset-to-debt ratio of defaulted banks (in equilibrium). By calculating first the asset-to-debt ratio of those banks which are not saved and consequently of those which are saved, we are able to optimise without the iteration of the asset-to-debt ratio after we have found the payment equilibrium. This is formally described in

[5] Even in normal times, however, since the GFC central bank and supervisory data collections have substantially improved allowing for better real-time surveillance of financial sector interconnectedness. For instance, the data set used in this paper relies on quarterly supervisory reporting that allows for constructing reliable interbank networks in real time.

sections 3.1 and 3.2

The policymaker has a budget constraint as an upper bound of capital infusion into the banks, we denote it by M. We will handle the problem by deciding which of the banks should be saved based on the amount of additional capital they should be provided.

The set of admissible intervention strategies is given by

$$\mathcal{A}_M = \left\{ \xi \in \mathbf{R}^n_+ \middle| \sum_i \xi_i \leq M \right\}. \tag{6}$$

The problem that has to be solved is

$$\min_{\xi \in \mathcal{A}_M} L(\xi). \tag{7}$$

The following assumption makes it possible to discretize the set of admissible intervention strategies.

Assumption 1. *There is no infusion into a bank which is defaulted under an infusion ξ^*. That is, if $R_i(\xi^*) < 1$ then $\xi_i^* = 0$. It can be interpreted as if the given bank can not be saved in the presence of an infusion constraint, we do not infuse any amount into them.*

Now assume that a subset $S \subseteq \mathcal{N}$ of banks are to be saved by infusion, then the admissible strategies are given by

$$\mathcal{A}_M^S = \left\{ \xi \in \mathcal{A}_M, i \in S : R_i(\xi) \geq 1, i \in \mathcal{N} \setminus S : \xi_i = 0 \right\}. \tag{8}$$

Proposition 1. *Assume that we have a solution in which a subset S of banks are saved by infusion. The solution to Problem 7 can be found by minimizing the loss function over all the possible subsets S of banks.*

Proof.

$$\min_{\xi \in \mathcal{A}_M} L(\xi) = \min_{S \subseteq \mathcal{N}, \xi \in \mathcal{A}_M^S} L(\xi), \tag{9}$$

where in the problem on the right side only a set of banks are saved by infusion. By exploring all possible subsets, the global minimum can be obtained. □

Now we will handle banks based on whether they are in the set S or not, differently. Since fire sales losses depend only on the asset-to-debt ratio of defaulting banks, $\delta_i = \delta_i(x_{j, j \notin S})$.

3.1 Banks not assumed to become solvent

In this case, the asset-to-debt ratio of banks in $\mathcal{N} \setminus S$ are not depending on ξ. Thus the equilibrium asset-to-debt ratio is given by the fixed point of

$$\Phi(x, S)_{i,i\in\mathcal{N}\setminus S} = \frac{(\gamma_i - \delta_i(x) + \sum_{j\in S}\pi_{j,i}l_j + \sum_{j\notin S}\pi_{j,i}l_j \cdot \min\{x_j, 1\})}{l_i}, \tag{10}$$

where we used that banks in S are able to pay their obligations and banks not in S may either default and pay only a proportion of their debt to their creditors. As proved in Lemma 1, this mapping also converges and we denote the equilibrium asset-to-debt ratio of bank i under infusion ξ by $R_i(\xi)$. It is clear that $R_i(\xi) = R_i^S$ for $i \in \mathcal{N} \setminus S$.

3.2 Banks assumed to become solvent

For banks guaranteed to be solvent, for $i \in S$ the asset-to-debt ratio is given by the fixed point of

$$\begin{aligned} &f(x, \xi, S)_{i,i\in S} \\ =&\frac{(\gamma_i + \xi_i - \delta_i(\gamma_i + \xi_i, R_j^S) + \sum_{j\in S}\pi_{j,i}l_j + \sum_{j\notin S}\pi_{j,i}l_j \cdot \min\{R_j^S, 1\})}{l_i}, \end{aligned} \tag{11}$$

where R_j^S are determined in section 3.1. In case of fire sales of type (B), $\delta_i = 0$ because the bank in S is not defaulting. Note that this function does not depend on x, therefore the minimal infusion that makes i solvent is

$$\underline{\xi}_i^S = \inf\{\xi_i | f(\cdot, \xi_i, S) \geq 1\}. \tag{12}$$

This is a non-linear equation system. For details on the solution, see appendix C.

Since it is the minimal amount of infusion needed, it is clear that for a given subset S, this is the only possible solution. If $\sum_i \xi_i > M$, then there is no solution for the given subset.

Example (N=2, no fire sales)**.** *Figure 2 depicts the methodology for an example of two banks. Both banks are initially defaulted, but only bank X has interbank payment obligations towards bank Y;* $l_{X,Y} = 100$*. Both banks' initial asset-to-debt ratio is below 1 as can be seen in the figure. The asset-to-debt ratio of X is 1/2, therefore it can pay only 50 out of 100, the interbank loss is 50. Y has no interbank obligations, but its liquidity is negative. The policymaker can choose to save either X or Y, or both*

X and Y with budget constraint $M = 50$. Since bank Y has no interbank obligations, it doesn't need to be saved, while saving X costs 50 which is equal to the budget constraint, therefore it is a solution. With an infusion of 50, the asset-to-debt ratio of X becomes 1, hence it is able to pay its obligations and there is no interbank loss. We note that in this simple example there was no need to iterate the mapping of the asset-to-debt ratio Φ.

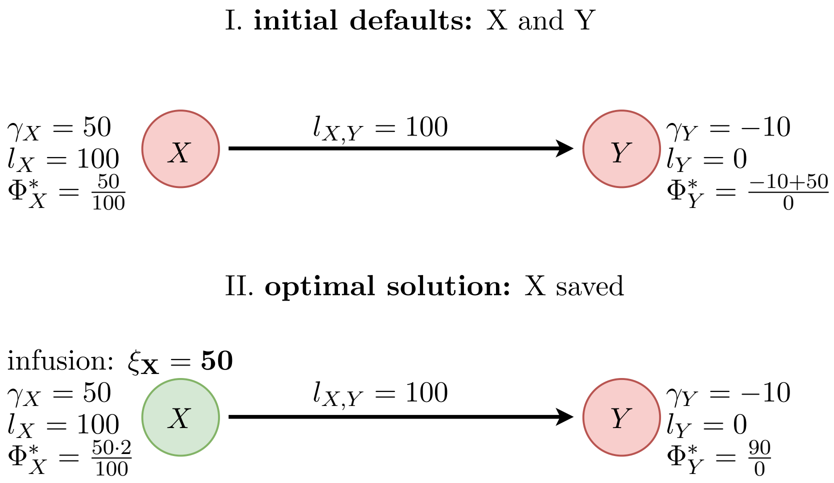

Figure 2: Example of optimisation for 2 initially defaulted banks.

3.3 Benefit from infusion

A main property of providing infusion is not only that it rescues a given bank, but it also stabilizes the whole system by preventing distress propagation. In the upcoming simulations and analysis we quantify prevented losses by the following measure called "benefit". This is defined as the percentage decrease in the sum of interbank and fire sale losses also taking into account the amount of infusion:

$$benefit = 1 - \frac{IB\,losses_{after} + FS\,losses_{after} + infusion}{IB\,losses_{before} + FS\,losses_{before}}, \qquad (13)$$

where subscripts "before" and "after" denote whether losses are calculated for the network before or after the infusion. This amount expresses how much higher order losses are avoided and is similar to a multiplier(-1). If it is close to zero, it means that the infusion was only able to eliminate first order losses; these losses immediately appear at counterparties. Intuitively, banks receive infusion by considering their capital shortfall. If losses after infusion are 0 and the benefit is also 0, it means that the original losses were very close to the sum of infusions. In this case, infusion could be considered as not worthwhile, as not being systemically necessary. If this benefit is larger than zero, it can be interpreted as a benefit of the (financial) economy. It is important to note that the $benefit$ can also be negative. That would happen if the amount of infusion is higher than the amount of initial shortfalls and interbank losses, e.g. the monetary cost of saving the banks exceeds the benefits if the only goal is to minimize contagion losses.

The "investment" can be done either by the banks themselves by holding additional capital or the policymaker as emergency liquidity assistance or private or public means of recapitalisation. For more details on ex-ante and ex-post evaluation of the infusion, see section 5 later.

4 Results

4.1 Data

We join two different datasets for the analysis of contagion losses and the optimal equity infusions to restrain such losses in the euro area banking sector. One dataset collects balance sheet elements for banks, while the second one is used to form the network.

First, we take capital levels for stressed banks from the output of the macroprudential perspective of the 2018 EBA stress test of the euro area banking sector (Budnik et al. 2019). The main contribution of this stress test is that it assumes a dynamic balance sheet reaction of banks in response to a shock and introduces a feedback loop between the banking sector and the real economy. The stress test found that the euro area banking sector was overall resilient and that only a small portion of banks would breach their minimum capital requirements, i.e. $c_i < c_{i,th}$. We take results for the adverse scenario of the stress test. We use the projections of capital levels for the period 2019Q1 to 2020Q4. Banks that are not participating in the EBA stress test or not part of the SSM's so-called SREP sample are not shocked in this setup.[6]

[6] Among the 119 significant institutions (SIs) that the ECB directly supervises, the 35 largest are part of the EBA stress test sample, while the ECB also conducts a parallel stress test for the rest of the SIs (the so-called SREP banks).

The second dataset is described in details in Covi, Gorpe, and Kok (2021). It contains bilateral exposures between banks, minimum capital requirements as mentioned above and amounts of high quality liquid assets (HQLA) as well as illiquid assets (non-HQLA) retrieved from FINREP and COREP templates. The reference date for this dataset is end of 2018. To construct the interbank network, we use bilateral exposures extracted from banks' reports of large exposures and large liabilities (C67). We focus on the interbank exposures reported, that is, non-financial corporations, general governments, central banks and households are omitted. An exposure is a large exposure if it exceeds 10% of an institution's eligible capital or its value is above EUR 300 million. The structure of the data is in line with the standard unsecured lending literature as financial institutions have to report collateral pledged for given exposures therefore one can simply calculate the amount of net exposures. These net exposures are used as $l_{i,j}$ in the model. Short-term exposures $s_{i,j}$ are obtained from exposures with less than 30 days maturity. Fire sale prices p_i of illiquid assets are calculated as weighted averages of haircuts applied to the portfolios of non-central bank eligible instruments.

Figure 3 shows a visual representation of our network of interbank exposures as of end-2018.

Throughout the analysis the default threshold of capital adequacy is assumed to be the sum of Pillar 1 requirement, Pillar 2 requirement and the capital conservation buffer (P1R+P2R+CCoB). The modelling approach allows for adjusting this threshold.

4.2 Analysis – Macro results

In this section we show the power of the methodology using the dataset described above. We identify several important properties of the framework. First, it is demonstrated that unconstrained infusions are able to eliminate all losses in the interbank network, even without infusing all banks. Secondly, we find that the benefit from infusion can become negative when saving a bank is not worth from a purely financial point of view. Lastly, we highlight non-linear properties of the existence of a budget constraint on the amount infusion, regardless of the fire sale mechanism assumed.

For illustrative purposes, we conduct the analysis for all quarters between 2019 Q1 and 2020 Q4 using the results of the macroprudential stress test as a starting point. The analysis is done by assuming simultaneous defaults in each quarter without feeding back the contagion results into the

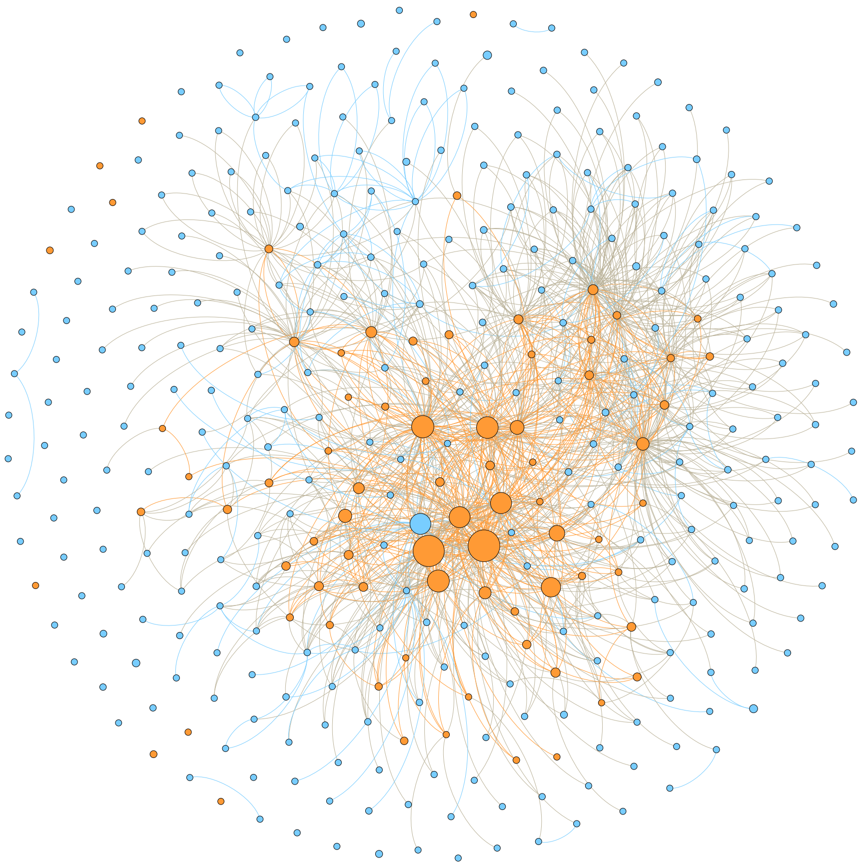

Figure 3: A visualisation of the large exposure network of banks in the eurozone
Orange nodes are banks subject to SSM supervision, blue nodes are banks subject to national supervision, sizes of nodes are proportional to the amounts of risk weighted assets

macroprudential stress test for the following quarter.[7] Tables 1-4 in Appendix D report the results for the case without fire sales and for the three different fire sale mechanisms. The reader can also find a detailed explanation of results in Appendix D, while this section focuses on the key findings of our analysis.

Losses without infusion. Figure 4 shows the distribution of interbank losses and fire sale losses across all quarters of the simulation horizon, assuming that there is no infusion at all ($\xi = 0$). The left panel shows boxplots of $L^{IB}(0)$, the right panel depicts boxplots of $\Delta(0)$. Both fire sale losses and interbank losses are different across the different fire sale assumptions: the higher the fire sale losses, the higher are interbank losses. This fact is a consequence of the payment equilibrium approach of our model. Since interbank payments are proportinal to the individual banks' asset-to-debt ratios and these ratios are depreciated by both fire sale losses and potentially non-performing interbank loans (equation (3)). Hence, higher fire sale losses decrease asset-to-debt ratios, this leads to higher interbank losses which further decrease the asset-to-debt ratios and thus increase fire sale losses. Consequently, we find similar interbank losses for the no fire sales (0), fire sales type (A) and fire sales type (C), as in this case fire sale losses are limited or even zero. There is a significant difference for fire sales type (B), the higher values of which causing outstanding interbank losses as well. In this specific fire sale assumption, short-term funding is withdrawn from defaulted banks resulting in high fire sale losses for these defaulted banks and given that the capital levels of defaulted banks are even further depleted, their ability to pay back interbank obligations is even weaker.

Unconstrained infusion. As reported in the second blocks of Tables 1-4, unconstrained ($M = \infty$) optimal infusions are able to completely eliminate contagion losses in the system. This is not surprising and in particular we will show in the following section that when there is no budget constraint, optimal infusions are identical across different fire sale assumptions. In order to quantify how much it is worth to save the initially defaulted banks, in Figure 5 we look at the functional relationship between our defined benefit measure (equation (13)) and the total amount of optimal infusions. Our most interesting observation is that the benefit from an infusion can also be negative. This is particularly the case when the sum of fire sale and interbank losses are smaller, e.g. for some quarters in fire sale types (0) and

[7] This could lead to an underestimation of overall losses, and hence the number of defaults at the end of the stress test horizon. This notwithstanding, the end-of-horizon results assume a simultaneous default of institutions and could be used as a worst case scenario.

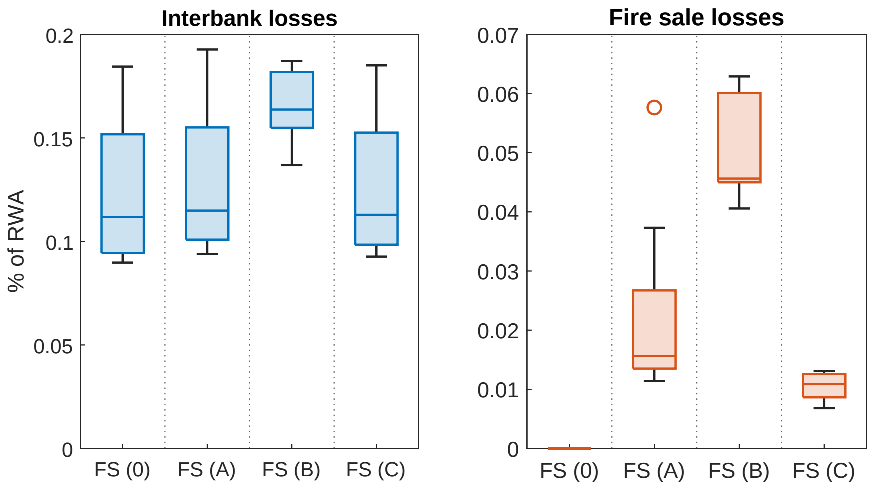

Figure 4: Distributions of interbank and fire sale losses without infusion, for the different fire sale assumptions (FS (0) denotes no fire sales).
Distributions are across all quarters: 2019Q1–2020Q4. Interbank and fire sale losses are reported in percentage of risk weighted assets of the stress tested banks.

(C) simulations, and for fire sale type (A) in one case. Though the levels of total infusions are equal across fire sale assumptions as seen in the figure (horizontal lines), benefits are lower when eliminated losses are too low compared to initial capital shortfalls. Equivalently, the higher the contagion losses across fire sale types, the higher benefit the policymaker can achieve by the infusion. In Appendix D in the second blocks of the tables it can also be seen that not all initially defaulting banks were needed to be saved only those banks need to be saved which create contagion losses.[8] Thus, optimization makes sense even if infusions are equal to the capital shortfalls of banks which create contagion losses.

Constrained infusion. In this exercise we set the budget constraint M to be equal to half of the total infusion needed without any constraint. These simulations show how contagion losses, infusion amounts and benefits change when there is a binding constraint on infusions. Figure 6 shows the distribution of total losses ($L(\xi^*)$, where ξ^* is the optimal infusion) across the quarters for all fire sale mechanisms. Still, fire sale type (B) delivers the highest contagion losses. Contrary to unconstrained infusions previously, we

[8] Obviously, there could be other reasons for wanting to save those banks. Here we only focus on the need to limit systemic effects due to interbank contagion losses.

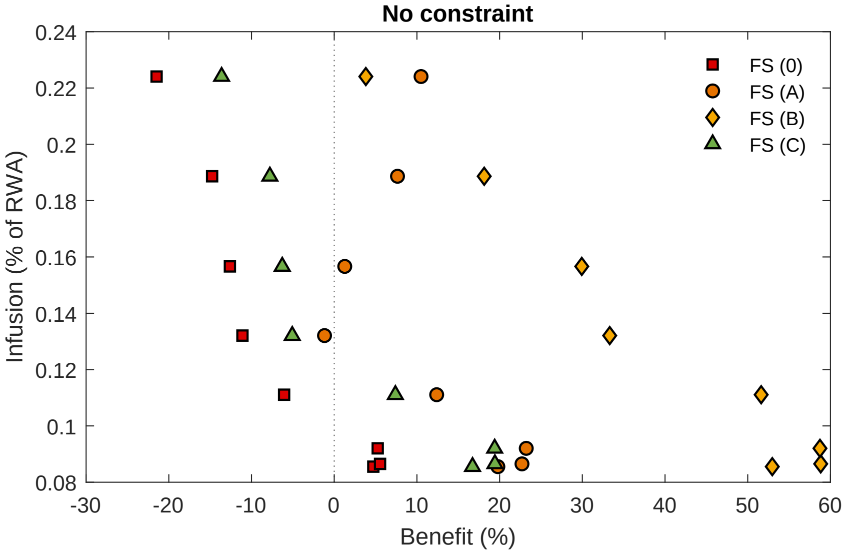

Figure 5: Relationship between our defined benefit measure and total amount of infusions for different fire sale assumptions. Infusions are unconstrained.

are naturally not able to eliminate all losses in the system as infusions cannot cover the total initial capital shortfalls of banks due to the budget constraint, thus we are not able to save all systemically important banks. This property implies major non-linearities of the modelling framework which are introduced via Figure 7. One would intuitively think if the budget constraint is decreased to one half, the amount of infusions in the optimal control problem and the benefits also become one half. However, as explained above, as we may not be able to save some systemically important banks due to the budget constraint, they may still trigger contagion in the network. Thus, the amount of infusion with half budget constraint is at most half of original infusions as the right panel of the figure confirms. On the other hand, our defined benefit measure is a function of infusion and contagion losses with and without infusion (equation (13)). We have seen that infusion becomes at most half of the original amount, and our simulations show that not saving some systemic banks results in more than half of contagion losses than originally (Tables 1-4). All this together implies that our benefit measures are also non-linear and the constrained benefit becomes at most half of the original benefit, as depicted in the left panel of Figure 7.

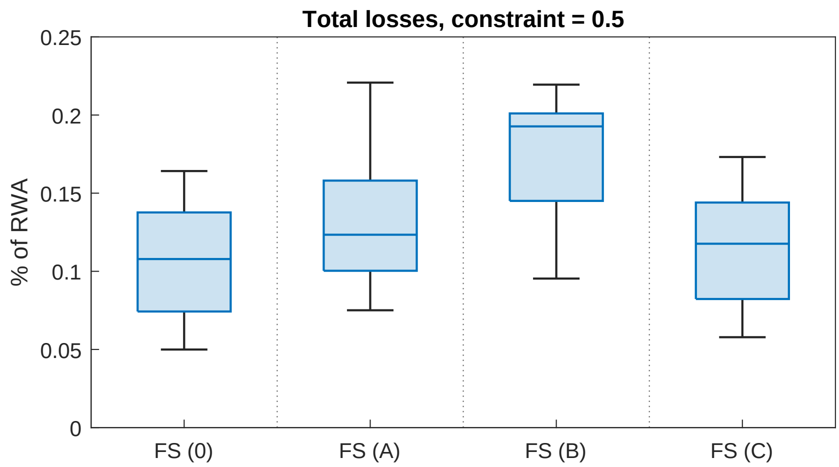

Figure 6: Distributions of total losses in case of constrained infusion for different fire sale assumptions.
Distributions are across all quarters between 2019Q1 and 2020Q4. Losses are reported in percentage of risk weighted assets of the stress tested banks.

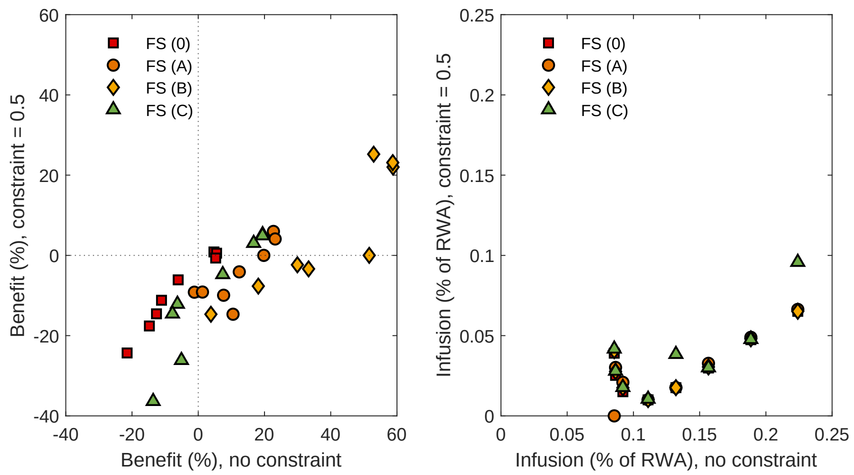

Figure 7: Non-linear properties of optimal infusions and benefits for the different fire sale assumptions.
All infusions and benefits for all quarters between 2019Q1 and 2020Q4. Infusions are reported in percentage of risk weighted assets of the stress tested banks.

In conducting the optimisation, the exploration all possible subsets of initially defaulted banks is a time-consuming exercise because the number of calculations increases exponentially with the number of defaults. We have found in practice for a banking system of size 373 that calculations up to 26 exogenous defaults can be completed within 6 hours for one setup, while 11 defaults need less than 0.2 minute running on one CPU core.[9]

4.3 Analysis – Bank-level results

In this section we aim to understand our capital infusion results by looking at the original capital shortfall $(c_{i,th} - c_i)$ of banks from the macroprudential stress test model and compare these to the infusions (ξ_i^*) needed from the solution of (7). By looking at bank-level results, we are also able to shed further light on macro results from the previous section. The main takeaway from this section are the following. Unconstrained infusions are identical to initial capital shortfalls regardless of the fire sale mechanism, while when a constraint is binding, solutions can be different. We also give an explanation

[9]Calculations could be boosted by using parallelization or GPU computing.

of the non-linear property of the infusion as a function of budget constraint.

Bank-level Figures 8 and 9 show capital shortfalls and infusions needed for the different quarters of the 2018 stress test. Figures in the left panel are the solutions when there is no constraint on the infusions, while the right panels show the constrained solutions (there was no solution for the constrained infusion for 2019 Q1). These figures confirm the theoretical reasoning in the previous section that when there is no constraint on the amount of infusion, capital infusions are equal for all types of fire sale losses. Furthermore, they are equal to the exact amounts of capital shortfalls. This result is not surprising, but we emphasize that not all banks necessarily need to be saved from a contagion perspective. Those banks which are not very active on the interbank market may not need to be rescued as they do not generate material contagion effects. In the constrained case, if there is a bank where an infusion is not possible, it might cause immediate interbank and fire sale losses to other banks in the network. These losses could further reduce the capital adequacy ratio of those banks that are being rescued and their capital shortfall would be higher than the initial shortfalls. Therefore, it is possible that infusions are higher than capital shortfalls from the stress test. Such examples can be seen in all figures for the constrained solutions. This happens for example in 2020 Q1 for bank 6 where fire sale type (C) infusion is above the capital shortfall, in 2020 Q3 for bank 6 and in 2020 Q4 for bank 7 and 9.[10] Finally, it can be noted that if there is a budgetary constraint, infusions can be different across fire sale types (right panel in Figures 8 and 9). These differences could not have been explored without the analytical approach presented here.

In addition to the previous results which focused on the practical properties of the methodology, in Appendix E we present a comparison to standard measures applied in network analysis or in the assessment of systemically important institutions. It is shown that our methodology provides added value.

[10]The numbers of banks are not IDs, just an enumeration of the initially defaulted banks.

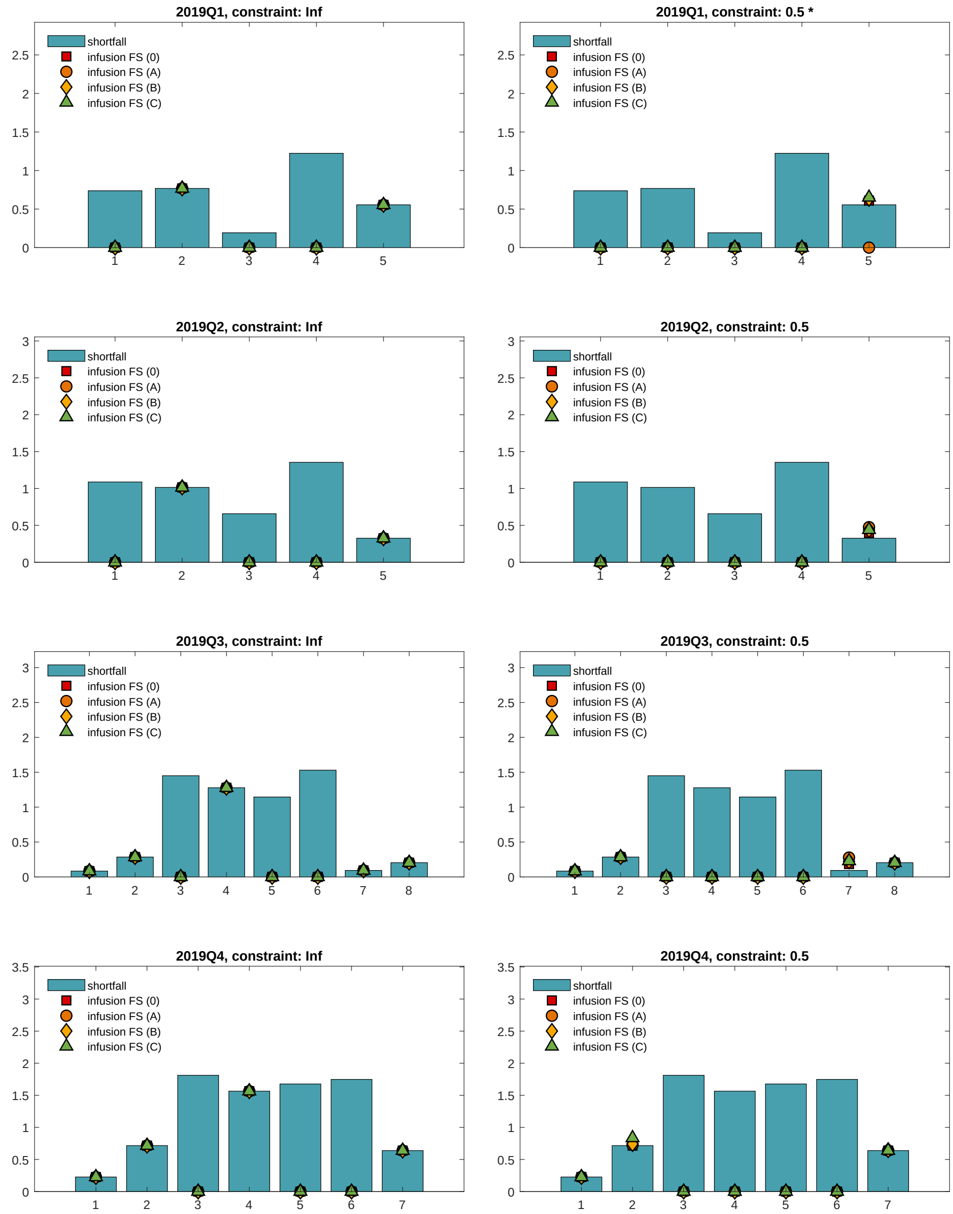

Figure 8: Capital shortfalls and infusions for different fire sale types in quarters of 2019

Measures are in percentage of risk weighted assets of the individual banks. * means that there was no solution for fire sale type (A)

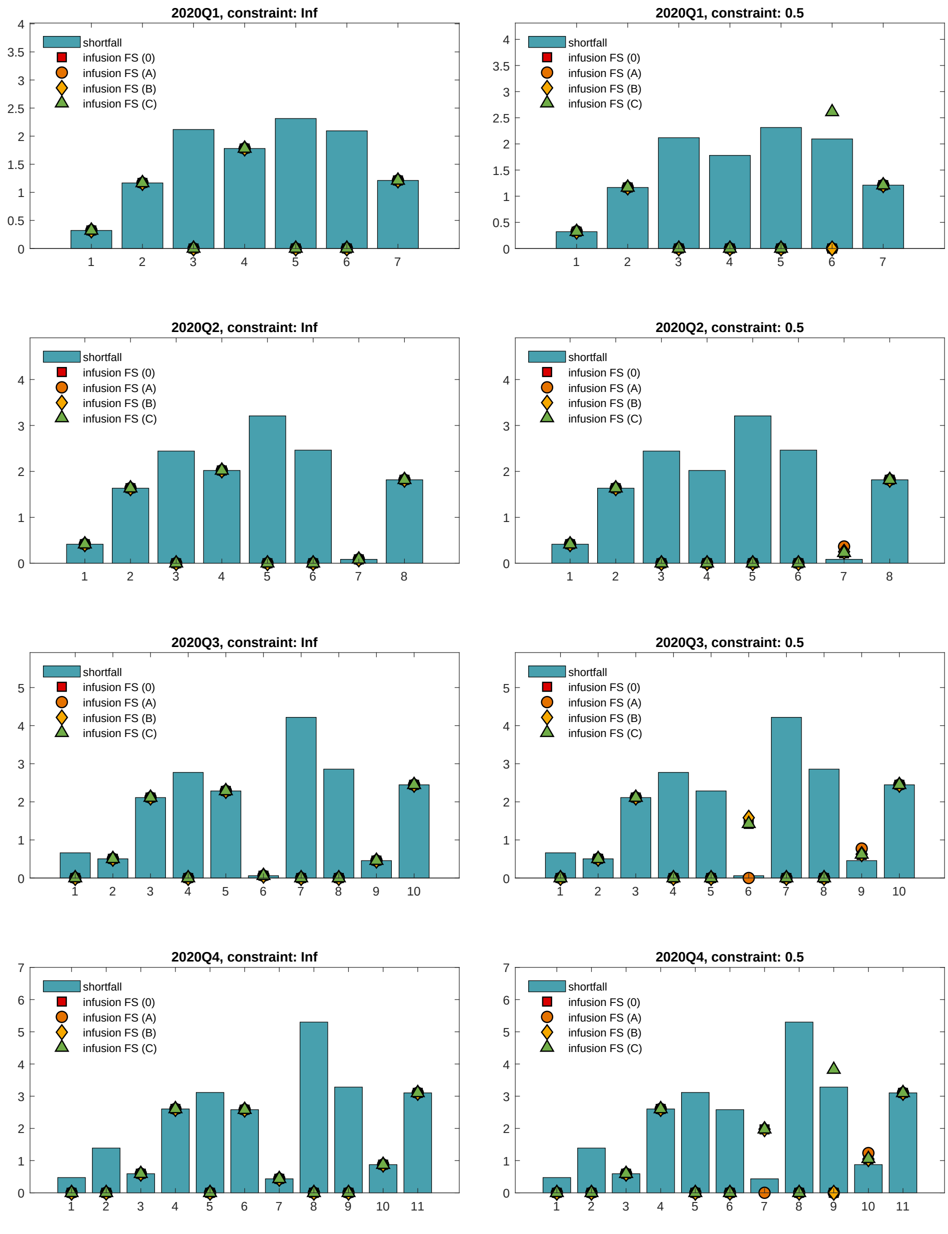

Figure 9: Capital shortfalls and infusions for different fire sale types in quarters of 2020.
Measures are in percentage of risk weighted assets of the individual banks.

5 Discussion: Policy applications

This section elaborates on the possible policy applications of the optimal contagion model. It is possible to apply the methodology in several ways and it will be useful to distinguish between 'ex post' crisis management actions and 'ex ante' preventive measures.

It should be emphasized that the model treats solvency and liquidity identically. In fact, both the capital buffer and the liquidity buffer of the bank is assumed to serve as a loss absorption capacity. Hence, the optimisation can be performed jointly within the model. This means that it is possible in theory that in a banking system there are both liquidity and solvency distressed banks. The optimization in this case would need the cooperation of the competent authorities. In principle, after running the contagion algorithm, one can easily identify which constraint (solvency or liquidity) was binding for a given bank. By doing so, it can be decided whether any given bank is in need of liquidity or additional capital.

5.1 Ex-post application: Crisis management

For what concerns ex-post crisis management the optimal contagion approach presented in this paper could be a valuable tool for decisions on whether to liquidate or bring into resolution a bank, or group of banks. By determining which banks needs to be saved or not if the aim is to minimise contagion losses in the financial system the tool can be used in 'public interest' assessments (i.e. whether failing or likely to fail banks should be resolved or liquidated). Furthermore, in the case of potential resolution the approach described here can be useful for assessments of whether financial stability would be endangered which may require targeted measures to cater for systemic liquidity shortages or situations where a solvent bank would be unable to raise capital privately in the markets. Such measures could include precautionary recapitalisation or government guarantees to issue new liabilities or to assess central bank funding (see Art. 32.4 of the Bank Resolution and Recovery Directive, BRRD).

According to the BRRD, the recapitalisation amount should be determined on the basis of an adverse scenario of a stress test and/or asset quality review. This, however, would typically ignore second-round effects such as direct and indirect (fire sale-related) contagion losses and thus potentially lead to biased estimates of the true recapitalisation needs in the system. The estimated infusion amount derived using the optimal contagion approach presented here, could therefore be used to inform the recapitalisation calibration by providing a more holistic perspective.

In principle, the optimal contagion tool could also be useful for assessing liquidity needs in an ongoing resolution, which could occur even if the bank had been recapitalised.[11] Such temporary liquidity support, assuming that private funding sources would not be available, could either be provided through emergency liquidity assistance (ELA) provided by the national central banks or from a government guarantee. Specifically, in case of ELA, a national central bank provides central bank money against suitable collateral for a bank which should be proven solvent or there should be a credible prospect of restoration of its capital position. Therefore, to apply our framework for such circumstances the solvency condition would be replaced by a liquidity condition for the given banks which are considered to receive ELA (the definition of γ_i does this already). Owing to the fact that banks are also expected to pledge collateral against ELA, the infusion constraint in the optimisation problem could be chosen considering the availability of central bank eligible collateral.

5.2 Ex-ante application: Crisis prevention

The framework could, however, also be applied by a macroprudential authority taking an ex-ante view with the aim of preemptively ensuring the resilience of the financial system to adverse shocks that may trigger system-wide contagion losses. Using this methodology, the policymaker is able to determine capital levels needed in the banking system in case of a stress event. These could be interpreted as macroprudential capital requirements to mitigate the risk of contagion. Capital requirements could be calibrated such as to minimise the need for capital infusions later on and our framework could be useful to inform such calibration.

For the purpose of making the financial system resilient to contagion, an ex-ante perspective could for instance be reflected in the calibration of additional capital requirements, such as the G-SII/O-SII buffer requirements targeting the systemicness due to interconnectedness of individual banks or a more broad-based Systemic Risk Buffer. Though the goal is clear, it is still not fully crystallized how capital requirements addressing risks due to interconnectedness should be optimally calibrated. Currently, G-SII and O-SII buffer calibration is implemented with simple size-based measures which do

[11] Experience suggests that banks in, or recently out of, resolution often may not be able to obtain sufficient liquidity to maintain critical operational tasks and meet margin calls. This could, for example, be due to a lack of adequate collateral to access market-based or central bank funding; see e.g. Amamou et al. (2020); Grund, Nomm, and Walch (2020).

not account for network dimensions. In the EU context, current O-SII guidelines EBA (2014) measure interconnectedness with a simple indicator-based method to derive a so-called O-SII score for interconnectedness. Capital calibration is then based on a simple mapping between scores and capital buffers. However, the suggested capital buffer calibration methods disregard any structural mechanism of the interconnected system and some recent studies have documented that O-SII scores to be only partially effective in quantifying contagion effects in Fink et al. (2016), Alter, Craig, and Raupach (2015) and Covi, Kok, and Meller (2018). Alter, Craig, and Raupach (2015) recalibrate capital requirements in the German banking system based on the centrality measures of individual banks in the interbank network also quantifying losses from correlated credit exposures. They find that this exercise leads to a more resilient system and identify some centralities that are the most effective. Fink et al. (2016) use a different approach allowing for losses triggered by the change in probabilities of defaults (PD) of banks in distress. These PDs give rise to losses in the credit valuations of the counterparties. They find that banking system level losses correlate with some centrality measures better than the O-SII scores themselves. Covi, Kok, and Meller (2018) show that network based measures of systemicness provide value added compared to size-based interconnectedness indicators.[12]

The framework presented in this paper, however, goes a step beyond the application of standard centrality measures and attempt instead to derive the optimal solution for contagion loss minimization purely based on theoretical considerations. This can then be used to determine the level of capital buffer that banks should hold to prevent systemic losses when tail events occur. In practice, the capital buffer needs being optimally determined by the model could therefore be used to complement standard-metrics (such as size-based indicators and network centrality measures) and help inform decisions on G-SII/O-SII or Systemic Risk Buffer calibrations. See also Appendix E for a comparison of standard interconnectedness metrics and the infusion results derived from our framework. Notably, the results presented in this paper are based on one specific adverse scenario outcome. To avoid scenario-dependency it might be more prudent to employ an array of scenarios and then evaluate the ranges of optimal contagion control derived using the model.

One deficiency of using the framework for calibrating capital requirements is that it does not take into account that banks' behaviour might

[12]The application of centrality measures is challenged theoretically and numerically in Fukker (2018) and in Siebenbrunner (2019).

change in response to additional requirements. To address this, one could apply a dynamic model of endogenous bank behaviour on the interbank market in the spirit of Hałaj and Kok (2015). Incorporating dynamic behaviour into our optimal contagion model is not straightforward, however, and remains a problem for possible future research.

5.3 Possible extensions

This section enumerates possible extensions or future challenges. As acknowledged previously, our current modelling framework does not take into account portfolio overlaps for in banks' balance sheets as we take fixed prices for marketable illiquid assets. A possible extension of the model could be to take into account these common exposures (Cont and Schaanning 2019).

Another interesting avenue was laid down by Barucca et al. (2020) and Bardoscia et al. (2019) who introduce a generalized network valuation of interbank claims incorporating uncertainty in banks' assets. These models encompass several other well-know interbank contagion models, thus, a next step could be the extension of the optimisation procedure to such classes of models.

A newer area in state-of-the-art stress testing is the inversion of models, also called reverse stress testing (Henry 2021). That is, we are interested in the severity of risk factors that lead to given level of capital shortfalls instead of calculating the outcomes of scenarios. Assuming that the endogenous variables in a stress test model are differentiable functions F of possibly lagged endogenous and exogenous variables, one can try to find the inverse of the model of the form F^{-1} by applying the Newton–Raphson method, see for example Hansen (2021). This method is useful when a model is based on linear regressions and therefore model equations are linear and of course differentiable. However, typically interbank contagion modelling equations are highly non-linear and non-differentiable and the researcher should approximate non-linear equations with smooth functions. An attempt on this approximation is done in Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015a) but a Taylor expansion is not proven to be efficient in numerical experiments (Fukker 2018). Also, these methods work robustly only if there is only one inverse. If the function F is not a bijection, there might be several possible inverses. Furthermore, in this kind of model inversion, there is no optimisation. For example, it could find a solution where there are no contagion losses, but that solution could be an upper bound of the optimal solution. Keeping these in mind, interbank contagion losses can be minimized rigorously by using methods similar to the one presented in this paper, and given the solution from this procedure can be fed into the Newton–Raphson method to invert a differentiable equation system (macroprudential stress test). This topic remains an avenue for possible future research.

6 Conclusion

We have studied the problem of the control of interbank contagion. This problem can be stated as the calibration of capital infusion in defaulted banks to mitigate contagion losses in the financial system. We have shown for this purpose a quantitative method that could also be used to calibrate either additional capital requirements or emergency liquidity assistance (ELA) provided to financial institutions. We also implemented three possible types of fire sale mechanisms to study the optimisation procedure. This optimisation can be done easily by the analytical solutions we derived. We found that the policymaker could be able to optimise its behaviour in a tail event of simultaneous banks under distress.

The method proved to be efficient in decreasing interbank and fire sale losses in an optimal manner. We have also seen that the unconstrained amount of infusion is independent from the fire sale mechanism itself and the possible benefit is very high when the assumed fire sale mechanism is conservative. Another important observation is that it may not be needed to save all of the defaulted banks to eliminate contagion losses, only banks with potential to amplifying contagion losses need to be saved. Introducing infusion constraints lead to different infusions depending on the fire sale mechanism and the potential benefit of an infusion becomes much lower. The drop in the benefit was explained by the fact that a budget constraint might be so binding that more contagious banks may not be saved and therefore the effect of those banks will not be proportional to the change in the constraint.

While we do not think that any model is able to perfectly describe the behaviour of an economic system, we find the approach a useful additional tool to analyse a problem from several points of view. Possible future work could be an introduction of endogenous network formation. Financial networks change endogenously (Hałaj and Kok 2015) in time and in response to shocks. Introducing capital requirements on interbank exposures would lower interbank market activity. Recalculating buffers with lower interbank activity would also lead to lower capital requirements.

A Proof of Corollary 1: Linking default thresholds

The initial asset-to-debt ratio of a defaulted bank i can be rearranged equivalently

$$
\begin{aligned}
r_i = \frac{\gamma_i + \sum_j \pi_{j,i} l_j}{l_i} &< 1 \\
\frac{\min(c_i - c_{i,th}, HQLA) + \sum_j l_{i,j} - \sum_j l_{j,i} + \sum_j \pi_{j,i} l_j}{l_i} &< 1 \\
\frac{\min(c_i - c_{i,th}, HQLA) + \sum_j l_{i,j}}{l_i} &< 1 \\
\frac{\min(c_i - c_{i,th}, HQLA)}{l_i} + 1 &< 1 \\
\frac{\min(c_i - c_{i,th}, HQLA)}{l_i} &< 0 \\
\Longleftrightarrow c_i - c_{i,th} &< 0
\end{aligned}
$$

where in the last step we used that HQLA and interbank liabilities are strictly non-negative while $c_i - c_{i,th} < 0$ for a bank under distress. □

B Proof of Corollary 2: Equivalence with endogenous recovery rate models

We show equivalence with Rogers and Veraart (2013). We denote by l the vector of interbank liabilities. The standard mapping determining the interbank payments is

$$
\phi(l)_i = \begin{cases} l_i & \text{if } l_i \leq \gamma_i + \sum_j \pi_{j,i} l_j \\ \gamma_i + \sum_j \pi_{j,i} l_j & \text{else,} \end{cases}
$$

where in the first line the bank is able to fulfill all of its obligations while in the second line the bank is able to pay only the available amount of liquid assets. The asset-to-debt ratio of the bank is $\frac{\gamma_i + \sum_j \pi_{j,i} l_j}{l_i}$ the value of which around 1 is a tipping point in the recovery rate, therefore by introducing $\min\left\{\frac{\gamma_i + \sum_j \pi_{j,i} l_j}{l_i}, 1\right\}$ the above mapping can be written like

$$
\phi(l)_i = \min\left\{\frac{\gamma_i + \sum_j \pi_{j,i} l_j}{l_i}, 1\right\} l_i = \min\{x_i, 1\} l_i,
$$

which shows the equivalence with the mapping of the asset-to-debt ratio in equation (1). ☐

C Finding the optimal solution

Analytical solution of the non-linear equation system (12). We have to find a ξ for which

$$
\begin{aligned}
&f(x,\xi,S)_{i,i\in S}\\
=&\frac{\gamma_i+\xi_i-\delta_i(\gamma_i+\xi_i,R_j^S)+\sum_{j\in S}\pi_{j,i}l_j+\sum_{j\notin S}\pi_{j,i}l_j\cdot\min\{R_j^S,1\}}{l_i}\geq 1.
\end{aligned}
$$

Actually, it is enough to find ξ_i for a fixed i. For fire sale type (B), $x_i = 1$ implies that $\delta_i = 0$. Note that the two sums on the left side are constants as they do not depend on ξ_i and all R_j^S are known from the iteration in section 3.2. After rearranging constants to the right side we get

$$
\xi_i-\delta_i(\gamma_i+\xi_i,R_j^S)\geq\overbrace{l_i-\gamma_i-\sum_{j\in S}\pi_{j,i}l_j-\sum_{j\notin S}\pi_{j,i}l_j\min\{R_j^S,1\}}^{E_i}.
$$

For fire sale type (B), $\xi_i = E_i$ is a solution. By following the definitions of δ functions,

$$
\xi_i-\min\left\{\frac{(\max\{\gamma_i+\xi_i,0\}-D_i)^-}{p_i},y_i\right\}(1-p_i)-E_i=0,
$$

where D_i is a fire sale type specific constant. We apply $\max\{\gamma_i + \xi_i, 0\}$ because if $\gamma_i + \xi_i < 0$, there would be instant fire sale loss even without interbank losses or withdrawn funding. Multiplying by $A_i = \frac{p_i}{1-p_i}$,

$$
A_i\xi_i-\min\{-\min\{\max\{\gamma_i+\xi_i,0\}-D_i,0\},p_iy_i\}-A_iE_i=0.
$$

Now depending on whether the first or second term applies in the min and max functions, one can find four different possible solutions:

$$
\begin{aligned}
\xi_{i,1}&=\tfrac{D_i+A_iE_i}{A_i}, &\quad \xi_{i,2}&=\tfrac{p_iy_i+A_iE_i}{A_i},\\
\xi_{i,3}&=E_i, &\quad \xi_{i,4}&=\tfrac{D_i+A_iE_i-\gamma_i}{A_i+1},
\end{aligned}
$$

from which the correct solution is easy to be identified for all i. ☐

D Detailed simulation results

Focusing first on the case without capital or liquidity infusions (Table 1-4, upper panels), the number of exogenous defaults (*"exodef"*) from the stress test increases from 5 to 11 during the time horizon. The number defaults after contagion is between 6 and 16 depending on the fire sale mechanism, i.e. the maximum number of contagious defaults is 5.

When there are *no fire sales*, there are 1-4 additional institutions that default due to contagion (*"defaults"* in Table 1). Losses on interbank exposures span between 0.09 and 0.19% of the total risk exposure amount of stress tested banks.

In the second block of the table (*"infused, no constraint"*) there is no constraint on the amount of infusion ($M = \infty$). We can see that obviously in this case a certain amount of equity infusion is able to completely eliminate interbank losses. We do not need to save all banks that are initially defaulted. Indeed, there are 0 interbank losses despite the fact that 3-5 banks remain defaulted. The reason for this is that there is no need to save banks which do not have any interbank obligations because saving them would not decrease interbank losses. In the last column *"benefit"* is computed as in equation (13). One can observe that as the amount of infusion needed inreases, the benefit decreases and when exceeding the amount of the corresponding contagion losses without infusion, the benefit even goes into negative. In these cases, it is not worth saving the defaulted banks because the size of their interbank positions do not justify the need for capital infusion.

In the third block (*"infused, constraint 1/2"*), in order to illustrate the importance of the budget constraint, we report results that are obtained with a constraint on infusion half the amount needed in the second block (without constraint). Benefits are close to but below the previous experiment, and it is clear that since the constraint is smaller than in the "no constraint" experiment, we still have defaults and interbank losses in the infused network. By the end of our time horizon, interbank losses in the presence of this constrained infusion increases to nearly the amount of losses without any infusion. We also note that the number of infused banks and the number of defaulted banks cannot be higher than the number of total defaults after contagion and without infusion.

The following Tables 2-4 quantify the same effects assuming the different fire sale mechanisms (A), (B) and (C). We can immediately infer that interbank losses are different across the different mechanisms. This is the consequence of the endogenous recovery rate as different fire sale mechanisms affect the balance sheets differently (as can be seen from equation (2)). Therefore, the asset-to-debt ratios driving the ability to pay the interbank obligations will be different across fire sale mechanism (A), (B), and (C), respectively.

As a consequence, in Table 2 the interbank losses are very close to Table 1 losses since fire sale losses are very small except for the last two quarters. This means that interbank losses in this case are so low that the value of banks' assets is just slightly deteriorated and they sell illiquid assets only to a limited extent except in 2020Q3 and 2020Q4 where fire sale losses increase to 0.04-0.06 percent of RWA of the stress tested banks. With unconstrained infusion, benefits are positive between 1 and 20 percent, except for 2020Q1 which means that in this quarter the initial capital shortfalls were higher than the contagion losses that would be contained by the infusion. But in general, results show that original contagion losses were so high that they dominated capital shortfalls, hence providing infusions up to the amount of shortfalls induce positive benefits in most cases for this fire sale type. The constrained infusions resulted in negative benefits in most of the cases. This also shows the non-linear nature of contagion. The provision of half the amount of infusions leads to at most half the amount of decrease in losses. We will point to the fact in the next subsection that a decrease in the budget constraint might immediately entail that it will not be possible to rescue banks with relatively high capital shortfalls and therefore the infusion benefit decreases by much more than the decrease in the amount of budget constraint. In this case, fire sale losses remain high in the constrained infusion experiment.

Table 3 shows results for fire sale type (B) where short-term funding is withdrawn from defaulted banks. Given that direct interbank losses are fairly limited in general, these type of fire sales lead to the highest amount of fire sale losses[13] because defaulted banks' capital shortfall is already high compared to the interbank losses and since short-term funding is withdrawn, the asset side is further depleted by the fire sales of illiquid assets of those banks which are already in default. These fire sale losses amount to 0.04-0.06 percent of total risk exposure amount. The benefit of infusion is highest in this case for unconstrained infusion clearly due to the fact that interbank and fire sale losses without infusion were the highest in this case and a smaller amount of infusion caused almost up to 60% benefits in the analysed quarters. Constrained results are less spectacular; for the first three quarters benefits are above 20%, and it goes into negative afterwards: the capital

[13]For the no infusion case

shortfalls are so high later that they go above the amount of contagion losses.

In Table 4, results are reported for fire sales (C) where short-term funding is withdrawn by the defaulting banks themselves. The results show that these withdrawals do not lead to further spread of contagion onto healthier banks, because the number of defaults do not exceed the number of defaults without any fire sales. Fire sale losses are therefore most limited in this case. The unconstrained infusions' benefit is slightly below 20% for the first three quarters and goes below zero from 2020 Q1. Constrained benefits are also mostly negative in this case.

These results confirm that infusions without a constraint are exactly the same for all fire sale mechanisms. The theoretical reason is the following. Since there is no constraint, infusions are enough to eliminate all losses in the network. In this case, there are no fire sale losses, therefore the amount of infusion is equal to the capital shortfall of banks. However, since we do not need to infuse all of the defaulted banks, it is still worth to explore all possible subsets of the defaulted banks. It is important to note that infusions are still fairly close comparing different fire sales mechanisms in the presence of a constraint. The reason is clear again for type (B) losses: fire sale losses of a bank are only the function of the given defaulted banks' asset-to-debt ratio which are going to be saved. Hence, if this bank is saved then there is no fire sale loss and the infusion is independent from other banks' fire sale losses. This explanation is not reasonable for type (A) and (C) losses and points to the fact that fire sale losses occur mostly at banks that are not saved by these infusions. Looking at the solution method in appendix C, it is obvious that providing the amount of capital shortfall is not a solution as defaulted banks might suffer additional fire sale losses which should also be covered. An example can be seen in Figure 9 in the 2020Q1 constrained chart in the top right corner. Bank 6 needed higher infusion than its initial shortfall for fire sale type (C). This is due to the fact that it suffers fire sale losses as a consequence of other banks' failure that could not be saved in the presence of the budget constraint. The following section confirms that constrained infusions are depending on the fire sale mechanisms and not just on the original capital shortfalls.

qtr	exodef	defaults	IB losses (RWA%)	FS losses (RWA%)	infusion (RWA%)	benefit
			no infusion			
19q1	5	6	0.0898	0		
19q2	5	6	0.0916	0		
19q3	8	10	0.0971	0		
19q4	7	10	0.1047	0		
20q1	7	10	0.1189	0		
20q2	8	12	0.1391	0		
20q3	10	12	0.1644	0		
20q4	11	13	0.1845	0		
			infused, no constraint			
	infused					
19q1	2	3	0	0	0.0855	4.73%
19q2	2	3	0	0	0.0865	5.54%
19q3	5	3	0	0	0.0920	5.26%
19q4	4	3	0	0	0.1111	-6.06%
20q1	4	3	0	0	0.1321	-11.10%
20q2	5	3	0	0	0.1566	-12.61%
20q3	6	4	0	0	0.1887	-14.76%
20q4	6	5	0	0	0.2241	-21.48%
			infused, constraint 1/2			
	infused					
19q1	1	5	0.0500	0	0.0390	0.86%
19q2	1	5	0.0659	0	0.0252	0.54%
19q3	4	5	0.0827	0	0.0151	-0.66%
19q4	3	6	0.1011	0	0.0100	-6.09 %
20q1	3	6	0.1146	0	0.0176	-11.18%
20q2	4	7	0.1295	0	0.0298	-14.56%
20q3	5	6	0.1458	0	0.0475	-17.59%
20q4	5	7	0.1642	0	0.0651	-24.32%

Table 1: Losses in the original and infused network, no fire sales.
Interbank losses, fire sale losses and infusions are reported in percentage of risk weighted assets of the stress tested banks.

qtr	exodef	defaults	IB losses (RWA%)	FS losses (RWA%)	infusion (RWA%)	benefit
			no infusion			
19q1	5	8	0.0939	0.0127		
19q2	5	8	0.0977	0.0143		
19q3	8	11	0.1041	0.0158		
19q4	7	11	0.1107	0.0161		
20q1	7	11	0.1191	0.0114		
20q2	8	13	0.1432	0.0155		
20q3	10	13	0.1670	0.0373		
20q4	11	16	0.1927	0.0576		
			infused, no constraint			
	infused					
19q1	2	3	0	0	0.0855	19.79%
19q2	2	3	0	0	0.0865	22.70%
19q3	5	3	0	0	0.0920	23.22%
19q4	4	3	0	0	0.1111	12.39%
20q1	4	5	0	0	0.1321	-1.17%
20q2	5	3	0	0	0.1566	1.28%
20q3	6	4	0	0	0.1887	7.65%
20q4	6	5	0	0	0.2241	10.50%
			infused, constraint 1/2			
	infused					
19q1	0 *	8	0.0940	0.0127	0	0%
19q2	1	6	0.0319	0.0091	0.0302	5.97%
19q3	4	6	0.0431	0.0112	0.0209	4.09%
19q4	3	7	0.0549	0.0149	0.0101	-4.11%
20q1	3	7	0.0632	0.0101	0.0176	-9.17%
20q2	4	7	0.0720	0.0109	0.0327	-9.14%
20q3	4	7	0.0815	0.0297	0.0489	-9.94%
20q4	4	10	0.0918	0.0499	0.0663	-14.67%

Table 2: Losses in the original and infused network, fire sales A.
Interbank losses, fire sale losses and infusions are reported in percentage of risk weighted assets of the stress tested banks. *: there was no solution

qtr	exodef	defaults	IB losses (RWA%)	FS losses (RWA%)	infusion (RWA%)	benefit
			no infusion			
19q1	5	7	0.1368	0.0450		
19q2	5	8	0.1523	0.0578		
19q3	8	11	0.1607	0.0623		
19q4	7	11	0.1667	0.0629		
20q1	7	10	0.1575	0.0406		
20q2	8	12	0.1786	0.0450		
20q3	10	12	0.1850	0.0454		
20q4	11	13	0.1871	0.0458		
			infused, no constraint			
	infused					
19q1	2	3	0	0	0.0855	52.97%
19q2	2	3	0	0	0.0865	58.82%
19q3	5	3	0	0	0.0920	58.73%
19q4	4	3	0	0	0.1111	51.63%
20q1	4	3	0	0	0.1321	33.31%
20q2	5	3	0	0	0.1566	29.93%
20q3	6	4	0	0	0.1887	18.14%
20q4	6	5	0	0	0.2241	3.82%
			infused, constraint 1/2			
	infused					
19q1	1	5	0.0727	0.0227	0.0406	25.22%
19q2	1	5	0.1010	0.0352	0.0277	22.01%
19q3	4	6	0.1183	0.0356	0.0175	23.13%
19q4	3	7	0.1607	0.0587	0.0102	0.01%
20q1	3	6	0.1508	0.0363	0.0176	-3.36%
20q2	4	7	0.1616	0.0367	0.0305	-2.35%
20q3	5	6	0.1631	0.0370	0.0480	-7.66%
20q4	5	7	0.1647	0.0373	0.0652	-14.67%

Table 3: Losses in the original and infused network, fire sales B.
Interbank losses, fire sale losses and infusions are reported in percentage of risk weighted assets of the stress tested banks.

qtr	exodef	defaults	IB losses (RWA%)	FS losses (RWA%)	infusion (RWA%)	benefit
			no infusion			
19q1	5	7	0.0927	0.0100		
19q2	5	7	0.0957	0.0117		
19q3	8	11	0.1012	0.0130		
19q4	7	11	0.1068	0.0131		
20q1	7	10	0.1189	0.0068		
20q2	8	12	0.1399	0.0075		
20q3	10	12	0.1652	0.0098		
20q4	11	13	0.1850	0.0122		
			infused, no constraint			
	infused					
19q1	2	3	0	0	0.0855	16.72%
19q2	2	3	0	0	0.0865	19.43%
19q3	5	3	0	0	0.0920	19.40%
19q4	4	3	0	0	0.1111	7.39%
20q1	4	3	0	0	0.1321	-5.08%
20q2	5	3	0	0	0.1566	-6.28%
20q3	6	4	0	0	0.1887	-7.78%
20q4	6	5	0	0	0.2241	-13.62%
			infused, constraint 1/2			
	infused					
19q1	1	5	0.0500	0.0079	0.0417	3.04%
19q2	1	5	0.0659	0.0080	0.0280	5.22%
19q3	4	5	0.0827	0.0080	0.0178	4.93%
19q4	3	7	0.1032	0.0119	0.0105	-4.75%
20q1	4	5	0.1146	0.0056	0.0384	-26.14%
20q2	4	7	0.1295	0.0057	0.0301	-12.13%
20q3	5	6	0.1458	0.0070	0.0477	-14.56%
20q4	6	6	0.1642	0.0090	0.0958	-36.36%

Table 4: Losses in the original and infused network, fire sales C.
Interbank losses, fire sale losses and infusions are reported in percentage of risk weighted assets of the stress tested banks.

E Optimal infusions and interconnectedness measures

Usual measures used in the identification and calibration of capital buffers are either some simple network centralities or size-based measures. All these kind of measures reflect the systemic importance of the institutions but as seen in the presentation of our methodology, contagion losses are driven not only by the network structure but also by the capital or liquidity shortfalls arising when a shock hits the banking system (or from the opposite perspective, conditional on how resilient the banks are to shocks). While previous theoretical studies point out that high levels of interconnections can have a stabilisation property, once the shock is severe enough dense interconnections can propagate the shock in the network. However, such detrimental propagation effects will only materialise if interconnected banks are severely hit by the shock to an extent that their solvency and/or liquidity positions deteriorate to a point where they fall below regulatory minima.

In order to illustrate how the optimised infusions derived from our model compares with traditional network metrics, we use the optimal infusion amounts using the results of the macroprudential stress test (at the last projection point, which indicates the highest number of defaults and capital shortfalls). We compare the optimal infusions to eigenvector centralities, in-degrees (interbank assets) and out-degrees (interbank liabilities) and OSII interconnectedness scores of the originally defaulting eleven nodes in 2020 Q4. The correlation of eigenvector centralities, and the different degrees is above 80%. OSII interconnectedness scores exist only for 55 banks in the consolidated bank network, but its correlation is only around 20% with the other network measures.[14] Interestingly, one can see from Figure 10 that regardless of the infusion constraint applied and the specific centrality metric, the optimal infusions have no functional relationship to the other measures. This corroborates previous findings of Fink et al. (2016), Alter, Craig, and Raupach (2015) and Covi, Kok, and Meller (2018) which suggest that simple network characteristics may only provide a partial perspective on the systemicness of individual institutions. Obviously, the optimal infusions derived here are a function of our modelling framework but also of the specific stress scenario imposed. Other scenarios may give a different picture. However, the point remains that to truly gauge the systemicness of individual banks it is crucial to go beyond simple metrics and interconnectedness scores.

[14] It should be noted however that these scores are calculated at country level and therefore banks from different countries are not comparable.

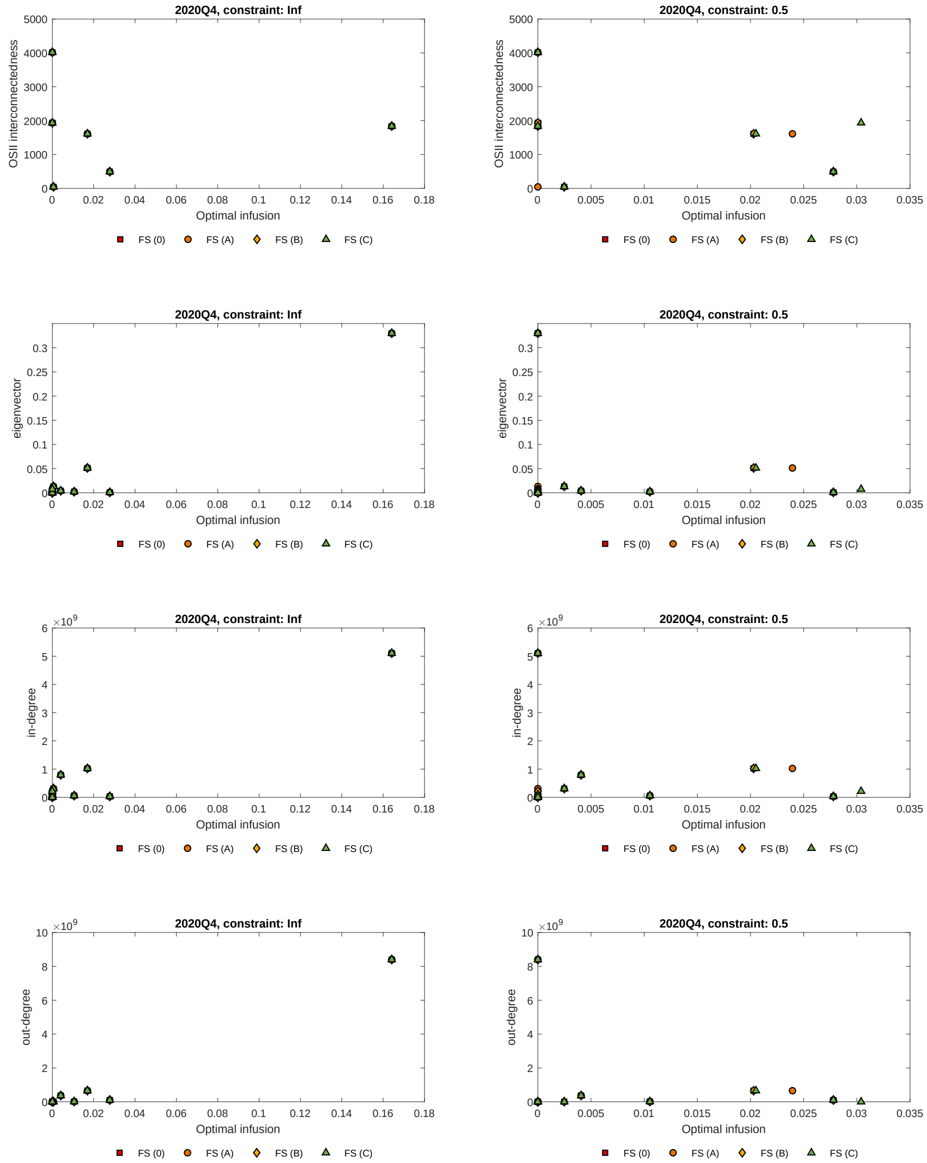

Figure 10: Optimal infusions compared to systemic importance measures in the different constraint and fire sale exercises.
The measures from top to bottom are OSII interconnectedness score, eigenvector centrality, in-degree (interbank assets) and out-degree (interbank liabilities). Optimal infusions are in percentage of total risk weighted assets of stress tested banks.

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Gábor Fukker
European Central Bank, Frankfurt am Main, Germany; Magyar Nemzeti Bank, Budapest, Hungary; email: gabor.fukker@ecb.europa.eu

Christoffer Kok
European Central Bank, Frankfurt am Main, Germany; email: christoffer.kok@ecb.europa.eu

Postal address 60640 Frankfurt am Main, Germany
Telephone +49 69 1344 0
Website www.ecb.europa.eu

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