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Switching-track after the Great Recession

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Abstract

We propose a theoretical framework to reconcile episodes of V-shaped and L-shaped recovery, encompassing the behaviour of the U.S. economy before and after the Great Recession. In a DSGE model with endogenous growth, negative demand shocks destroy productive capacity, moving GDP to a lower trajectory. A Taylor rule policy designed to reduce the output gap may counterbalance the shocks, preventing the destruction of economic capacity and inducing a V-shaped recovery. However, when shocks are deep and persistent enough, like during the Great Recession, they call for a downward revision of potential output measures, the so-called *switching-track*, weakening the recovering role of monetary policy and inducing an L-shaped recovery. When calibrated to the U.S. economy, the model replicates well the L-shaped recovery and *switching-track* that followed the Great Recession, as well as the V-shaped recoveries that followed the oil shock recessions.

Keywords: Economic Recovery, Endogenous Growth, Supply Destruction Prevention, Economic Capacity, Monetary Policy

JEL Codes: E12, E22, E32, O41, E52

Non-technical summary

The Great Recession that followed the 2008 financial crisis was a deep and prolonged downturn, with permanent scarring effects on the productive capacity of advanced economies. In this paper we focus on the dynamics of economic recovery after the Great Recession, in comparison with past episodes, to understand which factors affect recoveries and what role monetary policy plays in shaping them. Our framework is applicable to the majority of OECD countries, but we focus on the United States, where the level of GDP fell in conjunction with the Great Recession and then gradually resumed growing at a similar rate as before, resulting in a downward parallel shift in the path. This implies a permanent gap between the actual level of GDP and what it would have been had the recession not happened, giving rise to an L-shaped recovery. In contrast, past downturns were associated with a temporary acceleration of growth, allowing the economy to go back to its original growth path, a dynamic referred to as a V-shaped recovery.

The paper aims to build a model to replicate episodes of V-shaped and L-shaped economic recoveries in a single framework. We argue that an L-shaped recovery is not consistent with a fundamental modelling assumption commonly used to represent the technology behind GDP production in the literature aiming to model recessions through dynamic stochastic general equilibrium models (DSGEs). A Neoclassical production function (Cobb-Douglas) assumes diminishing returns to capital (*i.e.* the productive capacity of the economy), so that a negative shock affecting the capital stock would increase the return to capital, sustaining investment and bringing GDP back to its original growth trajectory. In this case, the level of potential output, *i.e.* the maximum level of production feasible, is not affected by the recession. However, a model with this production function does not explain L-shaped recoveries. Conversely, a production function featuring endogenous growth technology (AK) is only consistent with L-shaped recovery episodes because the return to capital is constant, so that a lower capital stock does not generate a higher return to capital and thus stimulus to invest after a shock. In this case, potential output switches track, permanently moving to a lower path.

The assumption chosen to model production technology has crucial implications for the dynamics of economic recovery, but also for the role played by policy interventions. We focus on monetary policy, due to the Fed's Taylor rule policy, aiming at closing the gap between

GDP and potential output (J. Taylor, 1993). This implies a systematic countercyclical policy response, providing stimulus when GDP falls below its potential level until the gap is closed. If the technology assumed in the model generates a V-shaped recovery (Cobb-Douglas case), the policy stimulus plays a supportive role. However, if production technology results in L-shaped recoveries (AK case), policy interventions are not only supportive, but can even be the force behind V-shaped recoveries, explaining past observed episodes as well as the L-shaped recovery that followed the Great Recession. This is a fundamental contribution of this paper.

We propose a novel framework to combine endogenous growth and DSGEs, and shows that a learning-by-doing technology (AK in aggregate) à la Romer (1986) is consistent with both V-shaped and L-shaped recovery episodes once the role of monetary policy is taken into account. The paper builds on Christiano, Motto, and Rostagno (2014), and introduces a novel amplification mechanism through bankruptcy spillovers on the depreciation rate of capital, so that an increase in risk and bankruptcy leads to the destruction of economic capacity. Moreover, the standard model is augmented by allowing for the *switching-track*, *i.e.* the revision of potential output estimates by the central bank. Calibration of the U.S. economy shows that a simple Taylor rule with a positive weight on the output gap could give enough stimulus to replicate the V-shaped recoveries that followed the oil crises. On the other hand, a large and highly persistent demand shock such as the Great Recession has a permanent negative effect on productive capacity. In order to generate this last result, allowing for the *switching-track* in the Taylor rule is key. If the central bank updates its beliefs on the level of potential output based on past observed GDP, as the latter falls due to the crisis, the estimate of potential output will follow, so that the output gap will shrink over time, and the policy stimulus will get increasingly weaker. The potential output *switching-track* in the Taylor rule replicates the timing of the potential output revisions carried out by the Fed. Moreover, we show that monetary policy acts a cushion in the model economy, preventing the destruction of productive capacity. This constitutes a novel transmission mechanism of monetary policy and another fundamental contribution of this paper.

In conclusion, this paper sheds new light on the drivers of economic recovery and the role played by monetary policy in shaping them, and offers a new tool to inform policy-makers.

1 Introduction

The Great Recession had a profound impact on the economic performance of OECD countries. Crucially, the crisis had a persistent level effect on GDP, which still remains below its pre-crisis path for the vast majority of advanced economies (Ball, 2014). Focusing on the United States, past recessions were typically followed by a temporary acceleration of growth, with GDP converging back to its pre-recession trajectory. This is commonly referred to as a V-shaped recovery. Conversely, after the 2008 financial shock we saw potential output *switching-track* in conjunction with the crisis, as the deep and persistent fall in economic activity led to a downward swerve of the GDP trend from its original path. Consequently, the output gap closed following the *switching-track* of potential output, rather than faster GDP growth, giving rise to a so-called L-shaped recovery (see Figure 1). This fact challenged the general consensus on the distinct relationship between trend, growth and business cycles. If recessions are not followed by recoveries, downturns will affect the long run path of GDP, implying that potential output cannot be represented by a stable trend in the productive capacity of the economy.

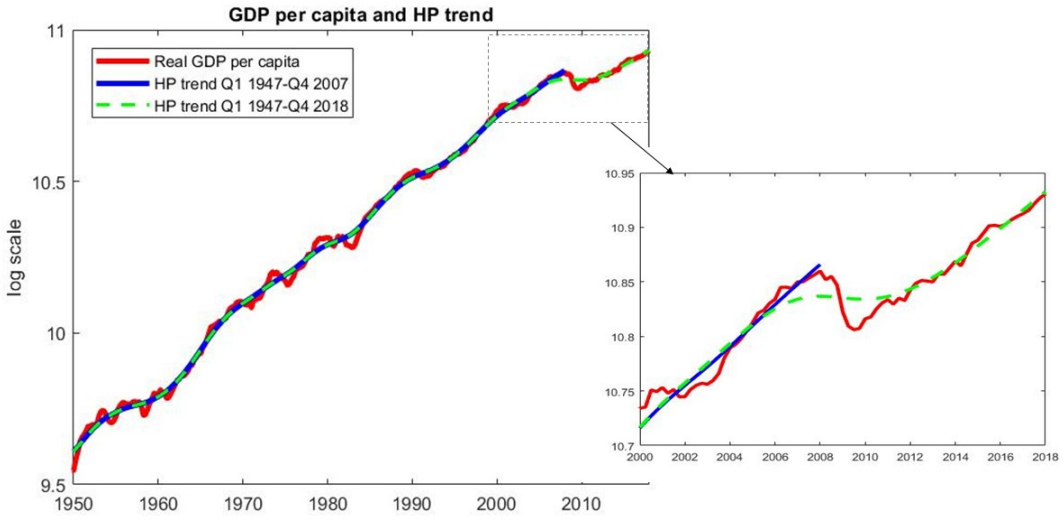

Figure 1: U.S. GDP per capita

This paper shows that the Great Recession can be seen as a large and persistent demand shock deeply reducing economic activity for an unusually long period, translating into a permanent depletion of productive capacity (a supply-side effect) casting the observed L-shaped

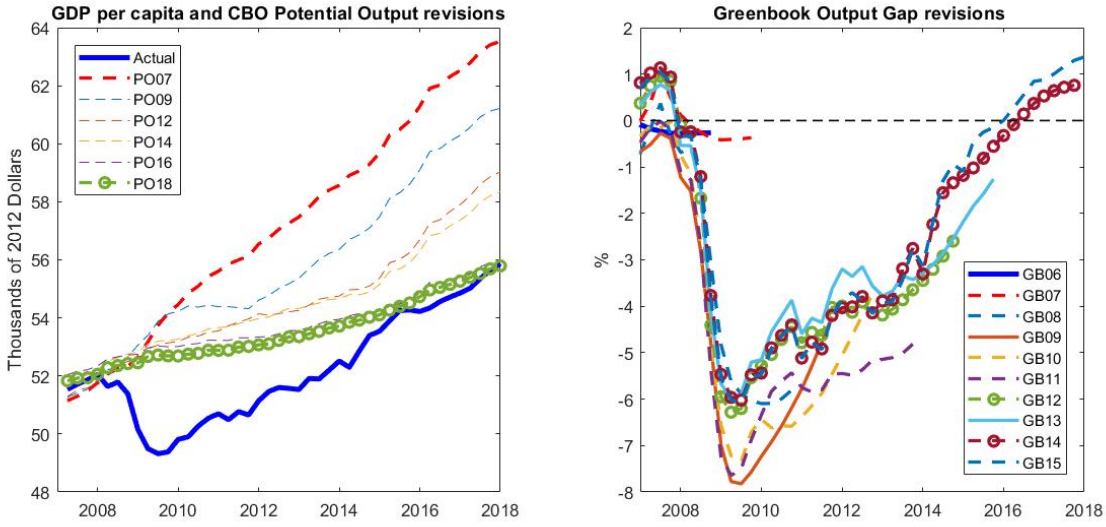

Figure 2: Switching-track

recovery. GDP moving down to a lower trajectory gradually induced a change in measured potential output. The economy then converged to its new lower potential output path, ending the recession without a full recovery. In this context, potential output is a measure of policy makers' beliefs about the level of GDP that can be feasibly sustained in the long run, a key indicator affecting monetary policy interventions, and ultimately the shape of the recovery.

Figure 2 shows that the output gap closed around 2016 following the *switching-track* process of potential output revisions instead of faster growth, weakening the strength of the policy intervention over time. The estimates of potential output published by the Congressional Budget Office (CBO) each year were revised down as the recession unfolded, leading to the closure of the output gap intrinsic to an L-shaped recovery –see the left panel of Figure 2. Consistently, the output gap estimates published in the Fed's Greenbook were revised over time, showing that the output gap also closed in 2016 –right panel of Figure 2.

To replicate the dynamics of U.S. GDP after the Great Recession, this paper relies on the combination of four key assumptions. First, we embed learning-by-doing à la Romer (1986) into a DSGE model with financial frictions à la Christiano et al. (2014). Under this assumption, aggregate technology faces constant returns to capital. Consequently, a negative shock to the capital stock would not allow the economy to return to its previous growth trajectory,

affecting the level of GDP permanently. Second, in line with Christiano et al. (2014), the Great Recession is modelled as a large negative shock to aggregate demand that induces a surge in bankruptcy. Third, the depreciation rate is assumed to be endogenous and positively related to the bankruptcy rate. A sudden increase in bankruptcy will then deplete the capital stock, inducing a permanent decline in output. Finally, the monetary authority is assumed to follow a standard Taylor rule, targeting inflation and the output gap, but we also assume that the monetary authority revises potential output when recessions are deep and persistent. This way, we introduce the *switching-track* in the model, consistently with the unprecedented downward revision of potential output estimates that followed the large and long lasting decline in GDP observed during the Great Recession.

Monetary policy plays a fundamental role in our model, by providing countercyclical stimulus to aggregate demand but also protecting the productive capacity of the economy by *preventing capital destruction*. The latter constitutes a novel transmission channel with potentially significant implications for the conduct of monetary policy and, more generally, the consensus concerning its role. The idea that large shocks can negatively affect the productive capacity of the economy permanently has gained traction in the aftermath of the Great Recession, and it is becoming even more prominent in the face of the COVID-19 crisis, especially for monetary policy-makers. During the press conference to announce measures in response the economic shock from Covid-19, Mark Carney (BoE) remarked: *"In this situation, it's disruption not destruction of supply. Part of our job is to make sure that that is indeed the case, and so that we're bridging, and that's very much part of the analysis."*[1] We aim to contribute to this debate and this paper is, to the best of our knowledge, the first to model the *destruction prevention channel* explicitly. Monetary policy operates as a cushion, protecting the economy from negative shocks that destroy the productive capacity of the economy.

We also show that in periods characterised by negative shocks that are not overly persistent, the standard Taylor rule is enough to generate a full recovery, making the economy converge to its past trajectory despite aggregate constant returns to capital. The Taylor rule is then

[1]Bank of England Press Conference, 11^{th} March 2020. For similar examples see the blog post by Christine Lagarde, President of the ECB, 9^{th} April 2020 and Fed's Chair Jerome Powell's speech on 9^{th} April 2020 "COVID-19 and the Economy".

sufficient to drive a V-shaped recovery in normal times. Furthermore, when calibrated to the recessions that followed the 1974 and 1990 oil shocks, the model replicates well the observed V-shaped recoveries. Our model can then reconcile the dynamics of the U.S. economy before and after the Great Recession. This represents another contribution of our work, as existing models provide insights on the possible mechanisms through which endogenous growth can lead to output shortfalls in line with the Great Recession, but such theories tend to remain silent concerning the drivers of recovery in the past.

In particular, Benigno and Fornaro (2018) build a Schumpeterian framework, in the spirit of Aghion and Howitt (1992), with nominal rigidities and a Taylor rule targeting the employment rate. Their model shows two stationary solutions, a good one and a *stagnation trap*, with low growth and unemployment. We can interpret the Great Recession in this framework as a fall in demand that moves the economy from the good stationary equilibrium to the stagnation trap. Pessimistic beliefs and the zero-lower bound (ZLB) are crucial to generate their results. In particular, the existence of a stagnation trap is a consequence of the ZLB constraint on monetary policy, which would otherwise restore full employment. When beliefs eventually become optimistic again, the economy escapes the stagnation trap, moving back to the good stationary equilibrium. Since the Taylor rule aims to restore the initial level of employment, but not the initial level of output, the production capacity lost during the recession is not recovered and the economy converges to a lower path, with the initial growth rate but a lower intercept. Similarly, in our paper we find that monetary policy is a key element in determining the level of economic activity after a demand shock bringing the economy to the ZLB. Differently from Benigno and Fornaro (2018), our model shares the standard property of the AK (endogenous growth) framework, featuring a unique stationary growth rate, which makes it suitable for a quantitative analysis capturing the permanent level effects on GDP that followed the Great Recession and the *switching-track* of potential output. Notably, the unique stationary growth rate determines the slope of the balanced growth path GDP trajectory. The intercept of the balanced growth path is in principle indeterminate, and set by the initial stock of capital, *i.e.* the productive capacity of the economy. Shocks that destroy productive capacity reduce the intercept of the balanced growth path, moving the economy to a lower trajectory. By preventing

destruction, monetary policy affects the new intercept of the balanced growth path and plays a role in shaping the recovery. When the economy hits the ZLB, the ability of monetary policy to prevent capacity destruction is hampered and this results in a lower intercept in the new steady sate.

Other papers focus on explaining the lack of recovery following financial crises in particular, and thus bring together endogenous growth and financial shocks. Bianchi et al. (2019) also propose a Shumpeterian model, adding financial frictions to explore the properties of different kinds of financial shocks, and their long lasting effects on the economy. Cozzi et al. (2017) propose an estimated Shumpeterian DSGE model and stress how the inclusion of endogenous growth leads to the amplification of financial shocks. Anzoategui et al. (2019) build on the pioneering work of Comin and Gertler (2006), proposing an endogenous growth model à la Romer (1990) with endogenous diffusion and financial frictions.[2] Their model generates an endogenous response of TFP to aggregate demand shocks through the R&D and diffusion channels. Their analysis also shows that this response is consistent with the observed cyclicality of the speed of diffusion of new technologies. Their estimation reveals that a shock equivalent to a risk shock can generate medium to long-term effects on the level of TFP, and thus output, due to a temporary slowdown in productivity enhancing investments and, most importantly, technology diffusion. Ikeda and Kurozumi (2019) choose a similar framework and, just like us, they propose a model where financial shocks can generate permanent shortfalls in GDP, resulting in a parallel downward shift in the level of economic activity. Queralto (2020) also contributes to this literature, by focusing on banking crises in an open economy expanding product variety framework. The paper shows that financing frictions can affect the introduction of new varieties, and thus endogenous TFP. Garga and Singh (2020) take a similar approach, but focus in the design of optimal monetary policies in their framework.

None of these papers discusses the suitability of their approaches to generate V-shaped recoveries in normal times. This paper aims to complement the literature by providing a framework that reconciles episodes of V-shaped and L-shaped economic recovery. To achieve this, we propose a novel way of incorporating endogenous growth into DSGE models with financial fric-

[2] Comin and Gertler (2006) analysed the link between short and medium-term variations in economic activity, drawing attention to medium-term business cycles and pointing out that recessions can have lasting effects.

tions, employing a learning-by-doing technology à la Romer (1986), where knowledge is crucial for economic growth, and its accumulation is directly linked to the capital stock. We choose this specification to show that a relatively small departure from a standard DSGE model can generate a rich departure from its predictions on the medium to long-run scars of large recessions. We picked the simplest growth model which allows us to explore the role of capital destruction in shaping the dynamics of potential output. We also introduce a new mechanism for capital depreciation and a Taylor rule that allows for the *switching-track*.

The rest of the paper is organised as follows: Section 2 discusses the role of capital destruction, Section 3 explains the model, Section 4 reports the calibration, Section 5 contains results for the Great Recession simulations, Section 6 describes normal times, Section 7 discusses the use of the Taylor rule at the Fed and Section 8 concludes.

2 Capital-Destruction: Evidence & Related Literature

In order to represent the dynamic of GDP during the the Great Recession we make the following modelling choices. First, we model the Great Recession as a demand shock that combines higher aggregate risk and lower consumer confidence, reducing investment and consumption demand. Like in Christiano et al. (2014), an increase in credit risk reduces credit and raises the probability of bankruptcy in equilibrium.

Secondly, the rise in bankruptcies that follows the demand shock causes capital destruction and thus a permanent supply effect. To generate this result, we augment the framework in Christiano et al. (2014) by positively linking the probability of bankruptcy to the depreciation rate of capital. During the Great Recession, a surge of bankruptcies thus leads to a depletion of the capital stock. The recession is then driven by a large shock to aggregate demand inducing supply effects. Our choice is motivated by empirical work by Hall (2015, 2016), who identifies the shortfalls in business capital and total factor productivity as the main drivers of the deviation of GDP from its previous trend. This paper represents the idea by associating the Great Recession with a substantial decline of the stock of capital and, through learning-by-doing, of total factor productivity.

Identifying capital destruction in the data is notoriously difficult. National Accounts esti-

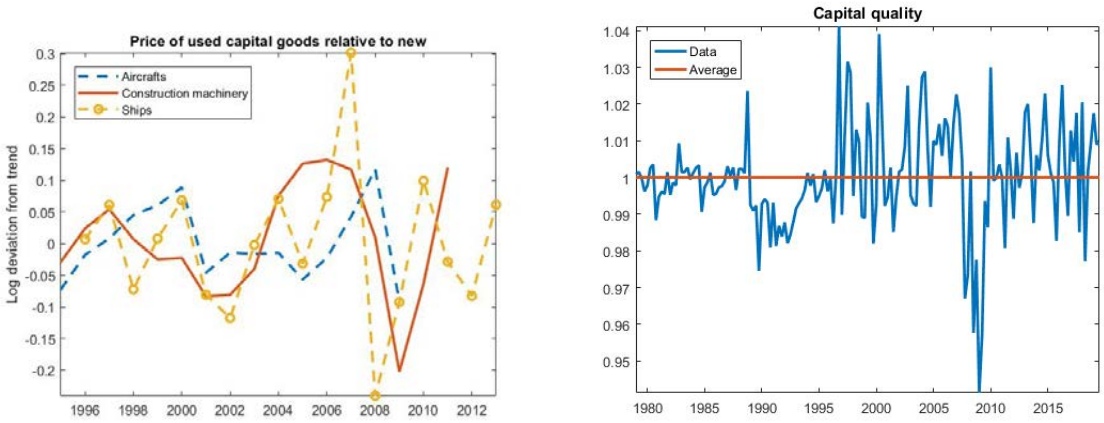

Figure 3: Capital destruction. Left panel: Log-deviation from trend of the price of used compared to new capital goods. Data made available by Lanteri (2018). Right panel: Capital quality. Constructed following methodology suggested by Kozlowski et al. (2020), all details in the Appendix.

mates of the capital stock are constructed using the perpetual inventory method, and typically employ geometric depreciation rates, held constant over time.[3] As a consequence, measurement of the capital stock in National Accounts reflects movements of investment, but cannot reflect cyclical capital destruction. Nonetheless, the idea that recessions destroy capital is not new, and we aim to provide new insights to the existing literature. For example, in a vintage capital framework, Caballero and Hammour (1994) provided a rational to the evidence stated by Davis and Haltiwanger (1990) that job destruction is much more cyclically responsive than job creation.[4] Which bring them to see recessions as cleansing periods, when the productive system eliminates outdated techniques and products. In their view, the associated physical capital becomes obsolete and is then scrapped. More recently, Gourio (2012) built a business cycle model with disaster risk, characterizing disasters as episodes of large capital destruction. He noted that economic downturns are associated with large reallocation of capital, leading to the loss of firm specific specialised capital goods as well as intangible capital. Recent theoretical work by Lanteri (2018) and Kozlowski et al. (2020) also contributes to this literature, and the authors provide empirical grounding for the idea of capital destruction following the Great Recession.

Lanteri (2018) analyses the dynamics of the second-hand market for equipment in the United

[3]For the United States, see Fraumeni (1997).

[4]For an analysis of job creation and destruction in vintage capital models, see Boucekkine, del Río, and Licandro (1999)

States.[5] He finds that quantities sold vary procyclically and that the price of second-hand equipment is procyclical and more volatile than the price of new equipment, showing a significant decline at the beginning of the Great Recession. This evidence indicates that the value of existing capital declined in real terms during the Great Recession (see left panel in Figure 3), suggesting a fall in productive capacity.[6] Additional evidence is provided by Kozlowski et al. (2020), who construct a measure of the quality of corporate non-financial assets. As can be observed in the right panel of Figure 3, their measure of capital quality was unusually low in conjunction with the Great Recession, which can be interpreted in their framework as an increase in the depreciation rate of capital (see Appendix C). Overall, these findings suggest that the productive capacity of capital goods fell during the Great Recession, and constitutes additional evidence indicating that downturns can be associated with increasing capital depreciation rates. Motivated by these measures, we introduce an endogenous countercyclical depreciation rate in the model, which results in a decline in the relative price of the second-hand market for capital and the destruction of its productive capacity following a negative shock.

3 The Model

We build a New Keynesian model of endogenous growth by introducing a learning-by-doing technology à la Romer (1986) in a Dynamic Stochastic General Equilibrium model with financial frictions. We build on the financial accelerator framework introduced by Bernanke, Gertler, and Gilchrist (1999) and draw from Christiano et al. (2014) for risk shocks. Households gain utility from consumption, dis-utility from labour, and are subject to confidence shocks, *i.e.* reductions in their marginal utility of consumption. The production sector is comprised of a labour union, final good producers, intermediate good producers, capital producers and entrepreneurs. The latter face financial frictions, since they need to fund part of their capital expenditure with external resources, exposing themselves to the possibility of bankruptcy. We model a risk shock as an increase in the probability of bankruptcy for entrepreneurs. The financial sector acts as an

[5] Lanteri (2018) collects data on quantities and prices from sales of second-hand capital, including commercial aircrafts, ships, and construction equipment, among others.

[6] In a recent paper, Gavazza and Lanteri (2021) analysed the market for vehicles, and found that the fall in the price of used cars was associated with a drop in upgrading during the Great Recession. As a consequence, the overall age, and thus quality and value of the vehicle stock declined.

intermediary between the production side and households, by providing loans to entrepreneurs and selling bonds to savers. The monetary authority sets the nominal interest rate according to a Taylor rule, factoring in the deviation of inflation from its target and the output gap. We also assume that potential output is measured as an average of past GDP values, so that large negative shocks lead to downward revisions. The following sections describe the behaviour of all agents in the model.

3.1 Labour Packer

Households, indexed by $\ell \in (0,1)$, supply differentiated labour input to a labour packing firm, *i.e.* a union, which then supplies an homogeneous labour input h_t^p to the production sector. The bundling technology is

$$h_t^p = \left(\int_0^1 h_t(\ell)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}\ell \right)^{\frac{\varepsilon}{\varepsilon-1}}. \tag{1}$$

The parameter $\varepsilon > 1$ represents the elasticity of substitution across types of labour, which are assumed to be gross substitutes, and $h_t(\ell)$ represents hours worked by household ℓ. Labour demand by the labour packer is

$$h_t(\ell) = \left(\frac{W_t(\ell)}{W_t} \right)^{-\varepsilon} h_t^p, \tag{2}$$

where $W_t(\ell)$ is the wage of labour variety ℓ. Substituting (2) into (1), we obtain the aggregate wage index

$$W_t^{1-\varepsilon} = \int_0^1 W_t(\ell)^{1-\varepsilon} \mathrm{d}\ell. \tag{3}$$

Thus total employment (measured in hours worked) is given by

$$h_t \equiv \int_0^1 h_t(\ell) \mathrm{d}\ell = h_t^p \int_0^1 \left(\frac{W_t(\ell)}{W_t} \right)^{-\varepsilon} \mathrm{d}\ell. \tag{4}$$

Defining $\int_0^1 \left(\frac{W_t(\ell)}{W_t} \right)^{-\varepsilon} \mathrm{d}\ell$ as a measure of wage dispersion across varieties implies that if the latter is larger than 1, then aggregate labour used in production is smaller than total employment. However, in this paper we will only consider symmetric equilibria, implying that the total labour supply $h_t^p = h_t$.

3.2 Households

There is a continuum of households of measure one. The ℓ-type household consumes, supplies labour, buys bonds and owns firms subject to wage frictions à la Rotemberg (1982);[7] when changing its wage, it incurs in a cost assumed to be proportional to aggregate output. Since households are assumed to be identical, differing only on the type of labour they offer, labour market equilibrium is symmetric. We will then omit index ℓ to simplify notation.

A representative household, offering a particular type of labour, maximises utility subject to its budget constraints and labour demand (2),

$$\max_{c_t, w_t, B_{t+1}} \quad u_t = E_t \left[\sum_{j=0}^{\infty} \beta^j \epsilon_t^c \left(\log \left(c_{t+j} - \chi c_{t-1+j} \right) - \psi \frac{h_{t+j}^{1+\nu}}{1+\nu} \right) \right] \tag{5}$$

$$\text{s.t.} \quad B_{t+1} + P_t c_t = R_{t-1} B_t + P_t w_t h_t - \frac{\chi^\omega}{2} \left(\frac{w_t}{w_{t-1}} - 1 \right)^2 P_t Y_t + D_t - \tau_t, \tag{6}$$

where c_t is consumption and h_t is labour, $\beta \in (0,1)$ is the time discount factor, $\chi > 0$ regulates the degree of habit formation, $\nu > 0$ is the inverse of labour supply elasticity, $\psi > 0$ is a parameter regulating labour hours in steady state and $\chi^w > 0$ regulates price frictions. B_{t+1} represent risk-less one period nominal bonds, purchased at time t, earning the risk-less nominal interest factor R_t.[8] Variable w_t represents the real wage rate, P_t is the price of the final good, D_t are profits redistributed to households by firms and τ_t are taxes. ϵ_t^c is a confidence shock, affecting the marginal utility of consumption, which follows an AR(1) process: $\log(\epsilon_t^c) = \rho_c \log(\epsilon_{t-1}^c) + \epsilon_{c,t}$, where $\rho_c \in (0,1)$ and $\epsilon_{c,t} \sim N(0, \sigma_c^2)$. This is a simple way of generating the fall in consumption demand as well as the nominal interest rate observed during the Great Recession.[9]

The FOCs for c_t and B_{t+1} in real terms are:

$$\lambda_t = (c_t - \chi c_{t-1})^{-1} \epsilon_t^c - \beta \chi E_t (c_{t+1} - \chi c_t)^{-1} \epsilon_{t+1}^c \tag{7}$$

[7] We choose this specification for price and wage frictions because they are better suited in the context of large shocks compared to the standard Calvo style. The letter imply a constant probability of resetting prices, which does not account for size effects. For a detailed discussion see Karadi and Reiff (2019).

[8] All nominal variables in this paper are defined in an arbitrary numeraire.

[9] Guerrieri and Lorenzoni (2017) provide insights into the underlying mechanism through the lens of a model with heterogeneous households. They show that when the economy's borrowing capacity is impaired, debtors reduce their demand for loans and creditors increase precautionary savings, resulting in lower demand and a fall in the nominal interest rate.

$$\lambda_t = \beta E_t R_t \frac{\lambda_{t+1}}{\pi_{t+1}}, \tag{8}$$

where λ_t is the Lagrange multiplier associated to the budget constraint and $\pi_t = \frac{P_t}{P_{t-1}}$.

The FOC for w_t gives the wage Phillips curve:

$$w_t = \frac{\varepsilon}{\varepsilon - 1} \frac{\psi h_t^{\nu} \epsilon_t^c}{\lambda_t} + E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \Omega_{t+1} Y_{t+1} \frac{1}{h_t} \right] - \Omega_t \pi_t \frac{Y_t}{h_t} \tag{9}$$

$$\Omega_t = \frac{\chi^w}{\varepsilon - 1} (\pi_t^w - 1) \pi_t^w \tag{10}$$

$$\frac{w_t}{w_{t-1}} = \frac{\pi_t^w}{\pi_t}, \tag{11}$$

where ε is the degree of substitution across labour types. The RHS of (9) shows that wages depend on the wage mark-up, the marginal rate of substitution and expectations. The expectation term implies that the labour supply is forward looking, and therefore will be less sensitive to contemporaneous shocks. Equation (11) is an identity to pin down the equilibrium.

3.3 Final Good Sector

The final good Y_t is produced under perfect competition and can be turned into consumption or investment, as well as used to cover the costs associated with financial, price and wage frictions. Production uses intermediate goods as inputs according to the following CES technology

$$Y_t = \left(\int_0^1 y_t(i)^{\frac{\theta-1}{\theta}} \mathrm{d}i \right)^{\frac{\theta}{\theta-1}} \tag{12}$$

with the elasticity of substitution $\theta > 1$. The associated demand function for intermediate goods is

$$y_t(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\theta} Y_t \tag{13}$$

with aggregate price index

$$P_t = \left(\int_0^1 p_t(i)^{1-\theta} \mathrm{d}i \right)^{\frac{1}{1-\theta}}. \tag{14}$$

3.4 Intermediate Good Sector

Each intermediate firm i, $i \in (0,1)$ operates under monopolistic competition. It employs capital services and labour by the mean of the following technology

$$y_t(i) = a_t K_t^{\eta} k_t(i)^{\alpha} h_t(i)^{1-\alpha}, \qquad \alpha \in (0,1) \tag{15}$$

where K_t represents a measure of knowledge, freely available to all firms and acquired through learning-by-doing. We assume that $K_t = k_t$, where $k_t = \int_0^1 k_t(i)\mathrm{d}i$. This implies that K is a pure externality that comes from the aggregate level of capital employed in the economy, and $0 \leq \eta \leq 1-\alpha$ represents the strength of the spillovers. a_t is an aggregate productivity shock, following

$$\log(a_t) = \rho_a \log(a_{t-1}) + \epsilon_{a,t},$$

$\rho_a \in (0,1)$ and $\epsilon_{a,t} \sim \mathrm{N}(0, \sigma^2)$. This is a moderate shock, typical of business cycle dynamics.

Solving the cost minimisation problem of the intermediate firm i, the FOCs for labour and capital services are

$$w_t = (1-\alpha) s_t \frac{y_t(i)}{h_t(i)}, \tag{16}$$

$$r_t^k = \alpha s_t \frac{y_t(i)}{k_t(i)}, \tag{17}$$

where r_t^k is the real rental rate of capital and s_t is the real marginal cost (Lagrangian multiplier) of producing $y_t(i)$, the same for all firms. Combining both FOCs

$$s_t = \alpha^{-\alpha}(1-\alpha)^{\alpha-1} \frac{w_t^{1-\alpha}(r_t^k)^{\alpha}}{a_t K_t^{\eta}}. \tag{18}$$

Intermediate good producers are monopolistically competitive and they face an adjustment cost when changing prices. Following Rotemberg (1982), the adjustment cost increases with the magnitude of the change in prices and the size of the economy. It is given by

$$\frac{\phi_p}{2}\left(\frac{P_t(i)}{P_{t-1}(i)} - 1\right)^2 Y_t, \tag{19}$$

where $\phi_p \geq 0$ is a measure of price rigidities. Using the demand function for intermediate goods

and assuming symmetry, the first order condition of the optimization problem yields the New Keynesian Phillips curve:

$$(1-\theta)+\theta s_t-\pi_t\phi_p(\pi_t-1)+\beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\pi_{t+1}\phi_p(\pi_{t+1}-1)\frac{Y_{t+1}}{Y_t}\right]=0. \tag{20}$$

3.5 Capital Producers

There is a unit mass of identical perfectly competitive capital producers. Each period t, the representative capital producer buys from entrepreneurs the current capital k_t and uses it to produce new capital k_{t+1} by combining it with investment i_t, and then sells k_{t+1} units to entrepreneurs at nominal price q_t. Since on average current capital depreciates at rate δ_t, $\delta_t \in (0,1)$, the average nominal price of second-hand capital is $q_t(1-\delta_t)$. Where note that $1-\delta_t$ relates to the relative price of second-hand to new capital in Lanteri (2018). The behaviour of the endogenous depreciation rate δ_t is modelled in the following subsection. The evolution law of raw capital reads

$$k_{t+1}=i_t\left(1-S\left(\frac{i_t}{i_{t-1}}\right)\right)+(1-\delta_t)k_t. \tag{21}$$

As in Christiano et al. (2014), investment is subject to the adjustment cost function $S\left(\frac{i_t}{i_{t-1}}\right)$. This assumption helps reducing the volatility of investment and tames inflation as the reaction of investment to shocks is smoother. As we will show at the end of Section 5, the Great Recession generates a temporary increase in δ_t, destroying productive capacity permanently.

Capital producers maximise their flow of profits, where $\hat{q}_t = q_t/P_t$ is the real price of capital, subject to the evolution law of capital above. The FOC reads:

$$1=\hat{q}_t\left(-S'\left(\frac{i_t}{i_{t-1}}\right)\left(\frac{i_t}{i_{t-1}}\right)+1-S\left(\frac{i_t}{i_{t-1}}\right)\right)+\beta E_t\frac{\lambda_{t+1}}{\lambda_t}\hat{q}_{t+1}S'\left(\frac{i_{t+1}}{i_t}\right)\left(\frac{i_{t+1}}{i_t}\right)^2, \tag{22}$$

3.6 Entrepreneurs and Financial Intermediation

This section closely follows Christiano et al. (2014), with a few differences designed to generate an endogenous depreciation rate depending on the fraction of firms going bankrupt. There is a unit mass of perfectly competitive entrepreneurs. At the end of any period t, each entrepreneur

has a net worth N, $N > 0$.[10] Even if the net worth of a particular entrepreneur is changing over time, we omit index t to simplify notation. At equilibrium, net worth is distributed $f_t(N)$ across entrepreneurs, with total net worth

$$\bar{N}_{t+1} = \int_0^\infty N f_t(N)\, \mathrm{d}N. \tag{23}$$

At the end of period t, entrepreneurs use their net worth N and loans B^N_{t+1}, $B^N_{t+1} \geq 0$, to acquire capital k^N_{t+1} from capital producers.[11] They pay price q_t for any capital unit they buy such that

$$q_t k^N_{t+1} = N + B^N_{t+1}. \tag{24}$$

At equilibrium, all capital is allocated to entrepreneurs, s.t.,

$$k_{t+1} = \int_0^\infty k^N_{t+1} f_t(N) dN. \tag{25}$$

As shown below, like in Christiano et al. (2014), the equilibrium debt-to-net-worth ratio does not depend on N, implying that loans and capital are proportional to it. At $t+1$, entrepreneurs use capital k^N_{t+1} to produce capital services ωk^N_{t+1} that they sell to intermediate firms at price $r^k_{t+1} P_{t+1}$, where ω is an entrepreneur specific productivity shock and P_{t+1} is the aggregate price index. Entrepreneurs draw the idiosyncratic productivity ω at period t after buying capital k^N_{t+1}. Idiosyncratic productivity ω is assumed to be i.i.d. across time and firms, drawn at t from the c.d.f. $\mathrm{F}_t(\omega)$, log-normally distributed, with unit mean and standard deviation σ_t.

An entrepreneur with net worth N obtains a loan B^N_{t+1} from mutual funds at the interest factor Z_{t+1}. The interest factor is contingent on the state of the economy in $t+1$ and, as shown below, it is independent of N at equilibrium. For this reason, index N is omitted. On top of aggregate risks, the debt contract has to take into account the presence of idiosyncratic risk, since entrepreneurs facing low realizations of the shock ω may be unable to repay the loan, and go bankrupt. Conversely, entrepreneurs with sufficiently high returns on their capital will repay

[10] As for financial frictions, the model mainly draws from Bernanke et al. (1999), so we assume that the entrepreneur employs its own net worth N as well as loans from financial intermediaries, *i.e.* mutual funds, to finance his venture.

[11] As usual, for an arbitrary asset X, X_{t+1} refers to the amount of this asset transferred from t to $t+1$.

their loans and make positive cash flow. For a given state contingent interest factor Z_{t+1}, let us define $\bar{\omega}_{t+1}$ as the state contingent productivity ω that zeroes the entrepreneur's cash flow at $t+1$, *i.e.*,

$$\Pi^N_{t+1}(\bar{\omega}_{t+1}) = \bar{\omega}_{t+1} R^k_{t+1}\, q_t k^N_{t+1} - B^N_{t+1} Z_{t+1} = 0, \tag{26}$$

where $\Pi^N_{t+1}(\omega)$ represents the entrepreneur's cash flow.[12] If $\omega > \bar{\omega}_{t+1}$, then $\Pi^N_{t+1}(\omega) > 0$, whilst if $\omega < \bar{\omega}_{t+1}$, the entrepreneur goes bankrupt. The fraction of firm that go bankrupt at period t is then $\mathrm{F}_{t-1}(\bar{\omega}_t)$.

At $t+1$, after production takes place, an ω-type successful entrepreneur sells its undepreciated capital $(1-\hat{\delta}_{t+1})\omega$ back to capital producers at price q_{t+1}. The depreciation rate of capital is $\hat{\delta}_{t+1} = \delta \left(\frac{\mathrm{F}_{t-1}(\bar{\omega}_t)}{\mathrm{F}(\bar{\omega})}\right)^{a_{\bar{\omega}}}$, $a_{\bar{\omega}} \geq 0$, where $\delta \in (0,1)$ is a parameter representing the depreciation rate at steady state. This specification includes negative network spillovers from entrepreneurs that went bankrupt in the previous period. It is a simplified way of representing the negative network effects on capital value associated with the disruptions in the production process generated by economic downturns and bankruptcy. A larger bankruptcy rate will make the capital of surviving firms less productive in the future due to the destroyed links. When the probability of bankruptcy is at its steady state value $\mathrm{F}(\bar{\omega})$, the impact of spillovers is normalised to 1. The ex-post return at $t+1$ to a unit of capital bought at t for an ω-type successful entrepreneur is ωR^k_{t+1}, with

$$R^k_{t+1} = \frac{r^k_{t+1} P_{t+1} + (1-\hat{\delta}_{t+1}) q_{t+1}}{q_t}. \tag{27}$$

In case of bankruptcy, the mutual fund pays a monitoring cost μ, $\mu \in (0,1)$, to appropriate the payments generated by the capital services provided to intermediate firms as well as the capital stock, which is then liquidated, subject to physical depreciation and obsolescence. Moreover, when an entrepreneur goes bankrupt, the steady state depreciation rate of capital is κ, $\kappa \in (\delta, 1)$ which is meant to capture both physical depreciation and obsolescence, obsolescence being measured by the difference $\kappa - \delta$. Negative network spillovers affect bankrupt entrepreneurs too, so that $\hat{\kappa}_{t+1} = \kappa \left(\frac{\mathrm{F}_{t-1}(\bar{\omega}_t)}{\mathrm{F}(\bar{\omega})}\right)^{a_{\bar{\omega}}}$ $a_{\bar{\omega}} \geq 0$. The ex-post return to capital for

[12] Like Z_{t+1}, $\bar{\omega}_{t+1}$ is independent of N at equilibrium. For this reason, index N is omitted.

bankrupt ω-type entrepreneur is ωR^f_{t+1}, with

$$R^f_{t+1} = \frac{r^k_{t+1} P_{t+1} + (1 - \hat{\kappa}_{t+1}) q_{t+1}}{q_t} \tag{28}$$

which in this case is appropriated by mutual funds.

At time t mutual funds issue bonds to households at the risk-less factor R_t to raise the resources needed to finance entrepreneurs. They also receive a transfer from the Monetary Authority. The transfer is financed by a tax on household's profits from entrepreneurial activity, and is thus proportional to the return to capital of successful entrepreneurs. The parameter ξ regulates the size of the transfer. This is a measure implemented to relax the cash constraint, aiming to promote credit provision in bad times, and will assure steady state values in line with U.S. data. For simplicity, let us assume that mutual funds specialise in entrepreneurs with net worth N and operate under perfect competition. Since the interest factor Z_{t+1} is state contingent, at each state of nature a zero profit condition holds, *i.e.*:

$$(1-\mu) q_t k^N_{t+1} \int_0^{\bar{\omega}_{t+1}} \omega R^f_{t+1} \mathrm{dF}_t(\omega) + \big(1 - \mathrm{F}_t(\bar{\omega}_{t+1})\big) B^N_{t+1} Z_{t+1} + \xi \int_0^{\bar{\omega}_{t+1}} \omega \mathrm{dF}_t(\omega) R^k_{t+1} q_t k^N_{t+1} = B^N_{t+1} R_t. \tag{29}$$

Divide both sides by $R^k_{t+1} q_t k^N_{t+1}$, use (26) and define leverage $L_t = \frac{N + B^N_{t+1}}{N}$ to get

$$(1-\mu)\frac{R^f_{t+1}}{R^k_{t+1}} \int_0^{\bar{\omega}_{t+1}} \omega \mathrm{dF}_t(\omega) + \bar{\omega}_{t+1}(1 - \mathrm{F}_t(\bar{\omega}_{t+1})) + \xi \int_0^{\bar{\omega}_{t+1}} \omega \mathrm{dF}_t(\omega) = \frac{R_t}{R^k_{t+1}} \frac{L_t - 1}{L_t}.$$

The zero profit condition above can then be used to find an expression for leverage

$$L_t = \left(1 - \frac{R^k_{t+1}}{R_t}\Big(\bar{\omega}_{t+1}(1 - \mathrm{F}_t(\bar{\omega}_{t+1})) + (1-\mu) H_t(\bar{\omega}_{t+1}) + \xi G_t(\bar{\omega}_{t+1})\Big)\right)^{-1}, \tag{30}$$

where

$$H_t(\bar{\omega}_{t+1}) = \frac{R^f_{t+1}}{R^k_{t+1}} \int_0^{\bar{\omega}_{t+1}} \omega \mathrm{dF}_t(\omega)$$

$$G_t(\bar{\omega}_{t+1}) = \int_0^{\bar{\omega}_{t+1}} \omega \mathrm{dF}_t(\omega).$$

$G(\bar{\omega})$, $G(\bar{\omega}) < 1$, represents unsuccessful entrepreneurs' contribution to the average ω, and $H(\bar{\omega})$, $H(\bar{\omega}) < G(\bar{\omega})$, is corrected by the ratio of unsuccessful to successful returns.

Notice that the loan contract (Z_{t+1}, B^N_{t+1}) can be also written as a contract on $(\bar{\omega}_{t+1}, L_t)$. Any pair $(\bar{\omega}_{t+1}, L_t)$ that satisfies (30) is a $(t+1)$-state contingent contract offered to entrepreneurs. As it will

become clear below, at equilibrium, the conditions of the loan contract $(\bar{\omega}_{t+1}, L_t)$ are the same for all entrepreneurs irrespective of their net worth N. On one side, for a given net worth N, choosing loan B^N is equivalent to choosing leverage L. On the other side, setting the nominal interest factor Z determines the cut-off productivity $\bar{\omega}$.

At time $t+1$, for any realization of the aggregate shocks, the debt contract $(\bar{\omega}_{t+1}, L_t)$ for an entrepreneur with net worth N is expected to generate the cash flow

$$\int_{\bar{\omega}_{t+1}}^{\infty} \Pi_{t+1}^{N}(\omega)\, \mathrm{dF}(\omega) = \Big(1 - \Gamma_t(\bar{\omega}_{t+1})\Big)\, R_{t+1}^{k} L_t N, \tag{31}$$

where $\Big(1 - \Gamma_t(\bar{\omega}_{t+1}\Big)$ is the expected share of total revenues retained for successful entrepreneurs, with

$$\Gamma_t(\bar{\omega}_{t+1}) = G_t(\bar{\omega}_{t+1}) + \bar{\omega}_{t+1}\Big(1 - \mathrm{F}_t(\bar{\omega}_{t+1})\Big) \tag{32}$$

being the expected share going to the mutual fund.

For any state of nature in $t+1$, the entrepreneur chooses the contract that maximises expected profit, which is equivalent to[13]

$$\max_{\bar{\omega}_{t+1}} \frac{1 - G_t(\bar{\omega}_{t+1}) - \bar{\omega}_{t+1}\big(1 - \mathrm{F}_t(\bar{\omega}_{t+1})\big)}{1 - \frac{R_{t+1}^{k}}{R_t}\Big(\bar{\omega}_{t+1}(1 - \mathrm{F}_t(\bar{\omega}_{t+1})) + (1-\mu)H_t(\bar{\omega}_{t+1}) + \xi G_t(\bar{\omega}_{t+1})\Big)} = \max_{\bar{\omega}_{t+1}} L_t(\bar{\omega}_{t+1})\big(1 - \Gamma_t(\bar{\omega}_{t+1})\big). \tag{33}$$

Thus the FOC pinning down the equilibrium $\bar{\omega}_{t+1}$ reads:

$$\frac{1 - \mathrm{F}_t(\bar{\omega}_{t+1})}{1 - \Gamma_t(\bar{\omega}_{t+1})} = \frac{\frac{R_{t+1}^{k}}{R_t}[1 - \mathrm{F}_t(\bar{\omega}_{t+1}) - \mathrm{G}_t'(\bar{\omega}_{t+1})\big(1 - \frac{R_{t+1}^{F}}{R_{t+1}^{k}}(1-\mu) - \xi\big)]}{1 - \frac{R_{t+1}^{k}}{R_t}\Big(\bar{\omega}_{t+1}(1 - \mathrm{F}_t(\bar{\omega}_{t+1})) + (1-\mu)H_t(\bar{\omega}_{t+1}) + \xi G_t(\bar{\omega}_{t+1})\Big)}. \tag{34}$$

This shows that the loss of accepting a higher threshold for entrepreneurs equals the benefit of higher leverage. The LHS is the elasticity of the share the entrepreneur keeps w.r.t. $\bar{\omega}_{t+1}$, whilst the RHS is the elasticity of leverage w.r.t. $\bar{\omega}_{t+1}$.[14] Also, notice that this specification implies that the elasticity of leverage is affected by credit subsidies.

It is easy to see that the equilibrium $\bar{\omega}_{t+1}$ does not depend on N. From (30), leverage does not depend on it either. Consequently, from (26), mutual funds set the same $(t+1)$-state contingent interest factor Z_{t+1} irrespective of net worth.

[13] Use (30) to substitute for L_t in (31), then divide by $R_{t+1}^{k} N$ to get (33).

[14] The FOC also shows that our framework reduces to the standard model in Christiano et al. (2014) in the case $R^k = R^F$ and $\xi = 0$.

Let us assume the standard deviation of $\mathrm{F}_t(\omega)$ follows

$$\log\left(\frac{\sigma_t}{\bar{\sigma}}\right) = \rho_\sigma \log\left(\frac{\sigma_{t-1}}{\bar{\sigma}}\right) + \epsilon_{\sigma,t},$$

with $\bar{\sigma} > 0$, $\rho_\sigma \in (0,1)$ and the risk shock $\epsilon_{\sigma,t}$ being i.i.d. Notice that a higher value of σ_t implies a higher probability of drawing a low value of ω. As the variance of the shock increases, the tails of the distribution get thicker, increasing the probability of tail events and modifying the threshold for $\bar{\omega}$, *i.e.*. increasing the probability of bankruptcy.

Let us finally assume that at the end of period $t+1$ (after entrepreneurs pay back to mutual funds their period t debt) a fraction $(1-\gamma)$ of successful entrepreneur's cash flow $\Pi^N_{t+1}(\omega)$ gets transferred to households, $\gamma \in (0,1)$. Moreover, each entrepreneur receives a transfer $w^e\, P_{t+1} k^N_{t+1}$ from households, $w^e \in (0,1)$, as a form of insurance, to compensate for risk taking, and assuring that bankrupt entrepreneurs will keep a strictly positive net worth allowing them to buy some capital for the following period.

Since liquidating the capital of failed entrepreneurs generates physical capital depletion, the depreciation of capital is endogenous and depends on the fraction of entrepreneurs going bankrupt. The aggregate depreciation rate at time t reads

$$1-\delta_t = (1-\hat{\kappa}_t)\,\mathrm{F}_{t-1}(\bar{\omega}_t) + (1-\hat{\delta}_t)\big(1-\mathrm{F}_{t-1}(\bar{\omega}_t)\big).$$

The intuition is that when bankruptcy happens some capital (tangible or intangible) is destroyed in the process, reducing the overall value of capital. The externality aims to capture negative disruptive spillovers generated by bankruptcy. These could be the result of some specialised machines not being reallocated as effectively as in normal times or knowledge embedded in intangible capital being lost. Moreover, we think of depreciation spillovers as (small) bankruptcy shocks with aggregate effects through the production network, resulting in an increase in the depreciation rate of capital for the whole economy. This interpretation draws inspiration from Acemoglu, Akcigit, and Kerr (2016), who document significant network-based propagation channels stemming from small shocks.

3.7 Aggregate Economy

The quantity of capital produced by capital producers must be equal to the capital purchased by entrepreneurs:

$$k_{t+1} = \int_0^\infty k^N_{t+1} f_t(N) dN.$$

From (24), (30) and the definition of leverage, $L_t = \frac{N+B^N_{t+1}}{N}$, the equation above becomes

$$q_t k_{t+1} = \frac{1}{1 - \frac{R^k_{t+1}}{R_t}\Big(\bar{\omega}_{t+1}(1 - \mathrm{F}_t(\bar{\omega}_{t+1})) + (1-\mu)\mathrm{H}_t(\bar{\omega}_{t+1}) + \xi G_t(\bar{\omega}_{t+1})\Big)} \bar{N}_{t+1}. \tag{35}$$

Consequently, the level of capital in the economy depends on aggregate net worth, as defined in (23), and financial conditions.

All intermediate firms face the same wage and capital cost, therefore, by symmetry:

$$\frac{k_t(i)}{h_t(i)} = \frac{k_t}{h_t}, \tag{36}$$

for all $i \in (0,1)$. Also, market clearing for capital and labour implies $\int_0^1 k_t(i)\mathrm{d}i = k_t$ and $\int_0^1 h_t(i)\mathrm{d}i = h_t$. Notice that if firms could change their prices in every period, they will choose the same price and produce the same quantity. In which case, $p_t(i) = P_t$ and $y_t(i) = Y_t$, hence aggregate production would become

$$Y_t = a_t k_t^{\alpha+\eta} h_t^{1-\alpha}. \tag{37}$$

Therefore, if $\eta = 1 - \alpha$ aggregate technology has an AK structure.

Aggregate profits of all entrepreneurs at the end of time t are $[1 - \Gamma_{t-1}(\bar{\omega}_t)]R^k_t q_{t-1} k_t$, so that aggregate net worth at $t+1$ is:

$$\bar{N}_{t+1} = \gamma[1 - \Gamma_{t-1}(\bar{\omega}_t)]R^k_t q_{t-1} k_t + P_t w^e k_t \tag{38}$$

Using the aggregate production function and $K_t = k_t$:

$$r^k_t = \alpha s_t a_t k_t^{\alpha+\eta-1} h_t^{1-\alpha} \tag{39}$$

$$w_t = (1-\alpha) s_t a_t k_t^{\alpha+\eta} h_t^{-\alpha} \tag{40}$$

$$s_t = \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \frac{1}{a_t k_t^\eta} w_t^{1-\alpha} (r^k_t)^\alpha \tag{41}$$

All bonds held by households must be equal to the amount of loans in aggregate, and transfers to mutual funds must equal taxes:

$$q_t k_{t+1} - \bar{N}_{t+1} = B_{t+1}$$

$$\xi \int_0^{\bar{\omega}_{t+1}} \omega \mathrm{dF}_t(\omega) R^k_{t+1} q_t k^N_{t+1} = \tau_t$$

Output is allocated to consumption and investment, but also to intermediary production aimed to

cover price and wage adjustment costs as well as monitoring costs, *i.e.*:

$$Y_t = \underbrace{c_t + i_t}_{\text{GDP}} + \mu \int_0^{\bar{\omega}_t} \omega d\mathrm{F}(\omega) R_t^F \frac{\hat{q}_{t-1}}{\pi_t} k_t + \frac{\phi_p}{2} (\pi_t - 1)^2 Y_t + \frac{\chi^w}{2} (\pi_t^w - 1)^2 Y_t. \tag{42}$$

GDP is then defined as the sum of consumption and investment, measuring aggregate demand.

3.8 Monetary Authority

The monetary authority uses the Taylor rule to set the nominal interest rate, subject to the zero lower bound constraint:

$$R_t^m = \bar{R} + \rho_\pi (\pi_t - \bar{\pi}^m) + \rho_y \log \left(\frac{\widehat{\text{GDP}}_t}{y_t^p} \right) \tag{43}$$

$$R_t = \max(1, R_t^m), \tag{44}$$

where R_t is the nominal interest factor, $\bar{R}$ and $\bar{\pi}$ are target values, $\rho_\pi > 0$ and $\rho_y > 0$ are policy parameters, and y_t^p

$$y_t^p = y_{t-1}^p + \rho \left(\frac{1}{n} \sum_{j=1}^{n} \widehat{\text{GDP}}_{t-4-j} - y_{t-1}^p \right) \quad 0 < \rho < 1 \tag{45}$$

is a measure of potential output. It is computed as a moving average of past GDP values, de-trended by the stationary growth rate of the economy g_z, so that $\widehat{\text{GDP}}_t = \frac{\text{GDP}_t}{(1+g_z)^t}$.

We define this measure to take into account the revisions carried out by central banks in the context of the Great Recession. We implicitly assume that the monetary authority does not have full information about the functioning of the economy, and it infers the underlying path from observed values of GDP. In particular, our measure takes into account the lag in potential output revisions, as we disregard the most recent 4 periods. This implies that the estimate of potential output is not very sensitive to negative shocks if they are small in size and duration. We set n equal to 10, to allow the central bank to consider a long enough period of time in the estimation of the underlying trend, but also react relatively quickly to shocks.

As we will show in the following sections, the
rule provides stimulus to the economy, inducing it to close the output gap after a negative shock. Despite the fact that the aggregate technology is AK, and because of the monetary policy intervention, GDP appears to fluctuate around a stable linear trend in normal times, appearing consistent with diminishing returns to capital. However, if the shock is large and prolonged, potential output will be revised downwards and the stimulus to the economy will lose strength over time. As a result, the economy will reveal its AK structure and the recovery will fail to materialise.

4 Baseline Calibration

We calibrate the model on quarterly data for the United States, considering the period 1980-2008 as our reference.[15] In order for the model to have an aggregate AK technology, we set $\eta = 1 - \alpha$. Returns to capital are then constant, generating endogenous growth. We consistently de-trend all non-stationary variables using the the stationary growth rate g_z, so that $\tilde{c}_t = \frac{c_t}{(1+g_z)^t}$, $\tilde{y}_t = \frac{Y_t}{(1+g_z)^t}$, and so on. Then we solve the stationarised system of non-linear equations, calibrating the model to match some key data moments.[16]

To characterise the stationary equilibrium, we target a labour share of 60.8%, in line with the average value of the share of labour compensation in GDP. This is achieved by setting $\alpha = 0.24$ and assuming a price mark-up of 25%. The latter implies a value of 5 for the elasticity of substitution across intermediate good, θ. We also calibrate the model to have zero inflation in steady state and a quarterly growth rate of 0.6%. The latter depends on the return to capital and the financial frictions. We set the scaling parameter $\psi = 16.8$ to labour hours in steady state be 0.2, in line with the average of annual hours worked by persons engaged. We set γ to 0.966 (close to Bernanke et al. (1999)). Concerning the depreciation rate, we set δ to 0.028 and κ to 0.04. These are somewhat higher than standard values, but we make this choice because our definition of capital is wider compared to National Accounts, including all categories of intangible capital. Although these stocks are not fully included in GDP measures, they do affect the outcome of production, and they depreciate faster than traditional measures of capital, so we account for their depreciation in our calibration. Monitoring costs are set to $\mu = 0.21$, in line with Christiano et al. (2014) and within the rage suggested by Carlstrom and Fuerst (1997). We set $\xi = (1-\mu)\left(1 - \frac{R^F_{ss}}{R^k_{ss}}\right)$ to obtain a steady state bankruptcy rate of approximately 1%, similarly to Christiano et al. (2014).

For Rotemberg adjustment costs, we follow the approaches proposed by Ascari and Rossi (2012) and Born and Pfeifer (2020), calculating them as follows:

[15] We choose data covering this time span to characterise the steady state of the model since we do not take a stand on whether an endogenous growth model has always been a good representation of GDP, as it is possible that that the U.S. economy evolved from a Neoclassical structure to an AK structure over time due to technological change. For example, the increasing importance of intangible capital, human capital and knowledge spillovers for production, as documented by Haskel and Westlake (2018), could have led to such a transformation in the production technology.

[16] Table 5 in the Appendix shows the comparison between the steady state of the model and U.S. data. For a description of the data, see Appendix A.

Parameter	Value	Description
Preferences		
β	0.99	Discount factor
χ	0.7	Habit formation parameter
ν	1	Inverse of Frisch elasticity
ψ	16.8	Scaling parameter for labour hours
Frictions		
θ	5	Substitution in final good
ε	21	Substitution in labour input
ϕ_p	46.6	Rotemberg price adjustment parameter
χ^w	141.7	Rotemberg wage adjustment parameter
S"	5	Investment adjustment costs
μ	0.21	Monitoring costs
γ	0.966	Fraction of profits going to entrepreneur
w^e	0.0005	Transfer from household
ξ	0.009	Transfer to financial intermediaries
Technology		
α	0.24	Private returns to capital
η	0.76	Knowledge spillovers in production
δ	0.028	Successful entrepreneurs' depreciation rate
κ	0.04	Bankrupt entrepreneurs' depreciation rate
$a_{\bar{\omega}}$	0.15	Strength of depreciation spillovers
Monetary Policy		
ρ_π	1.5	Policy response to inflation
ρ_y	0.125	Policy response to output gap
ρ	0.7	Speed of potential output revision
n	10	Number of past periods in moving average

Table 1: Quarterly calibration parameters

$$\phi_p = \frac{(\theta - 1)\theta^p}{(1 - \theta^p)(1 - \beta\theta^p)} \tag{46}$$

$$\chi^w = \frac{(\epsilon - 1)^{\frac{\theta-1}{\theta}}(1 - \alpha)\theta^w}{(1 - \theta^w)(1 - \beta\theta^w)} \tag{47}$$

where $\theta^w = \theta^p = 0.75$ represent the probabilities of not being able to reset prices and wages.

Finally, we set $a_{\bar{\omega}}$ to 0.15, which implies that a 1% deviation of the probability of bankruptcy from its normal value leads to an increase in the depreciation rate of approximately 0.1%. This is considerably smaller compared to the values in Lanteri (2018)'s data and capital quality shocks in Kozlowski et al. (2020), but our choice is conservative as the model is not expected to represent all dimensions of the data. Our view is that there is sufficient evidence to support the capital destruction channel as quantitatively relevant, but a precise estimate to base the calibration on is not available. We therefore choose the parameter $a_{\bar{\omega}}$ to bring our model simulations close to the fall in GDP observed in the data. The persistence of the confidence and risk shocks during the Great Recession, $\rho_c = 0.9$ and $\rho_\sigma = 0.97$ respectively, are taken from Christiano et al. (2014).

5 The Great Recession

Great Recession Shock. We model the Great Recession as the reaction of the economy to a large and persistent demand shock that combines a risk shock and a confidence shock, both hitting at the same time.[17] The magnitude of the risk shock is set to 15.7% increasing the bankruptcy probability from a baseline value of 1% to 4%, in line with the smoothed bankruptcy rate estimated by Christiano et al. (2014). We set the size of the confidence shock to 12.56%, in line with the decline of the HP-filtered University of Michigan's consumer sentiment for the U.S. at the start of the Great Recession.

The confidence shock brings the economy to the ZLB region. However, since our model does not capture the unconventional monetary policies implemented by the Fed, nor forward guidance, the model economy remains in the ZLB region only for a few quarters after the occurrence of the Great Recession shock. We use U.S. data to build an alternative measure of the nominal

[17] In order to appropriately account for the sizeable deviation of endogenous variables from the original steady state and the ZLB constraint, we solve the model non-linearly, employing a Newton-type method (Adjemian et al., 2011) (Juillard et al., 1996).

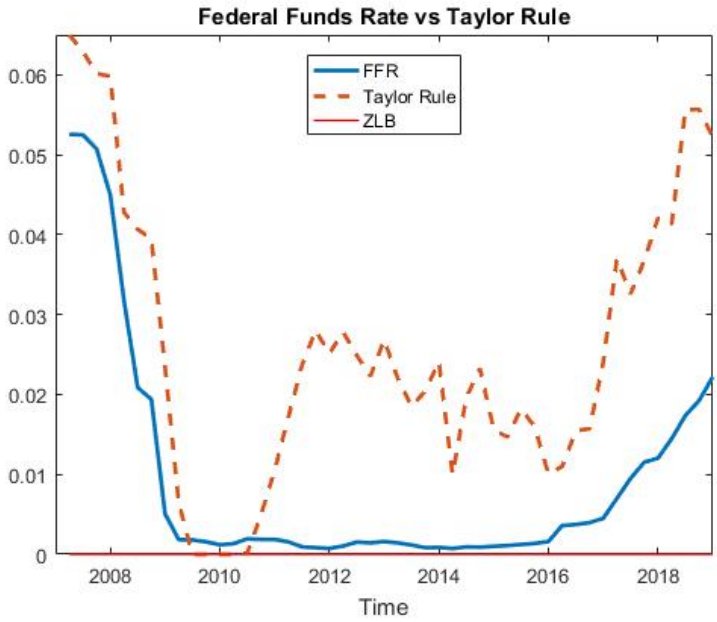

Figure 4: Taylor rule vs FFR

interest rate consistent with the standard Taylor rule, i.e.,[18]

$$\mathrm{TR}_t = \max\left\{0, \overline{TR} + 1.5\left(\pi_t - \bar{\pi}\right) + 0.5\,\log\left(\frac{\mathrm{GDP}_t}{y_t^p}\right)\right\},$$

setting the target nominal interest rate $\overline{TR}$ to 3.64%, the 2004-2007 average of the Federal Funds Rate (FFR), and the inflation target $\bar{\pi}$ to its official value of 2%. Quarterly GDP is measured in real terms and potential output y_t^p is real-time CBO data (in 2012 dollars).[19] Inflation is measured as the GDP deflator.[20] In line with our simulations, as displayed in Figure 4, the nominal interest rate TR_t predicted by the Taylor rule shows a much shorter ZLB episode compared to the Federal Funds Rate.

Summing up: The Great Recession shock combines a 15.7% risk shock and a 12.56% confidence shock. As a result, the bankruptcy rate increases in line with data, and the simulated nominal interest follows the hypothetical FFR predicted by the Taylor rule when applied to observed inflation and output gaps.

[18] A similar exercise was also conducted by the Board of Governors of the Federal Reserve System, among others, with comparable results. See link here.

[19] Real-time CBO data are constructed employing the estimates published by the CBO at the beginning of each year. It thus captures the unfolding of all revisions.

[20] Additional Details on data series used can be found in the Appendix.

Data Comparison. In order to compare the simulations with key data dynamics in the U.S., we follow the approach of Christiano, Eichenbaum, and Trabandt (2015) to estimate targets gap ranges from the data. We measure the deviation of variables from the path they would have followed, had the recession not happened, by calculating the percentage difference from a linear trend, fitted on past data. In order to select time intervals, we aim to follow the CBO's methodology for projections of potential output as closely as possible. *"Typically, a trend is considered to extend from at least one previous business cycle through the most recent quarter of data (because the peak of the current cycle is not known at the time of a forecast)."*[21] They consider full peak-to-peak business cycles, so we build our estimates by including data from the business cycles peaks preceding the Great Recession, up to the period before the downturn. We construct min-max targets by considering the intervals $[x : 2008Q2]$, with $x = (1990Q3, 2001Q1)$. We consider aggregate series retrieved from the FRED database for GDP, consumption, investment, credit and the Federal Funds Rate. Finally, in order to compare the *switching-track* of potential output in our model and in the data, we employ the measure of potential output we built to calculate the Taylor rule in Figure 4. More details on data sources can be found in the Appendix.

The Great Recession. Figure 5 shows impulse responses for some key variables in our model to the Great Recession shock. The fall in confidence slows down consumption demand, and the sharp rise in risk depresses investment demand and raises bankruptcy. As bankruptcy rates increase, capital depreciates faster resulting in capital destruction. Even when the shock subsides and financial variables converge back to the previous steady state values, due to capital destruction the negative effects on GDP, consumption and investment do not dissipate and the economy displays an L-shaped recovery. Our model fits the data quite well, especially for GDP and consumption, which both fall by almost 10% at the new steady state, converging monotonically in the same way as the data. As for investment, the simulation does not capture the full depth of the total shortfall, which was partially driven by the the real estate market. Nonetheless, results are more in line with private non-residential investment data. The model fits the data for credit well at the beginning of the sample, set aside the collapse of the credit

[21] From the report: 'Revisions of CBO's potential output since 2007', (Shackleton, 2014).

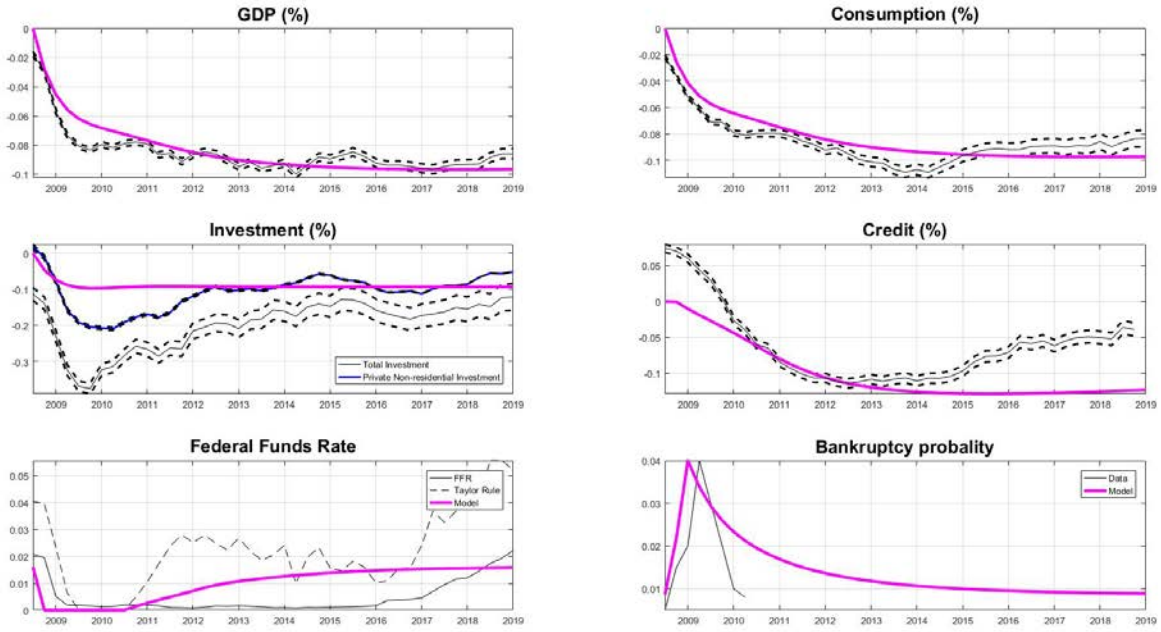

Figure 5: Model impulse responses to Great Recession shock vs U.S. data

boom that we do not model, and performs worse for later periods. This is likely because we are not modelling unconventional monetary policies providing credit easing. The behaviour of the nominal interest rate is in line with the Taylor rule estimated in Figure 4, and thus does not replicate the Federal Funds Rate after 2010, which remains for a long while at the ZLB, likely due to unconventional monetary policies.

Switching-Track. Figure 6 clearly shows that the model economy replicates quite well the *switching-track* of potential output in the data.[22] The simulated output gap shrinks with time, thus reducing its impact on the Taylor rule. Our simulations are in line with the U.S. experience, and our specification approximates well the timing of output gap dynamics observed in the data.

[22]The figure shows a slight increase in potential output in 2013, which is the result of the comprehensive revision of the national income and product accounts (NIPAs) to include new investment categories. This led to an increase in GDP value. We adjusted the potential output measures published, following the methodology indicated by the CBO in their 2014 report, but a small margin of error remains.

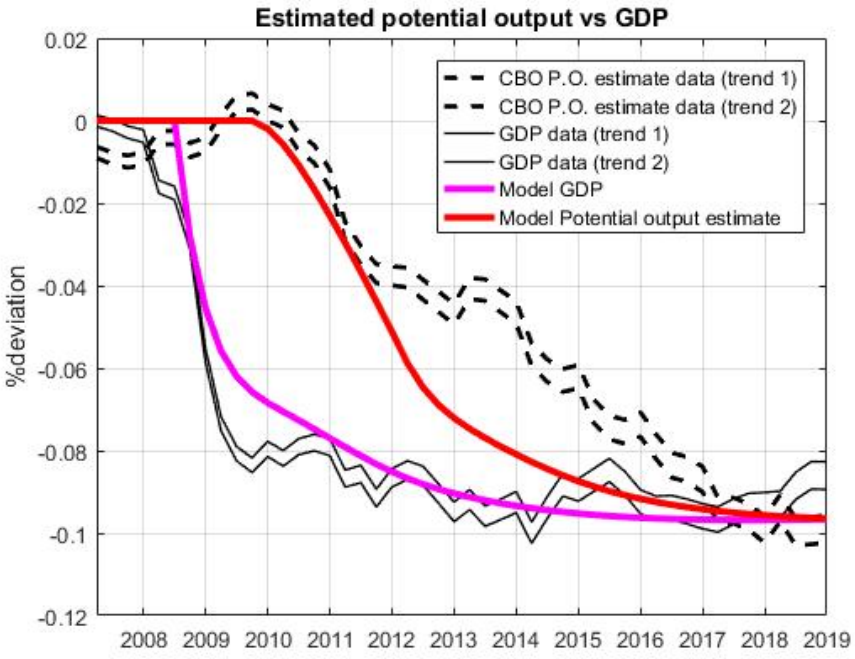

Figure 6: GDP and potential output

Model's Mechanics. In order to better understand the mechanics of the two shocks, Figure 19 in the Appendix includes the impulse response of other variables to the Great Recession shock. As it is common in financial crises, credit declines and net worth contracts, mirroring a stock market crash. Monetary policy is in place to mitigate the shock, but the ZLB constraint limits its effectiveness and, most importantly, the severity and the length of the recession put downward pressure on potential output estimates, giving rise to a *switching-track*. Without a strong policy intervention, the economy switches track moving to a lower GDP trejectory, revealing its AK nature. In Figure 20 in the Appendix, we compare the results of our baseline calibration with the case in which the Fed's measure of potential output is much less sensitive to past observed data. In this case, the policy remains much stronger, and the negative level effects on the trajectory of GDP are considerably reduced, showing the policy's potential to partially prevent the destruction of productive capacity and in shaping the the recovery. A slower revision of potential output (inducing a weaker *switching-track*) results in lower depreciation, directly preventing the destruction of productive capacity and sustaining consumption and investment

demand. As a result, inflation gets stronger and the real interest rate rises faster, promoting savings and a stronger recovery compared to the baseline simulation. The policy then induces households to consume and save more, and entrepreneurs to invest more, with the economy settling on a higher level.

Zero-Lower-Bound. Figure 21 in the Appendix highlights the role of the ZLB constraint in our results, showing that the level effect on GDP would have been less severe had the constraint not been binding

Risk Shock. The risk shock is the main driver of our results. Figure 22 represents the impulse response functions to the risk shock, once the confidence shock is removed. The economy does not reach the ZLB in this scenario, as the severe supply effect of the shock drives inflation up.

Capital Destruction Channels. In our model, bankruptcy destroys capital through two different channels: A *liquidation channel* operates directly through the partial destruction of the capital stock of entrepreneurs going bankrupt, and a *disruption spillovers channel* through the negative externality that bankruptcy has on the the productive capacity of capital for all entrepreneurs. The first captures the direct destruction of capital that follows bankruptcy, as we assume that bankrupt firms face a liquidation cost in terms of capital equal to $\kappa - \delta > 0$. These firms are hit by the risk shock the hardest, but their aggregate effect is not strong enough to generate a downturn as large as the Great Recession. Figure 7 shows that the same demand shock generates a permanent level effect on GDP even with no disruption spillovers, but the magnitude of the impact is considerably smaller. The second channel is the result of disruption spillovers, capturing the idea that in downturns most firms are worse off, not just the ones that exit.[23] This modelling choice is indirectly supported by Lanteri (2018) data, which shows that the price of used capital goods relative to new ones fell in the whole secondary market, and not just for bankrupt firms. This second channel is key to replicate the magnitude of the Great Recession in the model, as it is clear from Figure 7.

[23] In a similar spirit, Guerrieri et al. (2020) provide a theoretical argument in favour of a business exit multiplier in the context of the COVID-19 crisis, showing that the exit of some firms may lead to negative amplification effects in the economy.

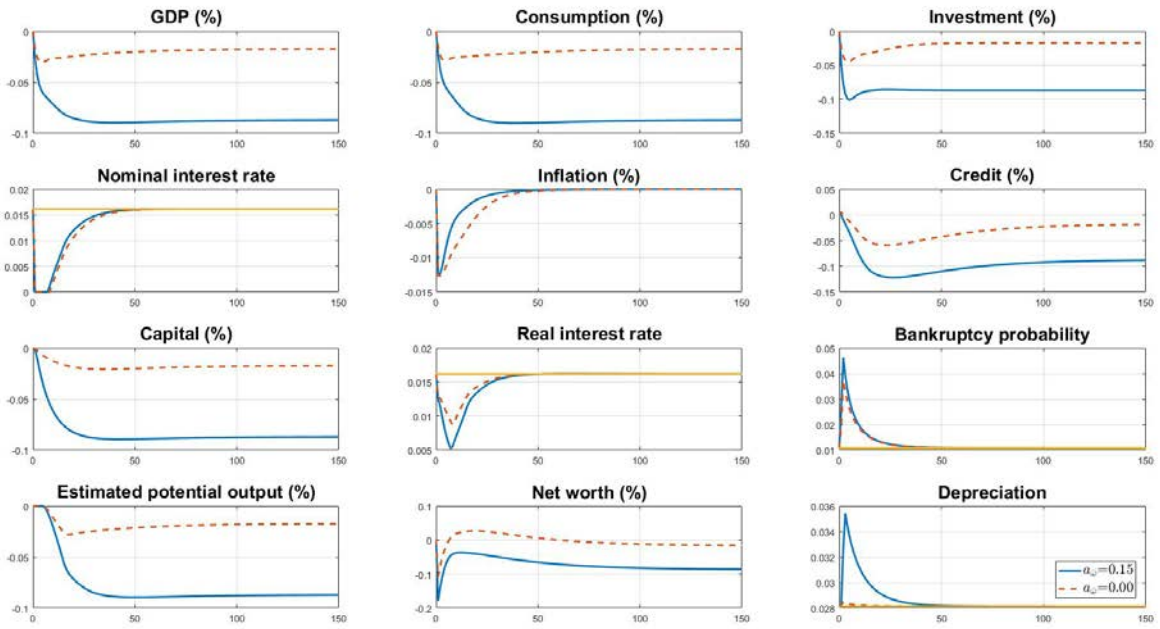

Figure 7: Baseline simulation vs case without disruption spillovers

Revision intensity	Faster than baseline	Baseline	Slower than baseline
ρ	0.9	0.7	0.04
Welfare losses	-7.3%	-6.8%	-6.4%

Table 2: Welfare losses for different intensity of potential output revision

Welfare Losses. In order to evaluate the impact of the policy, we calculate consumption equivalent welfare losses, by comparing the path of utility following the Great Recession shock to the path of utility at the initial steady state (where the economy would have stayed had the Great Recession not happened). We conduct the analysis for 500 periods. Moreover, we calculate welfare for different intensities of potential output revision (*switching-track*). The Baseline column in Table 2 gives the welfare losses produced by the Great Recession demand shock when output gap is revised following the baseline revision rule. The other two columns do the same for alternative revision rules. Comparing columns, we conclude that faster revision rules generate larger losses, which implies that a weaker policy intervention in the model wouldn't have been optimal.

In an AK world, when capital is destroyed output moves to a lower path. The optimal reaction to such a shock is to remain in the new lower balanced growth path. Any policy forcing households to save more than optimal in order to make capital return to its previous trajectory is suboptimal. Why is it then the case that a Taylor rule forcing output to go back to its previous track generates welfare gains? The fundamental reason lies in the *destruction prevention* channel of monetary policy. In an economy with a negative externality stemming from *disruption spillovers*, the cushioning effect of the policy intervention tames bankruptcy down and lowers depreciation, counteracting the shocks and allowing higher levels of consumption in the new steady state, without forcing excessive savings along the transition.

What if We Relax the AK Assumption? In the previous sections we assumed $\eta = 1 - \alpha$, which implies an aggregate AK technology. Is such a strong assumption necessary to generate our results? To test this, we simulate the model for lower values of η.[24] We pick $\eta = 0.06$ to get a quasi standard Cobb-Douglas technology. Figure 23 in the Appendix shows that when we subject this version of the model to comparable shocks to simulate the Great Recession, and we keep potential output constant to reproduce a more standard DSGE, the model generates a quick and full recovery.[25] This shows that the AK technology, and not the nature of the shocks,

[24] Note that in this case the model does not grow endogenously any more, implying that the growth rate of the economy converges to zero.

[25] The model would also generate a similar recovery if we reduced the weight of the output gap in the Taylor rule, as the recovery is a consequence of diminishing returns to capital.

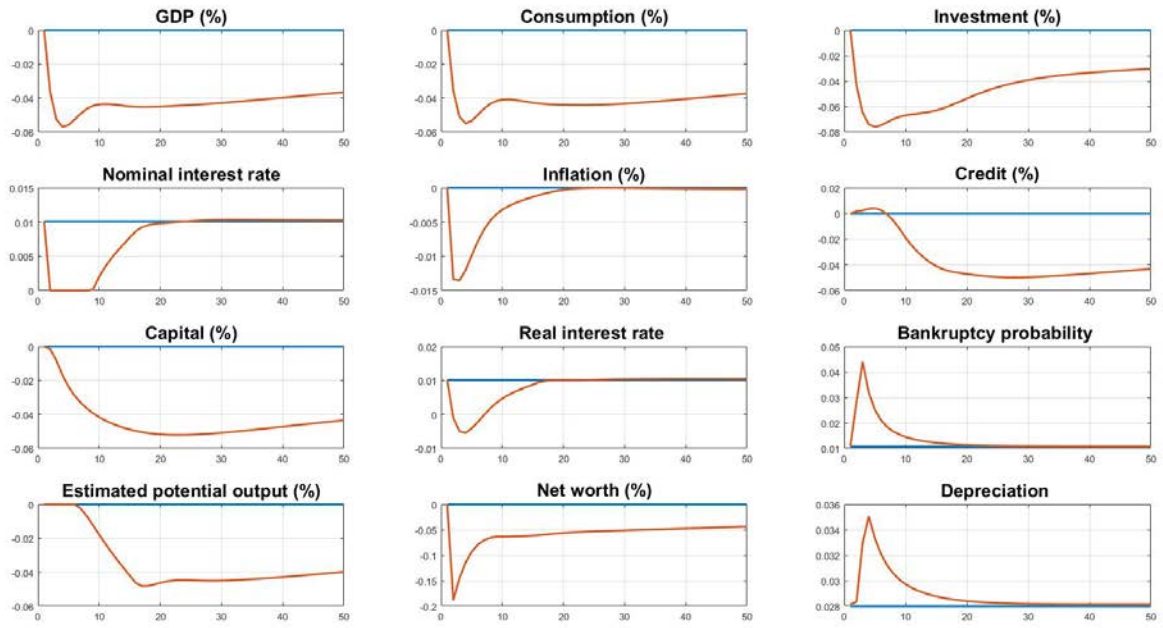

Figure 8: Great Recession shock for $\eta = 0.6$

is the source of the parallel downward shift of GDP in this model.

We then try an intermediate case, setting $\eta = 0.6$ to get closer to an AK model, and allowing potential output revisions. Figure 8 shows that the shocks result in a persistent recession of reduced depth. Although this version of the model could be a good representation of GDP dynamics in the presence of additional shocks, we prefer our baseline specification, as it captures key aspects of the Great Recession in a parsimonious way.

6 Normal Times and V-Shaped Recoveries

In this section, we begin by studying the mechanics of our model, and show that it can generate policy-driven V-shaped recoveries when shocks are not overly severe or particularly persistent. Moreover, we show that these mechanics can help our model replicate the dynamics of pre-Great Recession recoveries.[26] Throughout this section, we hold the value of potential output constant,

[26]Since in normal times the ZLB is never binding, we solve the model with perturbation methods.

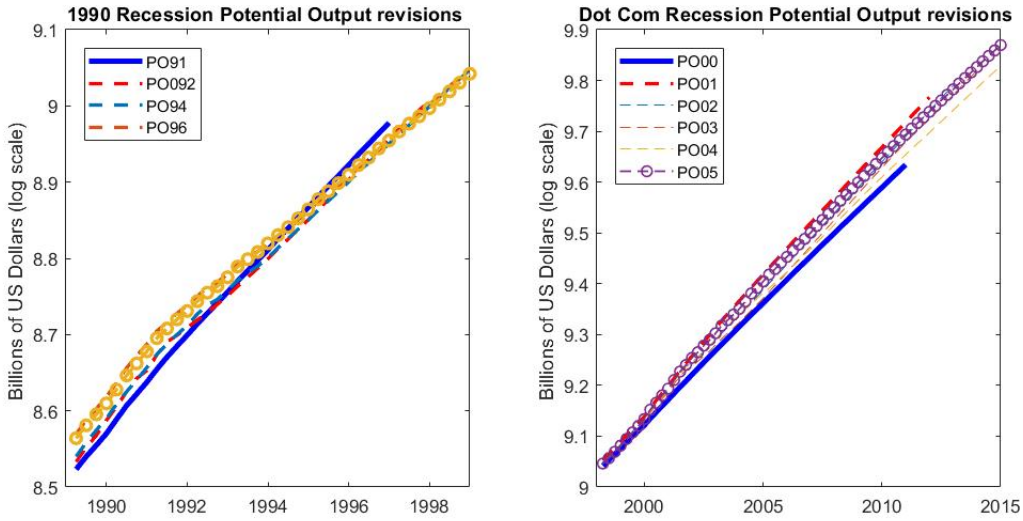

Figure 9: Potential output revisions in past recessions (CBO data)

so that the output gap measures the deviation of GDP from the initial steady state.[27] We do not update the potential output measure because the revision procedure adopted after the Great Recession started was the first of its kind in the U.S. monetary policy history, as documented by Coibion, Gorodnichenko, and Ulate (2017). Figure 9 illustrates this point for the two most recent recessions before 2008 using CBO historical data. These are the the 1990 recession and the 2000 dot.com recession. In both cases, the CBO revisions of potential output projections are minor. Older vintages are unfortunately not available, so we rely on the statement in Shackleton (2014) concerning the CBO's methodology: *"Recessions typically have little effect on historical estimates of potential output because the methodology aims to exclude cyclical effects"*. The Great Recession was the only exception to this rule.

[27] An alternative practice in the literature is to model potential output as the level of output that would prevail without nominal rigidities. We do not adopt this measure because the level of output at the balanced growth path in an AK economy is fundamentally indeterminate, and as such depending on the policy intervention itself. Moreover, central banks commonly use measures of potential output as deviations of output from trend. For example, see Edge et al. (2008) for a discussion of the Fed's FRB/US model or Vetlov et al. (2011), for the ECB's NAWM.

6.1 TFP Shocks

Let us first show that small negative TFP shocks are followed by fast V-shaped recoveries by implementing a textbook minus 1% TFP shock with persistence $\rho_a = 0.79$. The persistence of the shock is in line with the literature, and was selected to illustrate a case where the economy quickly recovers to the previous steady state.[28] In Figure 10, we plot simulation results for our baseline calibration as well as for a pure inflation targeting, *i.e.* following a Taylor rule with no weight on the output gap. This figure shows that small shocks have permanent effects on GDP when $\rho_y = 0$, whilst the economy recovers when $\rho_y = 0.125$. A positive weight on the output gap implies that the monetary authority will respond with stimulus to aggregate demand when the output gap is negative, by offering a lower nominal interest rate for each level of inflation. A negative TFP shock reduces the supply of output, which puts pressure on prices to increase, raising inflation. The presence of the output gap in the Taylor rule, compared to a scenario where the weight of the output gap is zero, puts additional pressure on prices by leading to higher consumption and investment.

More importantly, the presence of the output gap in the Taylor rule enables the *prevention destruction channel* of monetary policy to operate. The stimulus affects demand by lowering the wedge between the return to capital for successful entrepreneurs and the risk-less rate, thus effectively reducing the influence of financial frictions on the economy (see Figure 25 in the Appendix). The condition for leverage (30), clearly shows that a fall in the wedge implies that financial intermediaries will offer contracts with lower leverage for each feasible value of $\bar{\omega}$. In equilibrium, the entrepreneur will then find it optimal to pick a contract with lower leverage and lower $\bar{\omega}$, keeping a larger share of her returns. Bankruptcy is thus reduced, leading to less capital destruction. The spread between the interest rate on loans and the risk-less rate is also reduced as a consequence of the policy intervention, so we conclude that a positive weight on the output gap tames risk in the economy overall, sustains aggregate demand and prevents capital destruction, keeping output and savings higher until the output gap closes. By preventing capital destruction monetary policy leads to a V-shaped recovery.

[28] The Taylor rule systematically generates a full recovery after a TFP shock, but Figure 24 in the Appendix shows that the recovery is fast as long as the persistence of the shock is not too high.

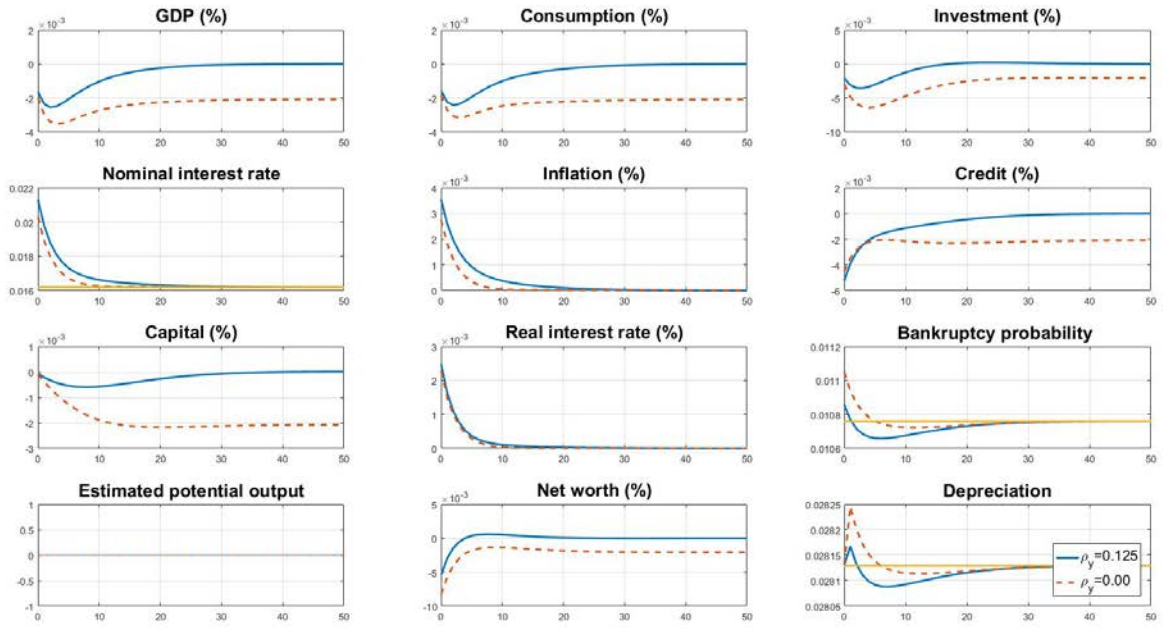

Figure 10: Effects of a 1% negative TFP shock

Weight on the output gap	$\rho_y = 0$	$\rho_y = 0.125$
150 periods	-0.22%	-0.08%

Table 3: Welfare losses of TFP shock for different weights on the output gap

Table 3 shows the welfare gains associated to monetary policy in this framework.[29] For a pure inflation targeting Taylor rule, with $\rho_y = 0$, a minus 1% transitory TFP shock generates sizeable 0.22% (consumption equivalent) welfare losses. When the output gap is added to the Taylor rule, with standard (quarterly measured) $\rho_y = 0.125$, we find that following the mandate of economic stability in monetary policy interventions unequivocally improves welfare, halving welfare losses of a recessions from 0.22% to 0.08%–see Table 3.

6.2 Small Demand Shocks

In this section, we aim to represent demand shocks in normal times, which we characterise as periods subject to shocks that are not as severe and persistent as during the Great Recession.

[29] In order to compute welfare gain and losses, we run the simulation with second-order perturbation methods.

We target an increase in the bankruptcy probability of approximately 0.5 percentage points, to mirror Christiano's estimation for the recessions preceding the financial crisis. Thus, we opt for a 5% increase in risk combined with a 4.5% confidence shock and we reduce the persistence of shocks to 0.7. This is close to pre-Great Recession estimates on confidence shocks documented by Angeletos, Collard, and Dellas (2018) and with the analysis of Christiano et al. (2015), who illustrate the increased persistence of financial shocks during the Great Recession compared to previous downturns. Figure 11 shows that monetary policy drives a V-shaped type of recovery in this case as well, although the economy will not converge back to the previous steady state within the 50 quarters period. Since we assume that potential output is never revised, the economy will eventually converge to the previous trajectory. In any case, the remaining output gap is small enough not to be perceived as a change of trajectory resulting from the demand shock.

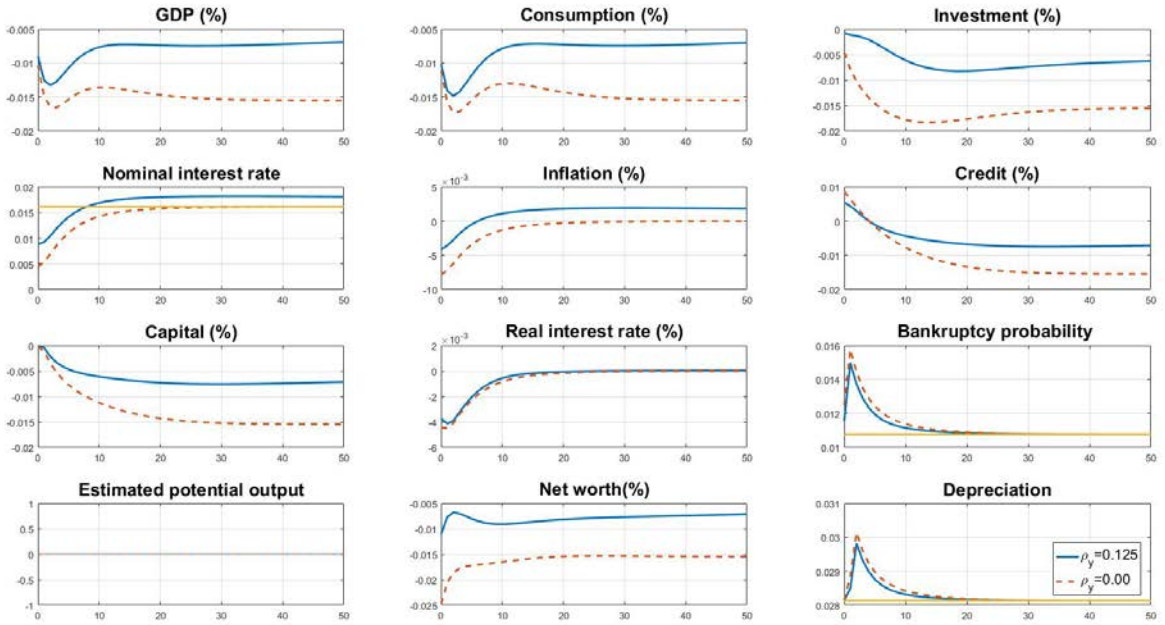

Figure 11: Effects of a small demand shock

In order to understand the mechanics behind the recovery role of monetary policy, it is useful to think about the policy in place as a force counteracting the negative shocks. As the demand

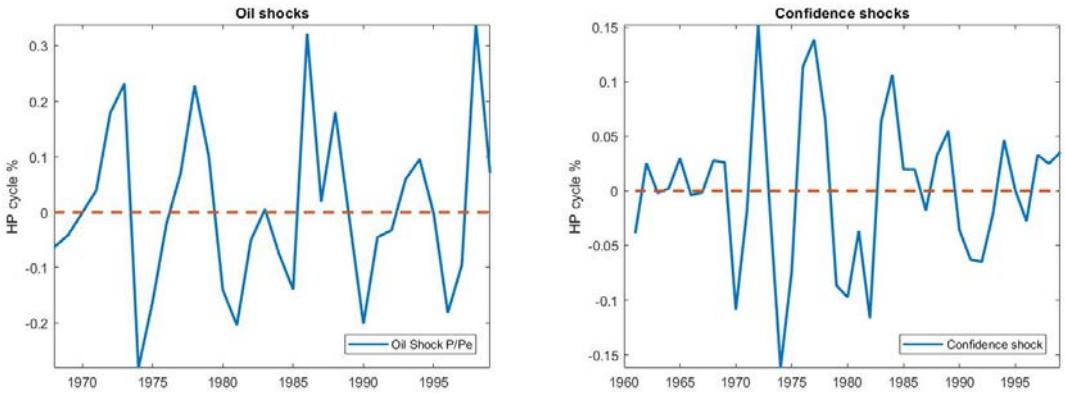

Figure 12: Oil and confidence shocks measure

shock hits and GDP falls, a larger output gap gives the policy strength, but as the recovery starts to materialise and inflation recovers, the monetary authority faces a trade-off between above target inflation and a negative output gap. The higher weight on inflation in the Taylor rule results in a slowdown in the recovery, although GDP will eventually go back to the initial steady state.

6.3 Historical V-Shaped Recoveries

This section exemplifies the ability of our model to generate V-shaped recoveries in line with the recovery episodes that followed the 1974 and 1990 oil shock recessions.[30]

Shocks. Both recessions followed large oil price increases, concomitantly associated with strong declines on consumers confidence. The left panel of Figure 12 represents the HP filter cyclical component of the ratio between the GDP deflator and the U.S. Crude Oil Composite Acquisition Cost by Refiners, both expressed as indexes with 2012 set equal to one. The confidence shocks are represented in the right panel of Figure 12 by the HP filter cyclical component of the Michigan Consumer Sentiment index. Both figures show large negative oil and confidence shocks in 1974 and 1990, larger in 1974 and more persistent in 1990.

[30]We don't aim to replicate the 1980 oil shock because the recession was driven by the disinflation effort initiated by Paul Volker. Similarly, we do not attempt to replicate the the dot.com recession as it was primarily driven by the collapse of the equity market.

The 1974 Oil Shock Recession. In October 1973, the Organization of Arab Petroleum Exporting Countries (OAPEC, the Arab majority of the OPEC) announced large cuts in oil production and an oil embargo affecting the U.S., among other countries. By March 1974, when the embargo ended, oil prices had tripled. Such a disruption in oil supply and increase in oil prices led to a deep recession, cumulating a large decline in U.S. GDP between the last quarter of 1973 and the first quarter of 1975, the size of the U.S. output gap reaching 5% according to Fed and CBO data.

In order to characterise the recovery that followed the First Oil shock, we construct a measure of output gap by de-trending real GDP consistently with the methodology we followed for the Great Recession.[31] Figure 13 shows that de-trended GDP follows closely the CBO output gap despite expected differences in methodology as well as differences driven by data revisions, as documented by Orphanides (2003). More importantly, Figure 13 shows for both measures of the output gap that the First Oil shock recession was followed by a V-shaped recovery, bringing GDP back to its previous track by the end of 1978 .

Let us here use our baseline model to replicate the reaction of the U.S. economy to the 1979 oil shock.[32] In line with Rotemberg and Woodford (1996) and Herrera, Karaki, and Rangaraju (2019), we include oil in the model as an input of production. In our framework oil shocks are equivalent to TFP shocks, resulting in rising production costs and a reduction in GDP. We thus represent oil price shocks as TFP shocks,[33] and set the size of the shock to generate a fall in GDP in line with data. Conversely, for the confidence shock we follow closely the result in Figure 12. We then model the First Oil Shock as a negative 18% TFP shock and a negative 16% confidence shock, setting the persistence of the shocks to 0.82 and 0.7, respectively, in line with the persistence of the shocks in the data (approx 3 years for the oil shock and 2 years for the confidence shock).

The left panel in Figure 13 compares the dynamics of the model with the CBO output Gap

[31] GDP is linearly de-trended including data from two previous business cycle peaks, in line with CBO methodology.

[32] Note that we did not modify our baseline calibration for this exercise to ease comparison with previous results, but also because some key steady state moments remained in line with the data even considering previous time periods (see Table 6 and Table 7 in the Appendix).

[33] See Appendix E for a formal argument in favour of using TFP shocks to represent oil price shocks in our framework.

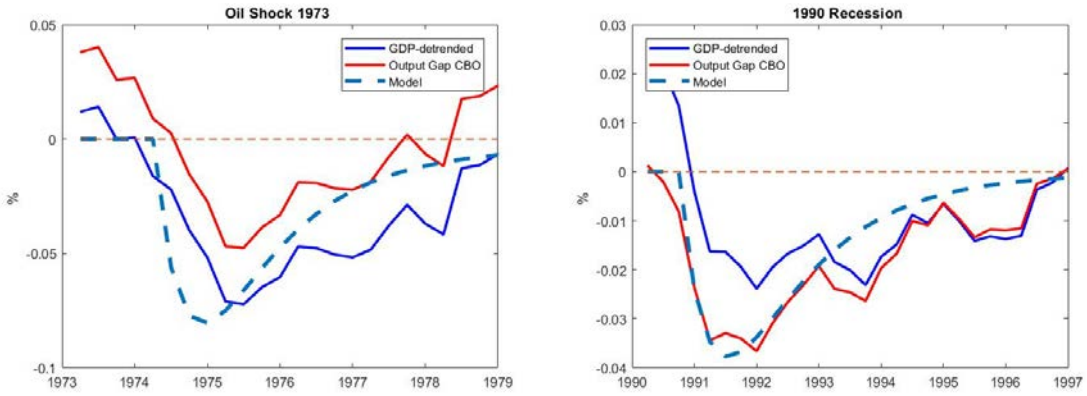

Figure 13: 1974 and 1990 Oil Shock Recessions - Model vs data

and our measure of de-trended GDP, and shows that the model replicates well the V-shaped recovery observed in the data.

The 1990 Recession. The 1990 recession was triggered by the Iraqi invasion of Kuwait in August 1990, lasting less than one year and leading to an increase in oil prices that was smaller and shorter than after the First Oil shock. However, the recovery was slow, as can be seen by comparing both panes in Figures 13, and jobless –see Schreft, Singh, et al. (2003). The fundamental reason is widely attributed to the persistent drop in confidence that followed the spike in oil prices. The Michigan consumer sentiment index shows longer persistence after the starting of the 1990 that after the start of the 1974 oil shock recession. Similarly to the 1974 excise, the right panel in Figure 13 compares to the CBO output gap and detrended GDP to the response of the model to a combined negative 15% TFP and 6% confidence shocks. The persistence of the shocks is 0.82 and 0.84, respectively, also in line with the persistence of the shock in the data (approx 3 years for the oil shock and 4 years for the confidence shock). The resulting dynamics follow closely the slow-V-shaped recovery in the data.

Overall, these findings show that our model can generate V-shaped recoveries in line with historical episodes.

7 The Taylor Rule and the Fed

The joint promotion of price stability and strong economic activity has been the main objective of the Federal Reserve System since its inception. Following the DSGE literature, this paper represents the decision process followed by the Fed, which ultimately aims at fulfilling its dual mandate of price and macroeconomic stability, by the J. Taylor (1993) rule.[34] In our model, the Taylor rule plays a critical role in shaping economic recoveries.[35] On one side, the existence of a V-shaped recovery relies on the assumption that the weight given to the output gap in the Taylor rule is positive and strong enough to bring the economy back to its original potential output track after a negative supply or demand shock. On the other side, the existence of an L-shaped recovery relies on the revision of potential output estimates during deep and persistent downturns, *i.e.* the *switching-track*. As a result, in our model the shape of economic recoveries critically depends on the specification of the Taylor rule and on the monetary authority's information set when measuring the output gap.

In this section, we provide a consistency check to substantiate the notion that the Taylor rule represents the Fed's policy choices, supporting our modelling strategy and validating our conclusions. We also discuss whether there are alternative rules consistent with the behaviour of the Fed that would call into question our conclusions on the dynamics of economic recovery.

Moreover, we conduct deeper analysis to define the Fed's information set and asses the validity of the indicators we use in the paper. The research staff at the Federal Reserve Board of Governors regularly prepares projections about how the U.S. economy will fare in the future. These projections are published in the Greenbook of the Federal Reserve Board before each meeting of the Federal Open Market Committee, and are part of the information set setting the ground for monetary policy decisions. Unfortunately, output gap protections in the Greenbook are made available with a six year lag and the corresponding time series are not as long as the Congressional Budget Office (CBO) data. For this reason, throughout this paper, we use

[34] For a detailed historical account of monetary policy objectives in the United States see Orphanides (2003).

[35] In an AK growth model, the intercept of the balanced growth path for GDP is indeterminate, and pinned down by the initial value of capital and capital accumulation in the transition to the stationary growth rate (the slope of the GDP balanced growth path). A monetary policy set to stabilise the output gap when facing negative supply or demand shocks, increases the intercept of the new balanced growth path by accelerating growth in the transition.

data from the CBO to proxy the changes in the Fed' beliefs about potential output. In this section, we use available data from the Greenbook to check for discrepancies between the two data sources, and test whether they matter in estimating the Taylor rule.

Does the Taylor Rule Summarise the Fed's Policy? In order to answer this question, we first build measures of the nominal interest rate consistent with the Taylor rule, and compare them with the Federal Funds Rate. We do that by measuring the nominal interest rate (in annual terms) emerging from the original J. Taylor (1993) rule[36]

$$\mathrm{TR}_t = \underbrace{r^*}_{\overline{\mathrm{TR}}-\overline{\pi}} + \pi_t + \phi_\pi \left(\pi_t - \overline{\pi}\right) + \phi_y \, x_t, \tag{48}$$

with $\pi_\pi = \phi_y = 0.5$. The real interest rate r^* is set to 2%. Inflation π_t is measured as year on year percentage changes in the quarterly GDP deflator, and $\bar{\pi}$ is set to 2%. The output gap x_t is one of the three following measures: the CBO's output gap (as reported in the February 2021 revision), the Greenbook output gap (as reported in February 2021), as well as a real time measure of the latter, i.e. using for each quarter the last historical estimate of the output gap published in the corresponding quarter. As it can be observed in Figure 14, the nominal interest rate series that emerge from the above Taylor rule, for the three different measures of the output gap, closely follow the Federal Funds Rate.[37] [38]

As an additional check, we estimate the policy parameters ϕ_π and ϕ_y of the standard Taylor rule, following the approach suggested by Kahn (2012), and considering the Federal Funds Rate as the policy instrument. To allow the target FFR, measured by $\overline{\mathrm{TR}}$ to change, we estimate (48) between 1960 and 2019 for the following sub-periods:

- 1960Q1-1979Q3: Great Relaxation (GRel)
- 1979Q4-1986Q4: Great Disinflation (GD, Volcker era)
- 1987Q1-2001Q1: Great Moderation (GM)

[36] Which is in line with the Board of Governors of the Federal Reserve System. See FRED Blog (St Luis Fed) link here.

[37] The discrepancies observed after 2010 are discussed in Section 5.

[38] Note that here CBO data corresponds to the time series published in 2021, whilst in Figure 4 we constructed a real-time estimate. As we could not build the same measure for the whole time period due to data limitations, we relied on the latest revision in this section.

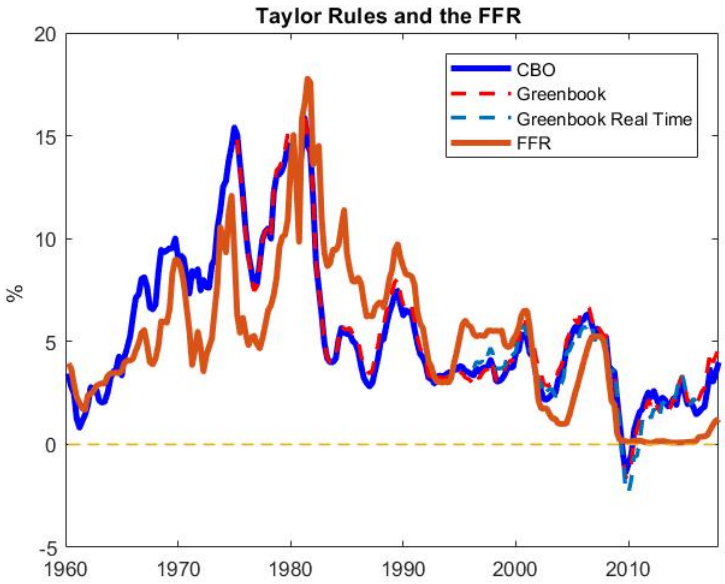

Figure 14: Taylor rule with alternative output gap measures

- 2001Q2-2019Q4: Great Deviation and Great Recession (GDe-GR).[39]

The results of the estimations in Table 4 show that the output gap is generally a strong predictor of the FFR, and coefficients are in line with the original J. Taylor (1993) rule. The only exception is the Volcker era, which comes at no surprise as the effort associated with reducing inflation levels likely generated a deviation from the standard Taylor rule during this period.[40] These findings support our modelling choices on the specification of the policy rule as well as our calibration.[41]

Did the Fed's Beliefs about Potential Output Switch-Track after the Great Recession? The left panel in Figure 2 clearly shows the switching-track in the CBO estimations of potential output during the Great Recession. However, did the Fed's beliefs about potential output, implicit in the output gap measures reported in the right panel of Figure 2, change

[39]Here we refer to J. B. Taylor (2011), who points out that policy makers started deviating form the TR already at the beginning of the century, giving rise to a Great Deviation ahead of the Great Recession.

[40]Similarly, the historical account of Orphanides (2003) found that the largest deviations from the Taylor rule occurred before the Great Relaxation and during the Volcker disinflation period.

[41]As a robustness check, we also estimated the policy parameters for the whole period, accounting for changes in the policy target through period dummies, and found comparable results.

	(GRel) FFR b/se	(GD) FFR b/se	(GD) FFR b/se	(GM) FFR b/se	(GM) FFR b/se	(GDe-GR) FFR b/se	(GDe-GR) FFR b/se	(GDe-GR) FFR b/se
Inflation Gap	0.823***	1.088***	1.090***	1.731***	1.422***	0.921***	1.138***	1.062***
	(0.062)	(0.111)	(0.111)	(0.102)	(0.094)	(0.185)	(0.195)	(0.236)
CBO	0.391***	0.078		0.682***		0.425***		
	(0.057)	(0.104)		(0.081)		(0.061)		
GB			0.024		0.522***		0.266***	
			(0.089)		(0.067)		(0.047)	
GB_RT								0.333***
								(0.068)
$\overline{\text{TR}}$	3.402***	7.453***	7.274***	5.462***	5.297***	2.217***	1.860***	2.391***
	(0.118)	(0.428)	(0.346)	(0.121)	(0.133)	(0.161)	(0.152)	(0.240)
Number obs.	79	29	29	57	57	75	71	59
R^2	0.762	0.812	0.810	0.763	0.753	0.621	0.588	0.612

Note: Robust standard errors in parentheses.

Table 4: Taylor rule estimation - Output Gap

consistently? Figure 14 shows that the output gap measures emerging from the CBO and the Greenbook data yield similar results when using the Taylor rule to estimate the nominal interest rate.[42] We then conclude that the implicit potential output measure in the Greenbook is consistent with the CBO potential output estimates, which is in strong support of the hypothesis that Fed's beliefs concerning potential output switched track after the Great Recession. Estimations of the output gap policy parameter in Table 4 confirm that CBO data can be considered a good proxy of the Fed' beliefs on potential output, and we can thus conclude that the *Switching-track* mattered for the Fed's policy choices during the Great Recession.

Alternative Rules. We showed that the classic Taylor rule can adequately summarize the Fed's policy making, supporting our claim that monetary policy interventions play a critical role in shaping economic recoveries. Nonetheless, the impact of economic activity on monetary policy, as represented by the Taylor rule, could be modelled in different ways. Two other measures of economic activity are widely considered as alternatives to the output gap in the Taylor rule: the unemployment gap and the growth gap. The former is measured as the difference between the unemployment rate and the non-accelerating inflation rate of unemployment (NAIRU), while the latter is measured as the gap between the actual and the long term growth rate. Could these different alternative specifications alter the key result of this paper, *i.e.* that the Taylor rule shapes economic recoveries? Is there an alternative rule, consistent with the historical behaviour

[42] In fact, Figure 26 in the Appendix shows that both output gap measures tend to be extremely close.

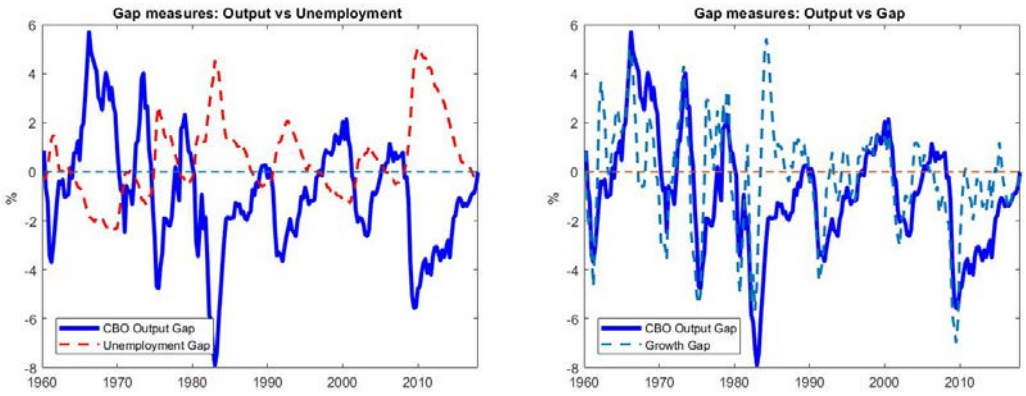

Figure 15: Alternative output gap measures

of the Fed that would not generate a V-shaped recovery in our model?

Firstly, it is important to stress that the NAIRU is estimated similarly to potential output, so that the unemployment gap in the data looks like the mirror image of the output gap, as shown by the left panel of Figure 15.[43] As a consequence, when accurately modelling unemployment, a Taylor rule targeting the unemployment gap will likely yield similar results as a Taylor rule targeting the output gap.[44]

As for the growth gap, we measure it as the distance between quarterly GDP growth, year on year, and the long term growth rate of 2.2%. The resulting growth gap is depicted in the right panel of Figure 15, and although it clearly moved in line with the output gap, it tends to shrink faster. Incorporating the growth gap in the Taylor rule in our model would then provide less stimulus after negative shocks, which might not be enough to generate V-shaped recoveries before the Great Recession. To check for this, we take two steps, detailed in Appendix D. Firstly, we check whether the growth gap can be considered as a good predictor of the FFR by estimating the relative policy parameters, and find that this was the case during the Great Disinflation and the Great Moderation. We then incorporate the growth rule in our model, and check how the model would react to the oil shocks considered in the previous section. As expected, the recovery in the model weakens compared to our baseline as a result of the policy change. GDP does not return to the original steady state, but the recovery remains V-shaped

[43] Unemployment data was retrieved from the U.S. Bureau of Labor Statistics, while the NAIRU estimate is taken from CBO data.

[44] We do not run this type of exercise using our framework since there is no unemployment in our model.

in both cases. To sum up, the growth gap is more likely to yield incomplete V-shaped recoveries compared to the output gap, but the differences are likely to be small in a context of moderate shocks.

Overall, the results presented in this section support our modelling choices and the idea that monetary policy interventions played a critical role in shaping recoveries in the United States.

8 Conclusion

Our paper contributes to the literature by showing that an endogenous growth model can reproduce the dynamics of U.S. GDP well, once the role of monetary policy is taken into account. In our framework, the differentiating factors between the Great Recession and previous recessions were the size and persistence of the shocks, the subsequent capital destruction, the binding of the zero lower bound and the introduction of the *switching-track*.

This paper opens several avenues for future research. Firstly, it would be interesting to explore whether the economy has in fact evolved from a Neoclassical technology to an AK model because of the rise of intangible capital. *"Because intangible investments, on average, behave differently from tangible investments, we might reasonably expect an economy dominated by intangibles to behave differently too."*[45] Secondly, we are keen to further investigate the destruction prevention channel of monetary policy and its implications for optimal monetary interventions. Lastly, this paper does not fully address the role of unconventional monetary policies implemented following the financial crisis. It would be interesting to explore this aspect, by evaluating further the impact of central bank credit policies on the recovery process in our model, following the example of Gertler and Karadi (2011).

[45] (Haskel & Westlake, 2018) page 10.

References

Acemoglu, D., Akcigit, U., & Kerr, W. (2016). Networks and the Macroeconomy: An Empirical Exploration. *NBER Macroeconomics Annual*, *30*(1), 273–335.

Adjemian, S., Bastani, H., Juillard, M., Karamé, F., Maih, J., Mihoubi, F., ... Villemot, S. (2011). Dynare: Reference Manual Version 4. CEPREMAP.

Aghion, P., & Howitt, P. (1992). A Model of Growth Through Creative Destruction. *Econometrica*, *60*(2), 323-51.

Angeletos, G.-M., Collard, F., & Dellas, H. (2018). Quantifying Confidence. *Econometrica*, *86*(5), 1689-1726.

Anzoategui, D., Comin, D., Gertler, M., & Martinez, J. (2019). Endogenous Technology Adoption and R&D as Sources of Business Cycle Persistence. *American Economic Journal: Macroeconomics*, *11*(3), 67-110.

Ascari, G., & Rossi, L. (2012). Trend Inflation and Firms Price-setting: Rotemberg Versus Calvo. *Economic Journal*, *122*(563), 1115-1141.

Ball, L. (2014). Long-term Damage from the Great Recession in OECD Countries. *European Journal of Economics and Economic Policies: Intervention*, *11*(2), 149-160.

Benigno, G., & Fornaro, L. (2018). Stagnation Traps. *Review of Economic Studies*, *85*(3), 1425–1470.

Bernanke, B. S., Gertler, M., & Gilchrist, S. (1999). The Financial Accelerator in a Quantitative Business Cycle Framework. *Handbook of Macroeconomics*, *1*, 1341–1393.

Bianchi, F., Kung, H., & Morales, G. (2019). Growth, Slowdowns, and Recoveries. *Journal of Monetary Economics*, *101*, 47 - 63.

Born, B., & Pfeifer, J. (2020). The New Keynesian Wage Phillips Curve: Calvo vs Rotemberg. *Macroeconomic Dynamics*, *24*(5), 1017–1041.

Boucekkine, R., del Río, F., & Licandro, O. (1999). Endogenous vs Exogenously Driven Fluctuations in Vintage Capital Models. *Journal of Economic Theory*, *88*(1), 161-187.

Caballero, R. J., & Hammour, M. L. (1994). The Cleansing Effect of Recessions. *American Economic Review*, *84*(5), 1350–1368.

Carlstrom, C. T., & Fuerst, T. S. (1997). Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis. *American Economic Review*, *87*(5), 893–910.

Christiano, L. J., Eichenbaum, M. S., & Trabandt, M. (2015). Understanding the Great Recession. *American Economic Journal: Macroeconomics*, *7*(1), 110-67.

Christiano, L. J., Motto, R., & Rostagno, M. (2014). Risk Shocks. *American Economic Review*, *104*(1), 27-65.

Coibion, O., Gorodnichenko, Y., & Ulate, M. (2017). The Cyclical Sensitivity in Estimates of Potential Output. NBER Working Paper 23580.

Comin, D., & Gertler, M. (2006). Medium-Term Business Cycles. *American Economic Review*, *96*(3), 523-551.

Cozzi, G., Pataracchia, B., Pfeiffer, P., & Ratto, M. (2017). How Much Keynes and How Much Schumpeter? An Estimated Macromodel of the U.S. Economy. Joint Research Centre, European Commission (Ispra site) 2017-01 Working Paper.

Davis, S. J., & Haltiwanger, J. (1990). Gross Job Creation and Destruction: Microeconomic Evidence and Macroeconomic Implications. *NBER Macroeconomics Annual*, *5*, 123-168.

Edge, R. M., Kiley, M. T., & Laforte, J.-P. (2008). Natural Rate Measures in an Estimated DSGE Model of the U.S. Economy. *Journal of Economic Dynamics and Control*, *32*(8), 2512–2535.

Fraumeni, B. (1997). The Measurement of Depreciation in the U.S. National Income and Product Accounts. *Survey of Current Business-United States Department of Commerce*, *77*, 7–23.

Garga, V., & Singh, S. R. (2020). Output Hysteresis and Optimal Monetary Policy. *Journal of Monetary Economics*, Forthcoming.

Gavazza, A., & Lanteri, A. (2021). Credit Shocks and Equilibrium Dynamics in Consumer Durable Goods Markets..

Gertler, M., & Karadi, P. (2011). A Model of Unconventional Monetary Policy. *Journal of Monetary Economics*, *58*(1), 17–34.

Gilchrist, S., & Zakrajšek, E. (2012). Credit Spreads and Business Cycle Fluctuations. *American Economic Review*, *102*(4), 1692-1720.

Gourio, F. (2012). Disaster Risk and Business Cycles. *American Economic Review*, *102*(6), 2734-66.

Guerrieri, V., & Lorenzoni, G. (2017). Credit Crises, Precautionary Savings, and the Liquidity Trap. *Quarterly Journal of Economics*, *132*(3), 1427–1467.

Guerrieri, V., Lorenzoni, G., Straub, L., & Werning, I. (2020). Macroeconomic implications of COVID-19: Can negative supply shocks cause demand shortages? No. w26918. National Bureau of Economic Research.

Hall, R. E. (2015). Quantifying the Lasting Harm to the U.S. Economy from the Financial Crisis. *NBER Macroeconomics Annual*, *29*(1), 71–128.

Hall, R. E. (2016). Why Has the Unemployment Rate Fared Better than GDP Growth? Manuscript.

Haskel, J., & Westlake, S. (2018). Capitalism without Capital: The Rise of the Intangible Economy. *Princeton University Press*.

Herrera, A. M., Karaki, M. B., & Rangaraju, S. K. (2019). Oil Price Shocks and U.S. Economic Activity. *Energy Policy*, *129*, 89-99.

Ikeda, D., & Kurozumi, T. (2019). Slow Post-financial Crisis Recovery and Monetary Policy. *American Economic Journal: Macroeconomics*, *11*(4), 82-112.

Juillard, M., et al. (1996). Dynare: a Program for the Resolution and Simulation of Dynamic Models with Forward Variables Through the Use of a Relaxation Algorithm. CEPREMAP Working Papers (Couverture Orange) 9602.

Kahn, G. A. (2012). Estimated Rules for Monetary Policy. *Economic Review*, *97*(Q IV).

Karadi, P., & Reiff, A. (2019). Menu Costs, Aggregate Fluctuations, and Large Shocks. *American Economic Journal: Macroeconomics*, *11*(3), 111–46.

Kozlowski, J., Veldkamp, L., & Venkateswaran, V. (2020). The Tail that Wags the Economy: Beliefs and Persistent Stagnation. *Journal of Political Economy*, *128*(8), 2839–2879.

Lanteri, A. (2018). The Market for Used Capital: Endogenous Irreversibility and Reallocation over the Business Cycle. *American Economic Review*, *108*(9), 2383–2419.

Orphanides, A. (2003). Historical Monetary Policy Analysis and the Taylor Rule. *Journal of Monetary Economics*, *50*(5), 983-1022.

Queralto, A. (2020). A Model of Slow Recoveries from Financial Crises. *Journal of Monetary Economics*, *114*, 1-25.

Romer, P. (1986). Increasing Returns and Long-run Growth. *Journal of Political Economy*, *94*(5), 1002-37.

Romer, P. (1990). Endogenous Technological Change . *Journal of Political Economy*, *98*(5), S71-S102.

Rotemberg, J. J. (1982). Sticky Prices in the United States. *Journal of Political Economy*, *90*(6), 1187–1211.

Rotemberg, J. J., & Woodford, M. (1996). Imperfect Competition and the Effects of Energy Price Increases on Economic Activity. *Journal of Money, Credit and Banking*, *28*(4), 549–577.

Schreft, S. L., Singh, A., et al. (2003). A Closer Look at Jobless Rcoveries. *Economic Review-Federal Reserve Bank of Kansas City*, *88*(2), 45–73.

Shackleton, R. (2014). Revisions to CBO's Projection of Potential Output Since 2007. Congressional Budget Office Report 45150.

Taylor, J. (1993). Discretion Versus Policy Rules in Practice. *Carnegie-Rochester Conference Series on Public Policy*, *39*, 195 - 214.

Taylor, J. B. (2011). Macroeconomic Lessons from the Great Deviation. *NBER Macroeconomics Annual 2010, Volume 25*, 387-395.

Vetlov, I., Pisani, M., Hlédik, T., Jonsson, M., & Kucsera, H. (2011). Potential Output in DSGE Models. European Central Bank Working Paper Series 1351.

Appendix

A Model-data Comparison and Data Sources

Taylor Rule: GDP is Real Gross Domestic Product, Billions of Chained 2012 Dollars, Seasonally Adjusted Annual Rate, inflation is Gross Domestic Product: Implicit Price Deflator, Index 2012=100, Seasonally Adjusted (percentage change from a year ago), FFR is the Effective Federal Funds Rate, Percent, Not Seasonally Adjusted. Potential output historical estimates were retrieved from the Congressional Budget Office website.

Data for Model Simulation Comparison: all data was retrieved from the FRED database. GDP is Real Gross Domestic Product, consumption is Real Personal Consumption Expenditures, investment is Real Gross Private Domestic Investment and private non-residential investment is Private Nonresidential Fixed Investment, divided by its deflator (retrieved from BEA). All variables are expressed in billions of chained 2012 dollars, quarterly, seasonally adjusted annual rates and in per capita terms. The bankruptcy probability is the estimated risk shock in Christiano et al. (2014). Credit is Total Credit to the Non-Financial Corporations, adjusted for breaks, in billions of U.S. Dollars, divided by the GDP deflator. The nominal interest rate is the Effective Federal Funds Rate (quarterly averages).

Data for Model Steady State Comparison: All values are averages over the period 1980-2008Q2 or 2008 when data was available annually. The labour share is the average of the share of labour Compensation in GDP, and labour hours are the average of the time series average annual hours worked by persons engaged for United States, calculated as a percentage of total hours in the year. GDP growth per capita was retrieved from FRED (BEA data) and the investment to GDP ratio is calculated in real terms from BEA data. The real interest rate data was retrieved from the World Bank database. The interest spread is the corporate bond credit spread constructed by Gilchrist and Zakrajšek (2012), i.e., the average difference between the interest rate on firm specific loans in COMPUSTAT and the rate the U.S. government would have paid for a comparable maturity. Leverage is

proxied by the average of real debt over real assets for the Non-financial private sector in COMPUSTAT, where total assets were deflated using the BEA investment deflator, whilst total debt was deflated using the GDP deflator.

Moment	Model	U.S. Data
Labour share (1980-2008)	60.8%	61%
Labour hours (1980-2008)	20%	20.5%
Quarterly GDP growth pc (1980-2008Q2)	0.6%	0.5%
Investment share(1980-2008)	12%	15%
Real interest rate (1980-2008)	1.6%	0.6 %
Interest spread(1980-2008Q2)	1.2 pp	1.6 pp
Leverage(1980-2008)	0.5	0.5

Table 5: Aggregate data and model steady state values

Moment	Model	U.S. Data
Labour share (1950-1973)	60.8%	63.4%
Labour hours (1950-1973)	20%	22%
Quarterly GDP growth pc (1947-1973Q3)	0.6%	0.6%
Investment share(1947-1973Q3)	12%	11%
Real interest rate (1961-1973)	1.6%	0.6%

Table 6: Aggregate data and model steady state values - 1947-1973

Moment	Model	U.S. Data
Labour share (1980-1990)	60.8%	61.2%
Labour hours (1980-1990)	20%	20%
Quarterly GDP growth pc (1990-1990Q3)	0.6%	0.5%
Investment share(1980-1990Q3)	12%	14%
Real interest rate (1980-1990)	1.6%	1.68%

Table 7: Aggregate data and model steady state values - 1980-1990Q3

B Taylor Rule Mechanics

With a positive weight in the output gap, the Taylor rule implies that a negative output gap will put pressure on the nominal interest rate to fall. However, in our simulations we find that adding to the Taylor rule a positive weight on the output gap generates higher nominal interest rates and inflation in equilibrium, compared to pure inflation targeting (see Figures 10 and 11). More importantly, the real interest rate is also higher, depressing economic activity instead of promoting it. This appears to be puzzling. In order to understand how a positive weight on the output gap can generate an increase in both the nominal interest rate and inflation, we propose a simple exercise of comparative statics. In our model, as in the standard New Keynesian model, the nominal interest rate and inflation are determined by the intersection of the Euler equation and the Taylor rule. These can be represented by the following linear relationships between inflation and the nominal interest rate, ignoring the ZLB constraint,

$$\text{Taylor rule}: R = \frac{1+gz}{\beta} + 1.5\ (\pi - \bar{\pi}) + 0.125\ \log\left(\frac{G\hat{D}P_t}{G\bar{\hat{D}}P}\right) \tag{49}$$

$$\text{Euler equation}: R = \frac{1+gz}{\beta}\frac{\hat{\lambda}_t}{\hat{\lambda}_{t+1}}\frac{\pi_{t+1}}{\pi_t}\ \pi. \tag{50}$$

At any period t, equilibrium inflation π_t and nominal interest rates R_t are the pair $\{\pi, R\}$ that solves (49) and (50). The straight-lines crossing point SS in both plots in Figure 16 represent (49) and (50) at the non-stochastic steady state.

How would equilibrium change if we considered the response of the economy to a TFP shock? To answer this question, we use values of $\frac{\hat{\lambda}_t}{\hat{\lambda}_{t+1}}$, $\frac{\pi_{t+1}}{\pi_t}$ and $\frac{G\hat{D}P_t}{G\bar{\hat{D}}P}$ from our baseline TFP shock simulation at times 1 and 5 to plot (49) and (50). The left panel in Figure 16 represents both equations in a pure inflation targeting economy –when zeroing the coefficient of the output gap in (49). As the shock hits, the Taylor rule does not move but the Euler equation moves upwards, and then gradually comes back to the initial value. The transition is consistent with the IRFs in FGDe-GRigure 10, as the TFP shock results in a temporary increase in the nominal interest rate and inflation. The real interest rate goes up during the transition, since the slope of the Taylor rule is larger than one.

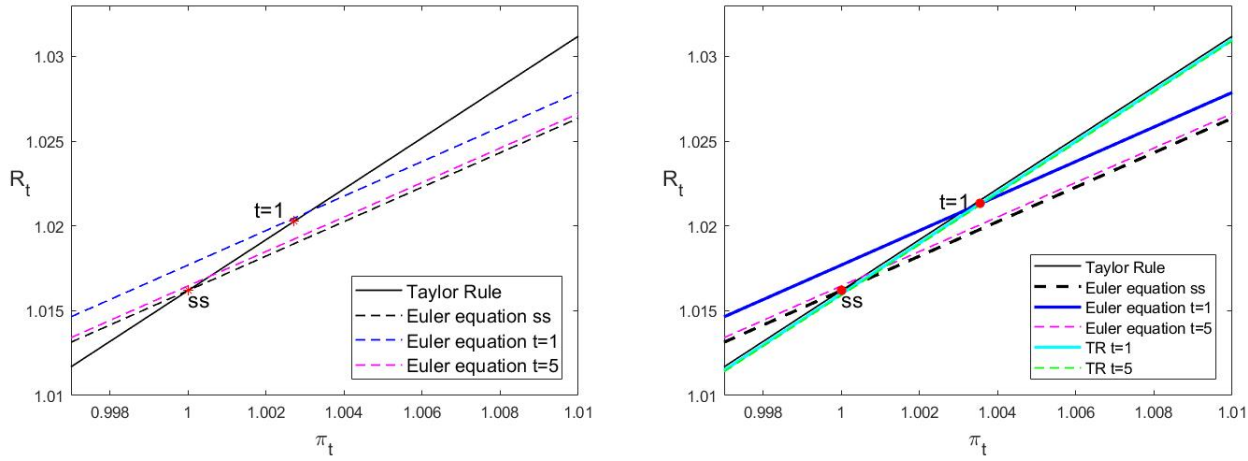

Figure 16: TFP shock mechanics with no weight on the output gap

We repeat the exercise for the scenario in which the weight on the output gap is positive. The right panel in Figure 16 shows that in this case the Taylor rule shifts to the right as the output gap pushes the intercept down, so that R_t and π_t are higher in equilibrium at time 1, compared to the pure inflation targeting case, consistently with our results in Figure 10. More importantly, the effect on the real interest rate relative to the pure inflation targeting scenario depends on the slope of the Euler equation, in turn effected by aggregate shocks and the policy intervention. This simple exercise shows that a positive weight on the output makes the the monetary authority offer a lower nominal interest rate for each level of inflation, stimulating substitution from savings to consumption through the Euler equation. The resulting dynamic can entail a larger interest rate compared to inflation targeting, but this equilibrium result does not imply a contractionary policy impact.

C Capital Quality

In this section we compare the depreciation rate in our model for the Great Recession to a measure of capital quality constructed by Kozlowski et al. (2020). They employ the Fed's Flow of Funds data for non-financial assets held by corporations, at market value (MV) and historical cost (HC). We replicate their methodology with quarterly series, assuming the depreciation rate to be at our steady state value. They define a capital quality shock as a decline in the productive

value of installed capital, which is equivalent to a rise in depreciation in our model. Assuming: $k_t = \phi_t \hat{k}_t$, the capital used in production k_t depends on the installed capital $\hat{k}_t$ and the shock ϕ_t, equal to 1 in normal times.

Let us denote by k_t the stock of capital at the end of period t, and by x_t and p_t real investment and the price of capital at time t, respectively. Past capital (net of depreciation) and current investment cumulate in $\hat{k}_t$ that suffers a quality shock ϕ_t before becoming the end of period capital k_t, *i.e.*:

$$\hat{k}_t = x_t + (1-\delta)k_{t-1} \qquad \text{and} \qquad k_t = \phi_t \hat{k}_t. \tag{51}$$

Kozlowski et al. (2020) assume that historical capital is measured following

$$\text{HC}_t = p_{t-1}x_t + (1-\delta)\text{HC}_{t-1} \tag{52}$$

Then, assuming $p_t k_t = \text{MV}_t$, from (51) and (52) we can recover

$$p_{t-1}\hat{k}_t = (1-\delta)p_{t-1}k_{t-1} + p_{t-1}x_t = (1-\delta)\text{MV}_{t-1} + \text{HC}_t - (1-\delta)\text{HC}_{t-1}. \tag{53}$$

then using the non-residential investment deflator from BEA:

$$\phi_t = \frac{k_t}{\hat{k}_t} = \frac{p_t k_t}{p_{t-1}\hat{k}_t}\frac{PriceIndex_{t-1}}{PriceIndex_t} \tag{54}$$

The resulting measure has an average value of 1.01 in the interval we consider, and displays a large negative realization in conjunction with the Great Recession. This mostly captures variations in the market value of structures, as the methodology behind the data adjusts the market value of commercial real estate. In this sense, this series complements Lanteri (2018)'s data, which was informative as to the value of equipment.

Mapping into our model, $\phi_t = \frac{1-\delta_t}{1-\delta}$. We normalise the data to average 1 to ease comparison, and Figure 17 shows that our model is consistent with the dynamics of the data. The magnitude of the model's fall in capital value is considerably smaller compared to the constructed measure, confirming our conservative approach.

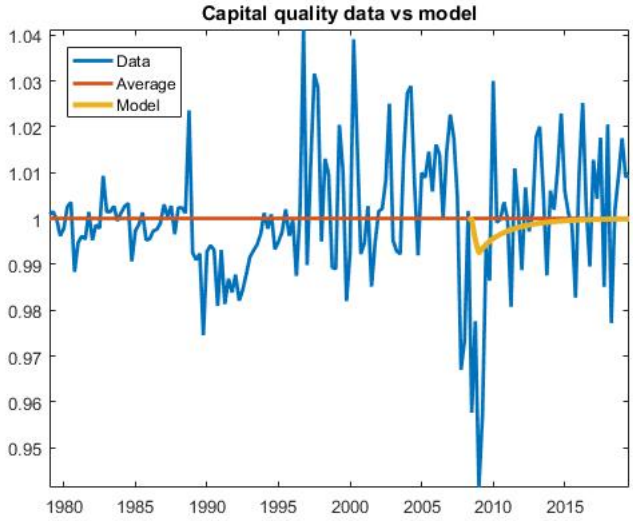

Figure 17: Capital quality measure from Kozlowski et al. (2020) in the data and the model

D The Growth Gap and the Taylor Rule

In order to evaluate the role of the growth gap for monetary policy in the United States, we repeat the steps followed in Section 7, quantifying the growth gap as the distance between quarterly GDP growth, year on year, and the 2.2% trend. Table 8 shows the estimation of the Taylor rule using the growth gap instead of the output gap. The growth gap performs similarly to the output gap during the Great Moderation, but much worse after the year 2000, as the coefficient turns negative.

We then check how the growth gap would affect the behaviour of our model. If we assume that policy makers were following the growth gap before the Great Recession, can we still generate V-shaped recoveries in line with data? To do this, we substitute the Taylor rule (43) in the model by[46]

$$R_t = \bar{R} + \rho_\pi(\pi_t - \bar{\pi}^m) + \rho_y \log\left(\frac{\widehat{\text{GDP}}_t}{\widehat{\text{GDP}}_{t-4}}\right). \tag{55}$$

We then run the model to replicate the 1974 and the 1990 oil driven recessions. Figure 18 shows that the model still generates a V-shaped recovery, but this is incomplete, as GDP fails

[46]Output is de-trended in steady state in our model, hence growth is null. Consequentially, the year on year growth gap is equivalent to the deviation of the log of de-trended GDP from its fourth lag.

	(GRel) FFR b/se	(GD) FFR b/se	(GM) FFR b/se	(GDe-GR) FFR b/se
InflationGap	0.782***	1.266***	1.615***	1.724***
	(0.074)	(0.075)	(0.149)	(0.212)
GrowthGap	0.118	0.283**	0.441***	-0.167*
	(0.076)	(0.115)	(0.083)	(0.096)
$\overline{\text{TR}}$	3.632***	6.712***	5.063***	1.453***
	(0.134)	(0.284)	(0.187)	(0.135)
Number of obs.	79	29	57	75
R^2	0.634	0.855	0.550	0.492

Legend: GRel= Great Relaxation: 1960Q1-1979Q3; GD=Great Disinflation: 1979Q4-1986Q4; GM= Great Moderation: 1987Q1-2001Q1; GDe-GR= Great Deviation-Great Recession: 2001Q2-2019Q4. Note: Robust standard errors in parentheses.

Table 8: Taylor rule estimation - Growth Gap

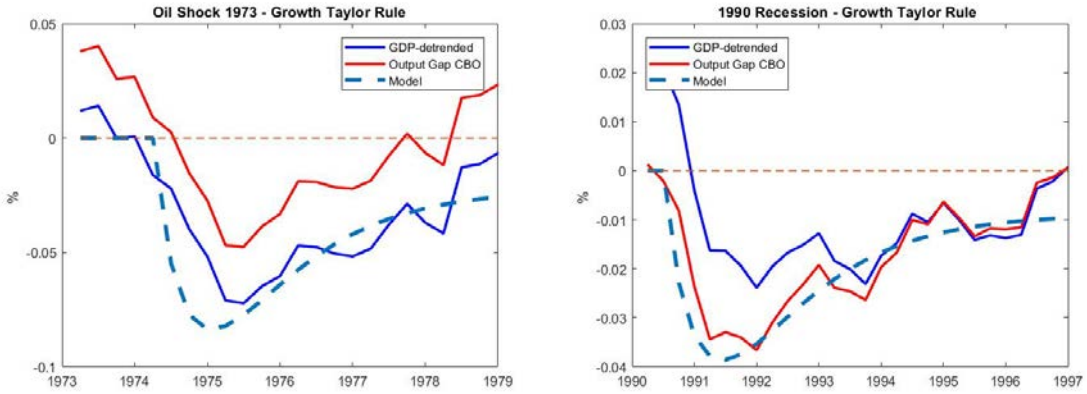

Figure 18: 1974 and 1990 Oil Shock Recessions - Model vs data

to reach its previous steady state. Nevertheless, the distance is small. If we were to increase the persistence of the shocks then model results would start to diverge more significantly. Overall, these findings suggest that a growth Taylor rule and an output gap Taylor rule are almost equivalent with moderate shocks consistent with historical data, validating the thesis that V-shaped recoveries were policy driven.

E Energy

In order to account for oil price shocks in our model, we draw inspiration from papers in the literature who explicitly model energy as a production input (see for example Rotemberg and

Woodford (1996) and Herrera et al. (2019)).

Let energy be a production input for intermediate producers, with $e(i)$ representing the consumption of energy by firm i –let us omit the time index t to simplify notation. The supply of energy is infinitely elastic at price p_e. Let us then add energy $e(i)$ to (15), so that production technology becomes

$$y(i) = AaK^{\eta}e(i)^{\beta}\Big(\underbrace{k(i)^{\alpha}h(i)^{1-\alpha}}_{x(i)}\Big)^{1-\beta}, \tag{56}$$

where $\beta \in (0,1)$ represents the energy share in production and $A > 0$ is an arbitrary constant. We will solve the problem in two stages. In the first stage, and following the same logic as in the main text, there exist a marginal cost $p_x = \alpha^{-\alpha}(1-\alpha)^{1-\alpha}(r^k)^{\alpha}w^{1-\alpha}$ that minimizes the cost of producing one unit of $x(i)$. Notice that $x(i)$ represents the value added generated by i and p_x represents its shadow price. Since capital and labour services are homogeneous, p_x is the deflator of value added.

In the second stage, and following a similar logic, firm i chooses $e(i)$ and $x(i)$ in order to minimize the cost of producing output $y(i)$. From the first order conditions of the minimization problem

$$\frac{e(i)}{x(i)} = aK^{\eta}\left(\frac{p_x}{p_e}\right)^{\beta}x(i),$$

under the normalisation assumption that $A = \left(\frac{\beta}{1-\beta}\right)^{-\beta}$. After substitution of this equation in (56), production of good i becomes

$$y(i) = \underbrace{a\left(\frac{p_x}{p_e}\right)^{\beta}K^{\eta}}_{\text{TFP}}\ \Big(\underbrace{k(i)^{\alpha}h(i)^{1-\alpha}}_{x(i)}\Big).$$

We can then interprete shocks to the energy price (relative to the GDP deflator) as TFP shocks. Consequentially, an oil price shock will reflect in a TFP shock, bringing about a fall in GDP and a rise in production costs. To replicate the reaction of the model to the oil shock recessions, we then impose a TFP shock, combined with and confidence shock to capture the observed fluctuations in consumer sentiment.

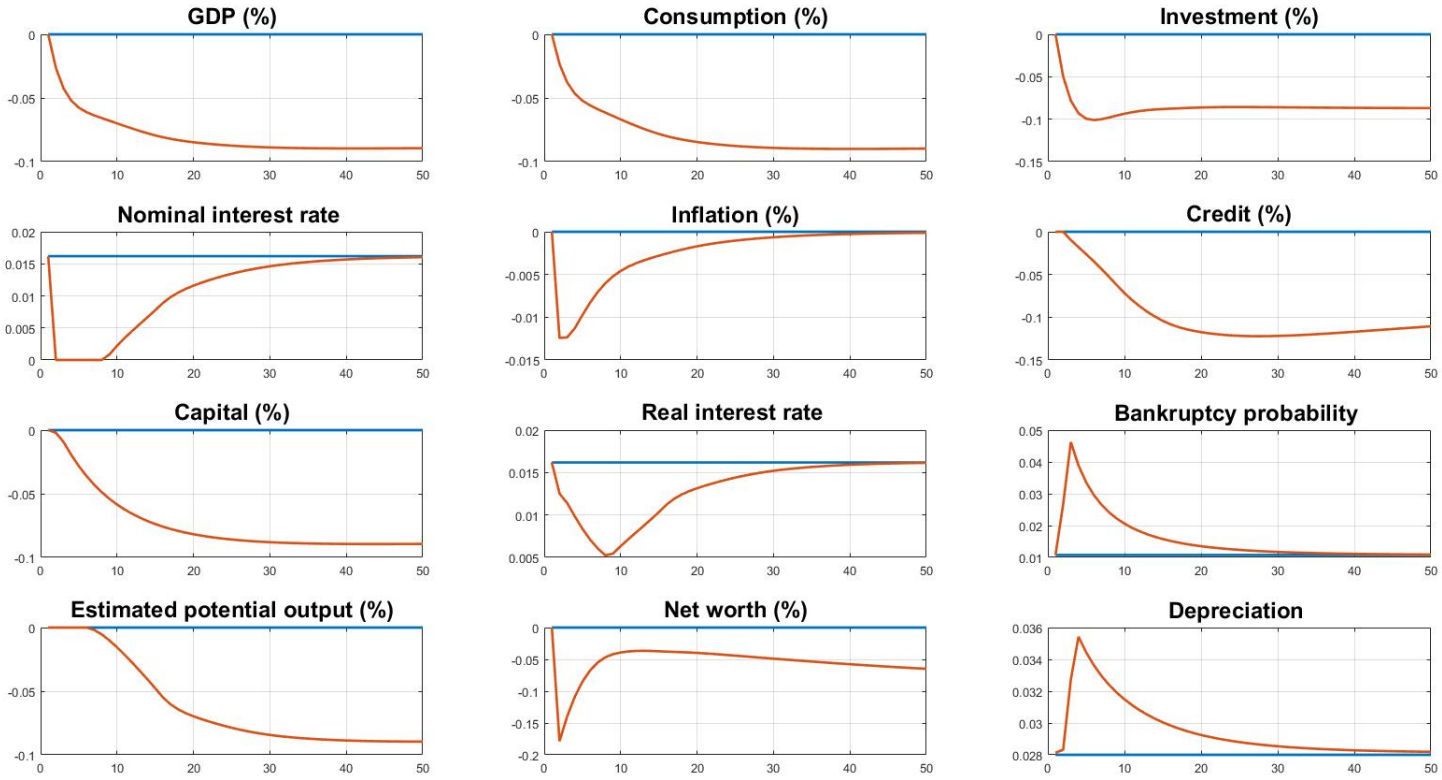

Figure 19: The Great Recession (baseline): Quarterly model simulations

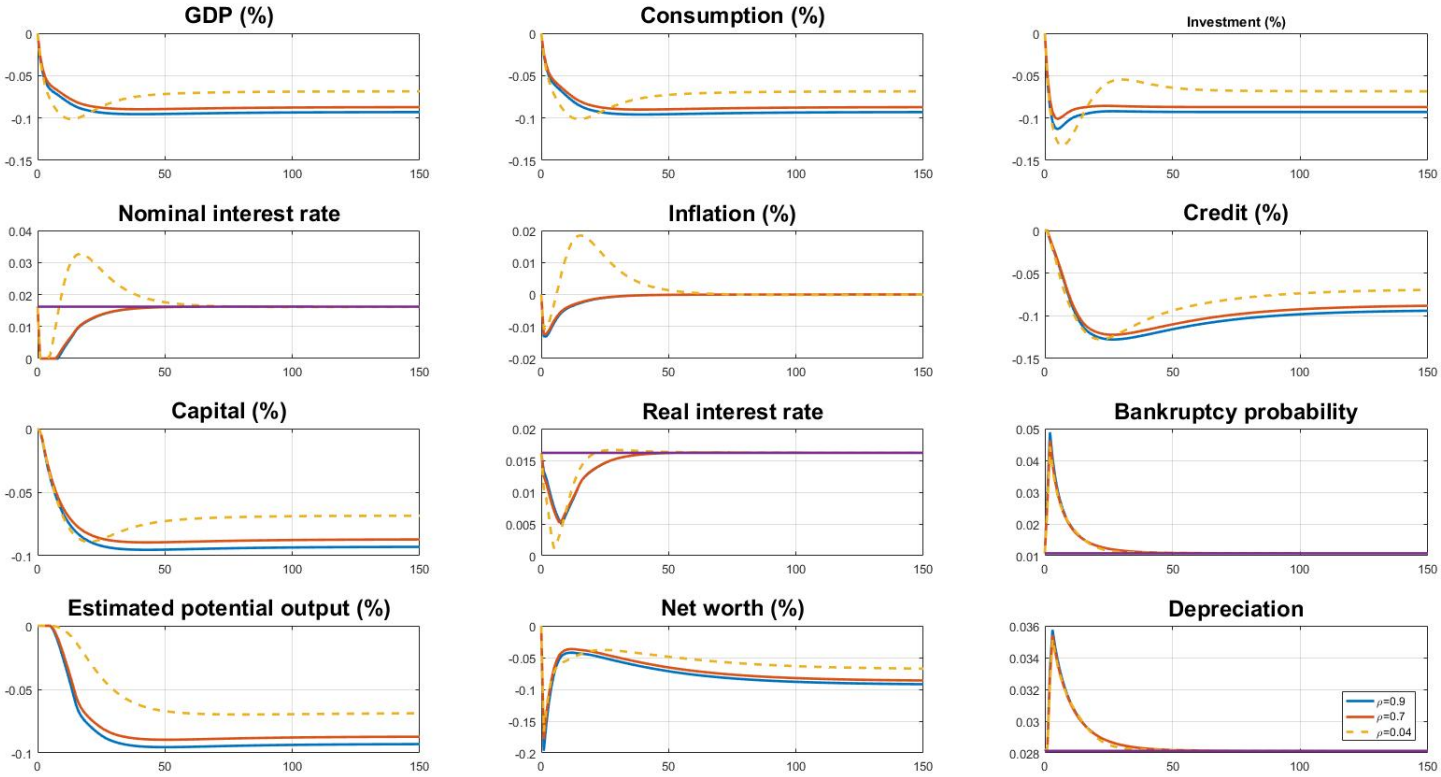

Figure 20: The Great Recession for different values of potential output revision intensity

Note that GDP, consumption and investment fall more as the shock hits when the revision of potential output is slower than baseline, *i.e.* the policy intervention is stronger. This is a consequence of the ZLB binding, as stronger demand stimulus generates higher inflation compared to the baseline, resulting in a sharp fall in the real interest rate with a binding ZLB. The latter provides stimulus to the marginal utility of consumption, but results in a sharper contraction in the labour supply, negatively affecting the level of economic activity. As soon as the nominal interest rate leaves the ZLB, the real interest rate re-bounces and monetary policy generates a partial recovery.

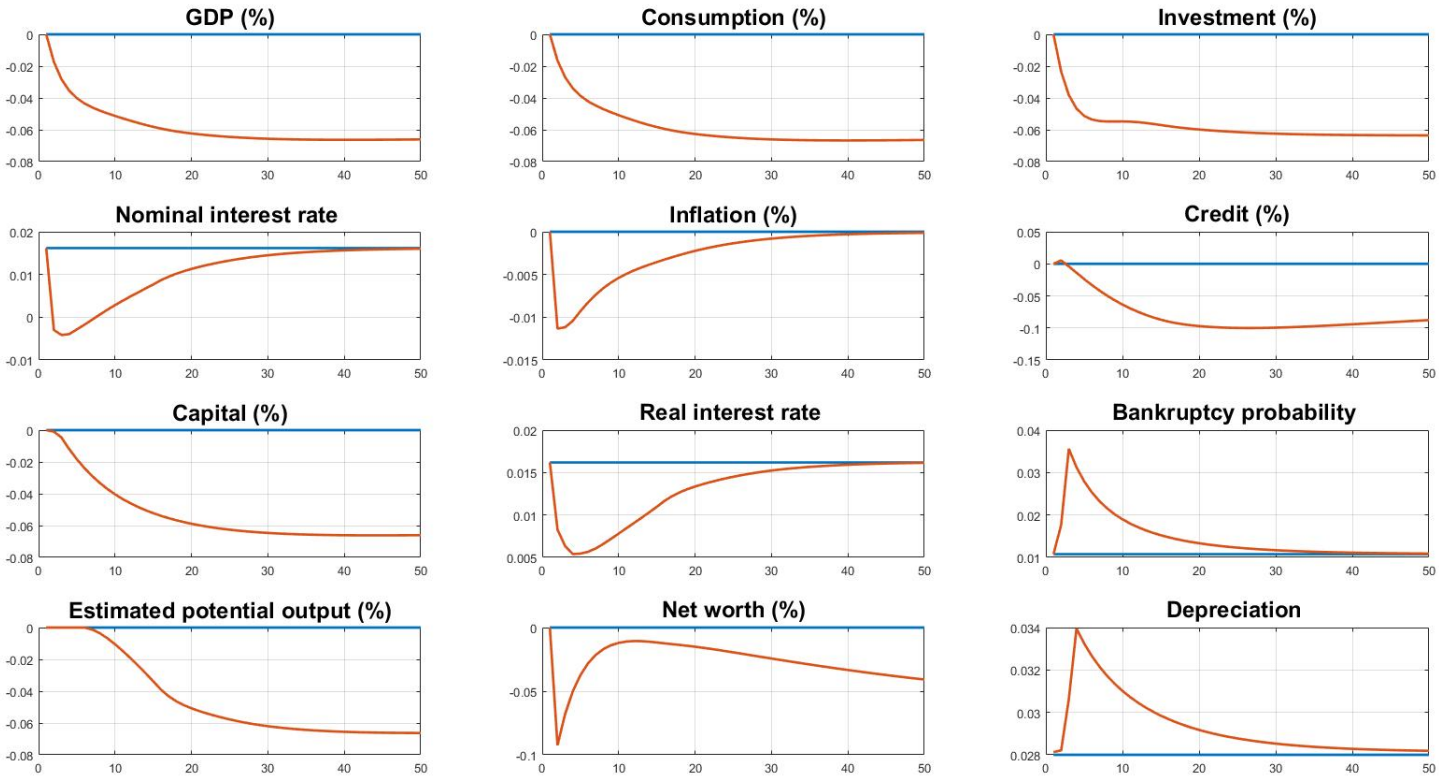

Figure 21: The Great Recession: Quarterly model simulations, without the ZLB constraint

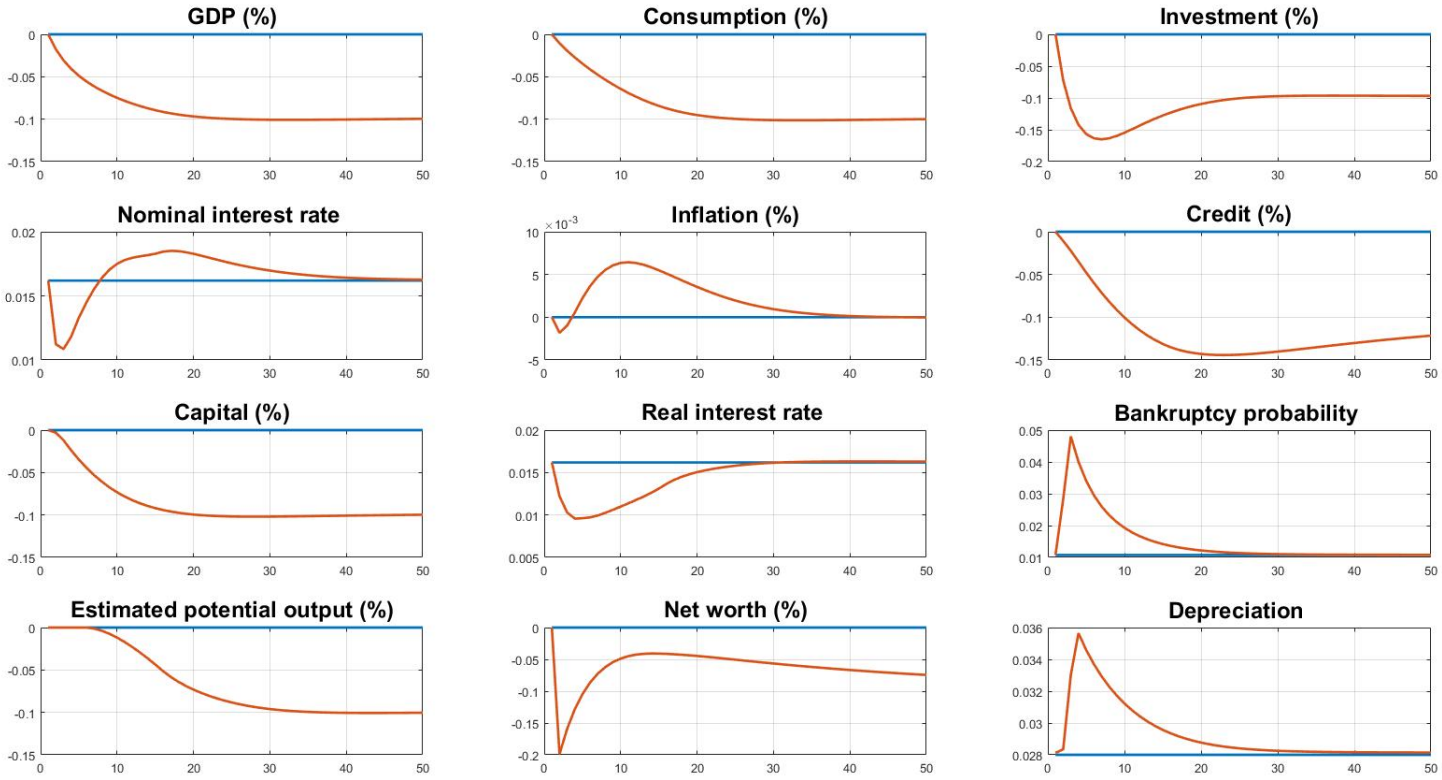

Figure 22: The risk shock: Quarterly model simulations

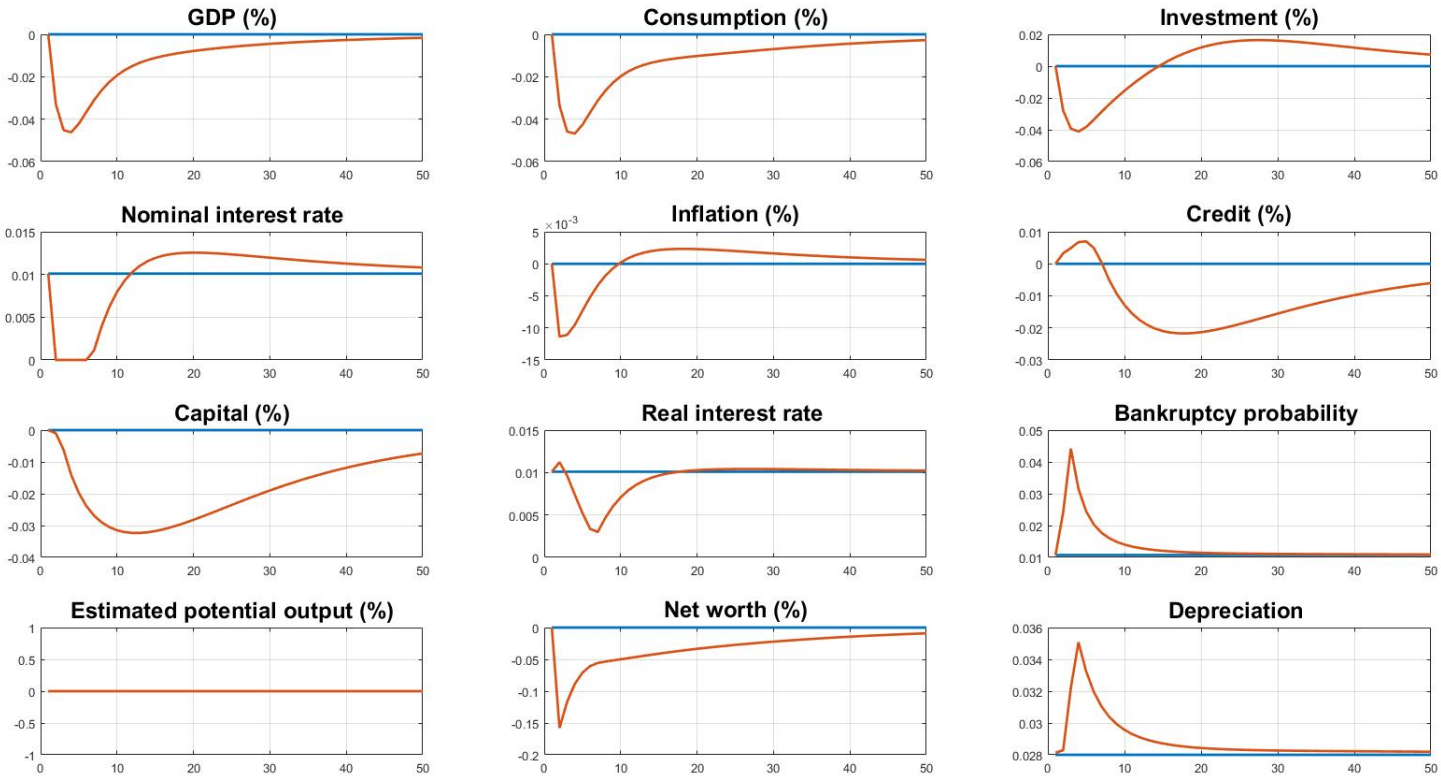

Figure 23: The Great Recession shock with a quasi standard Cobb-Douglas technology and no potential output revisions

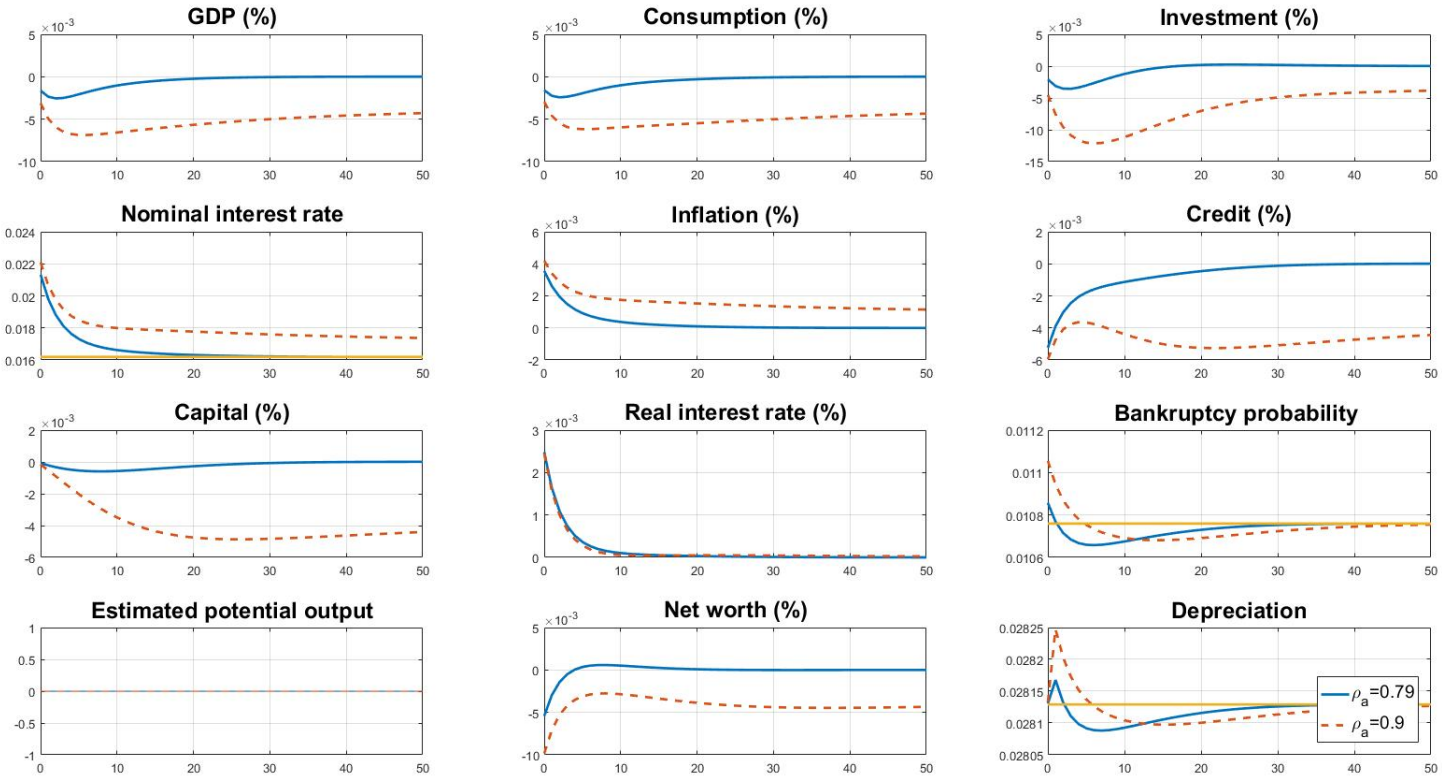

Figure 24: TFP shock for different values of persistence: Baseline calibration

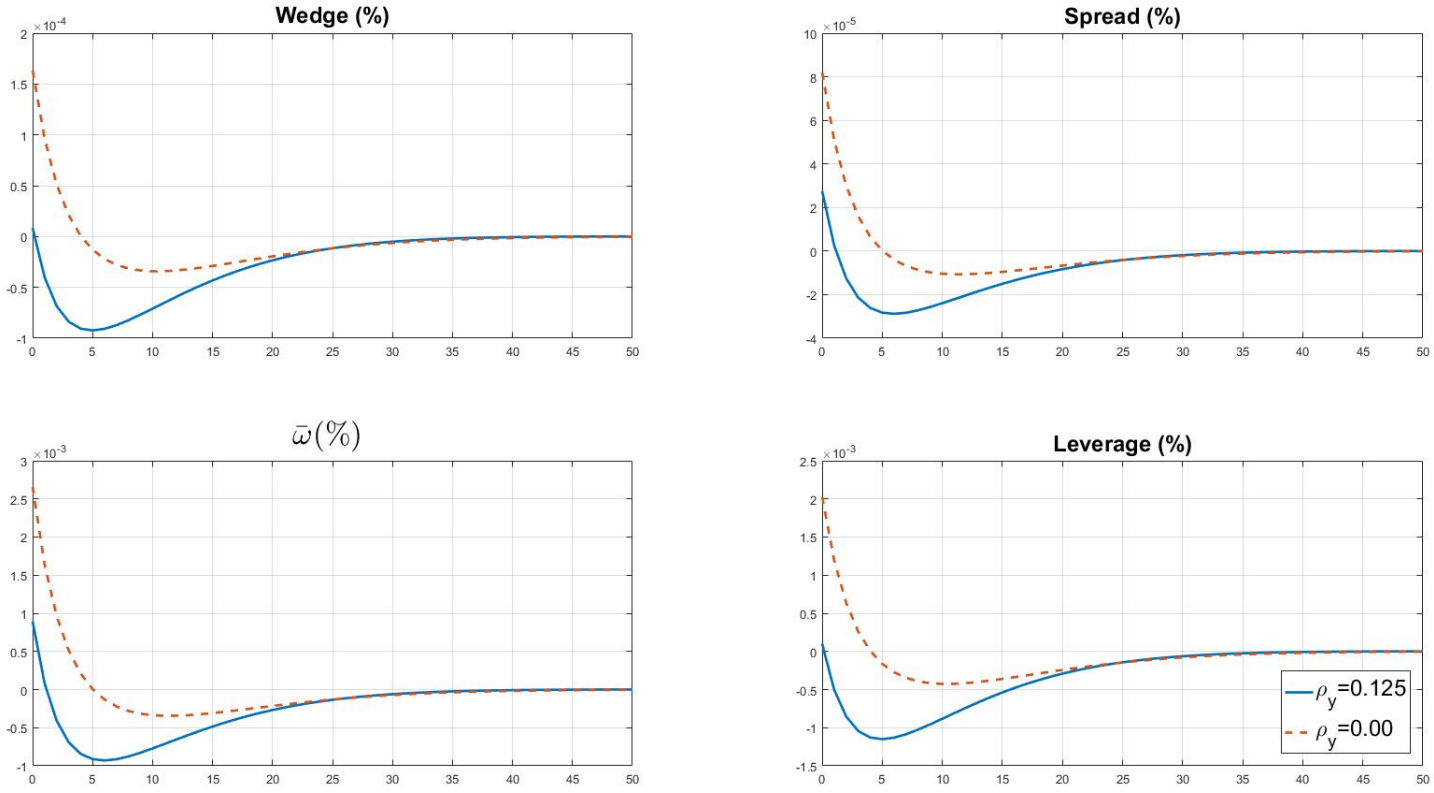

Figure 25: TFP shock: Additional variables

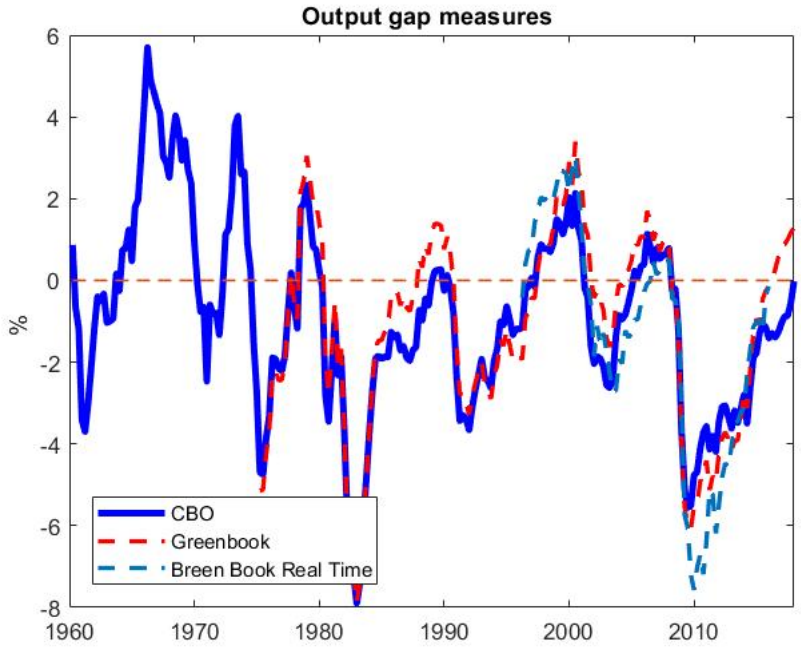

Figure 26: Alternative output gap measures

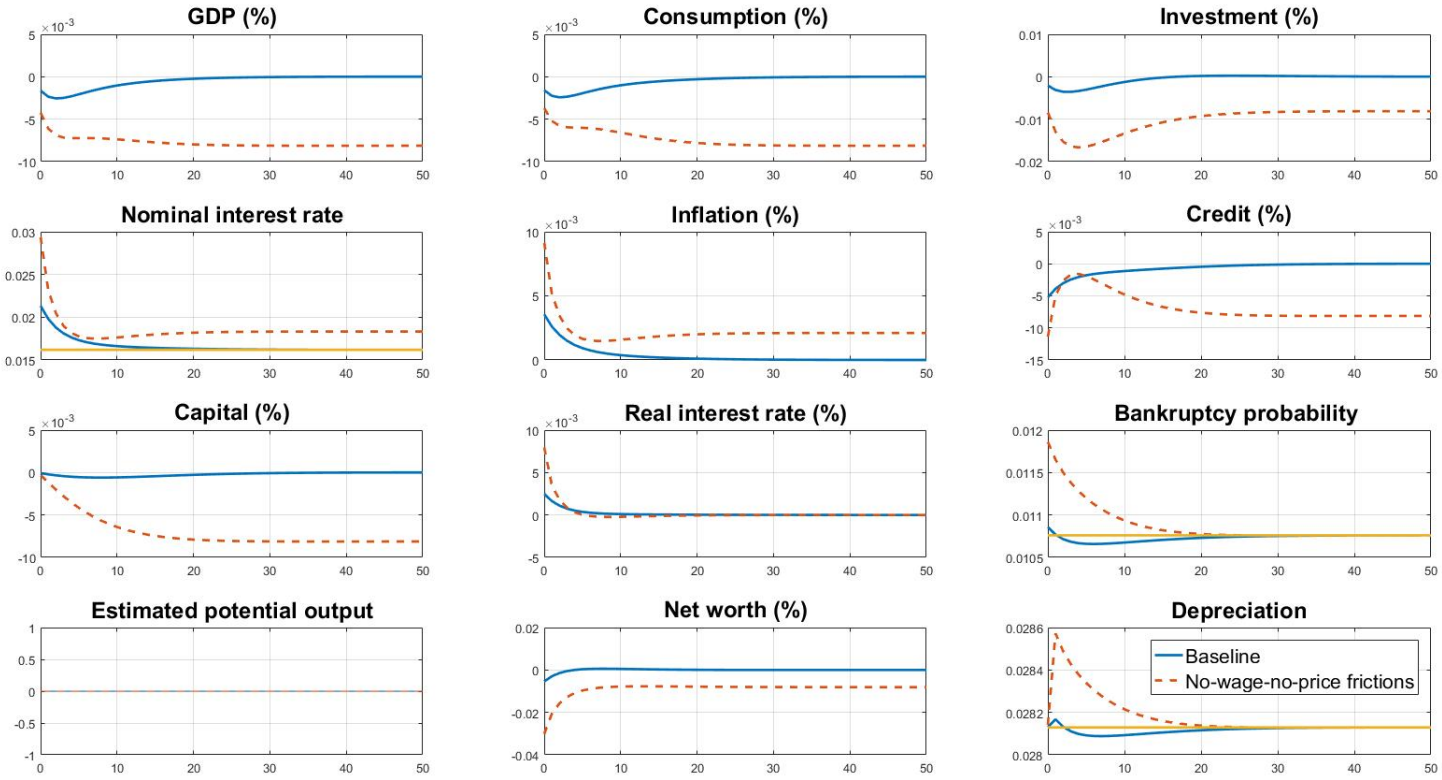

Figure 27: TFP shock: Baseline vs No-wage-no-price frictions

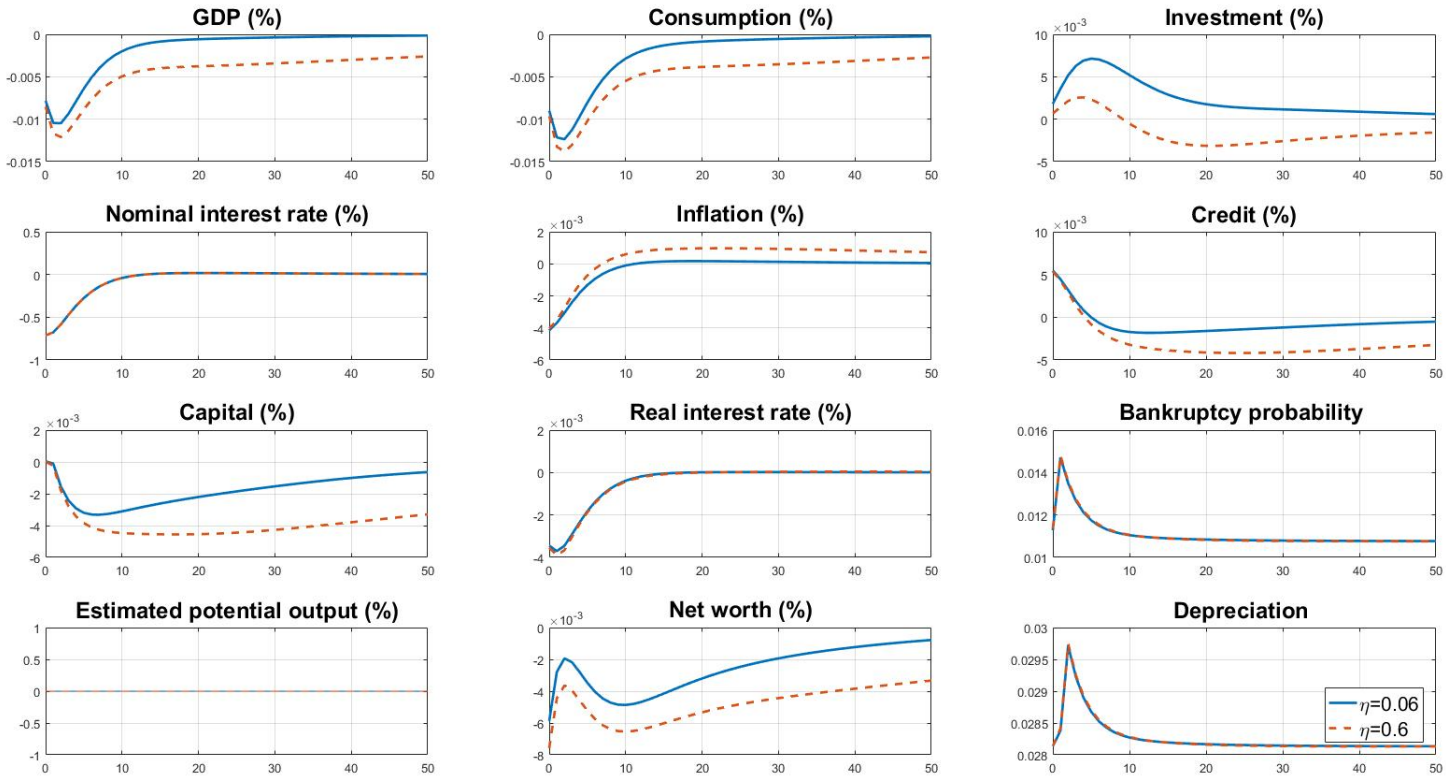

Figure 28: Relaxing the AK assumption: small demand shock for different intensities of knowledge spillovers

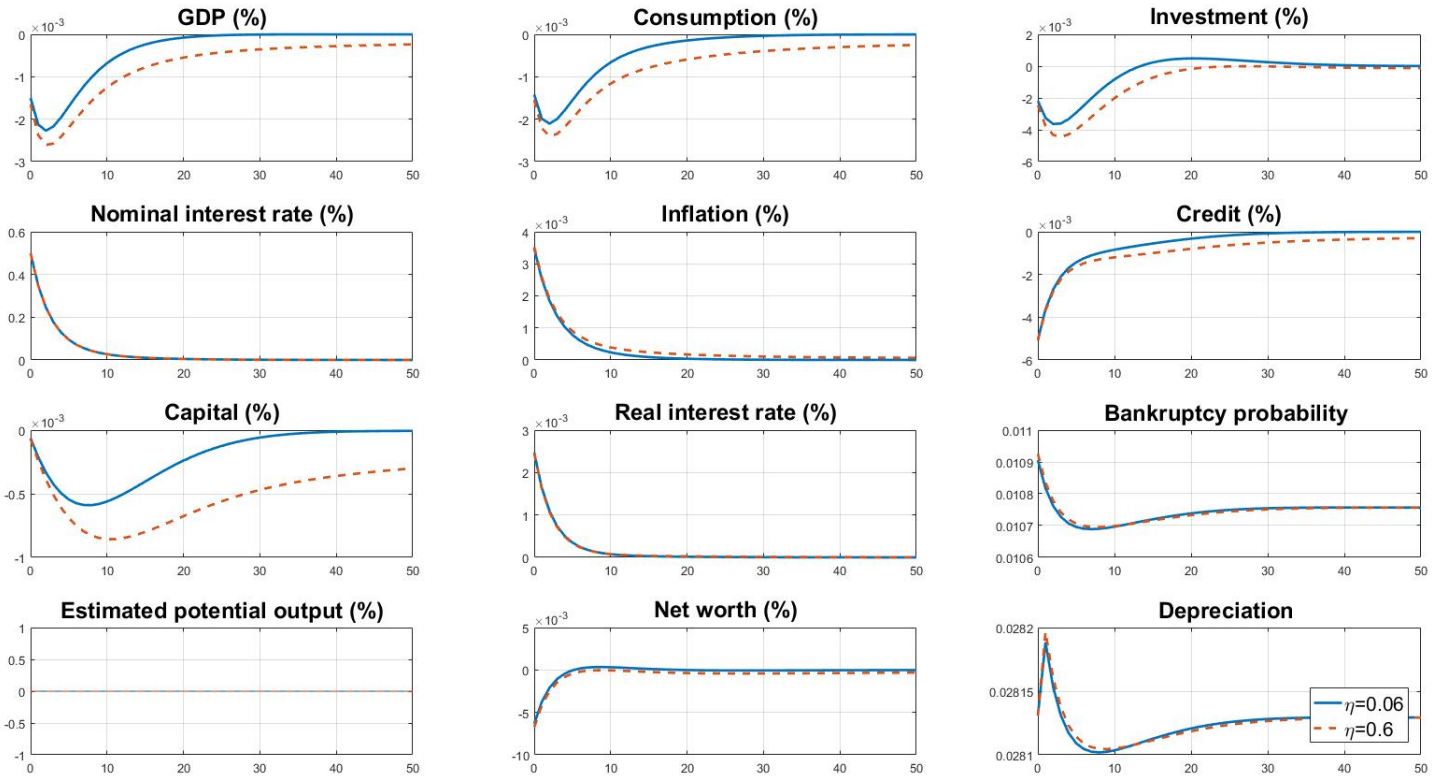

Figure 29: Relaxing the AK assumption: TFP shock for different intensities of knowledge spillovers

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