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Green and brown returns in a production economy

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Challenges for Monetary Policy Transmission in a Changing World Network (ChaMP)

This paper contains research conducted within the network "Challenges for Monetary Policy Transmission in a Changing World Network" (ChaMP). It consists of economists from the European Central Bank (ECB) and the national central banks (NCBs) of the European System of Central Banks (ESCB).

ChaMP is coordinated by a team chaired by Philipp Hartmann (ECB), and consisting of Diana Bonfim (Banco de Portugal), Margherita Bottero (Banca d'Italia), Emmanuel Dhyne (Nationale Bank van België/Banque Nationale de Belgique) and Maria T. Valderrama (Oesterreichische Nationalbank), who are supported by Melina Papoutsi and Gonzalo Paz-Pardo (both ECB), 7 central bank advisers and 8 academic consultants.

ChaMP seeks to revisit our knowledge of monetary transmission channels in the euro area in the context of unprecedented shocks, multiple ongoing structural changes and the extension of the monetary policy toolkit over the last decade and a half as well as the recent steep inflation wave and its reversal. More information is provided on its website.

Abstract

Does it pay to invest in green companies? In countries where a market for carbon is functioning, such as those within the European Union, our findings suggest that it should be beneficial. Using a sample of green and brown European firms, we initially demonstrate that green companies have outperformed brown ones in recent times. Subsequently, we develop a production economy model in which brown firms acquire permits to emit carbon into the atmosphere. We find that the presence of a well-functioning carbon market could account for the green equity premium observed in our data. Incorporating a preference for green financial assets is also unlikely to overturn our results.

JEL: E32, Q51, G18

Keywords: Asset Pricing, Composite Habits, Equity Premium, General Equilibrium, Monopolistic Competition

Non-technical Summary

Does it pay to invest in green companies? According to this article, if carbon is appropriately priced, the long-term answer is yes. Using a sample of green and brown European companies, we initially observe that from 2013 to 2022, companies with lower emissions have earned higher returns compared to high-emission firms. This finding suggests the existence of a green equity premium.

Given that Europe hosts the world's largest carbon market, the European Union's Emission Trading System (EU ETS), we examine whether carbon pricing can account for documented differences in returns. Indeed, the EU ETS aims to establish a price for carbon emissions by requiring brown companies to acquire pollution permits or emissions allowances to release greenhouse gases into the atmosphere.

Our argument relies on the premise that a well-designed carbon market generates pro-cyclical fluctuations in the carbon price. As emissions are influenced by economic activity, they tend to increase during economic booms and decrease during downturns. Since the carbon price should reflect the social cost of emissions, it should rise during booms when emissions are higher. As documented in our analysis, we find that the price of allowances in Europe is indeed positively correlated with the business cycle, aligning with theoretical predictions.

How does this impact the returns of green and brown firms? A procyclical price of carbon implies that the cost borne by brown companies to purchase permits increases during periods of economic strength. Consequently, a procyclical price of carbon helps to smooth the profits and dividends of brown companies, as their costs rise in good times and decrease during recessions.

From a risk perspective, carbon pricing acts as an insurance mechanism by reducing the procyclicality of dividends in the brown sector. With a carbon market in place, the decline in dividends received by an investor holding a brown portfolio during economic downturns, characterized by low consumption and high marginal utility, is attenuated. Consequently, compared to investing in green companies unaffected by such regulation, this effect should reduce the compensation demanded by investors to hold brown stocks.

Incorporating a preference for green financial assets is unlikely to overturn our results, as an explanation based solely on preferences for assets cannot exclude bonds from the analysis. In a

model encompassing both equities and bonds, we show that introducing an aversion for brown financial assets generates offsetting effects on relative returns, making an explanation solely based on preferences an unlikely candidate.

1 Introduction

Does it pay to invest in green companies? According to this article, if carbon is appropriately priced, the long-term answer is yes. Using a sample of green and brown European companies, we initially observe that from 2013 to 2022, companies with lower emissions have earned higher returns compared to high-emission firms. This finding suggests the existence of a green equity premium.

However, a green equity premium is difficult to reconcile with conventional explanations. If investors are indeed hesitant to invest in brown companies, this reluctance should lead to higher returns for these firms as investors demand greater compensation to hold stocks of companies they dislike. Based on basic asset pricing principles, we would expect to observe a carbon premium instead, where brown stocks outperform green stocks.

Given that Europe hosts the world's largest carbon market, the European Union's Emission Trading System (EU ETS), we examine whether carbon pricing can account for documented differences in returns. Indeed, the EU ETS aims to establish a price for carbon emissions by requiring brown companies to acquire pollution permits or emissions allowances to release greenhouse gases into the atmosphere. Assuming the existence of a well-functioning market, our main contention is that the green equity premium observed in the data can be explained by environmental regulation.

Our argument relies on the premise that a well-designed carbon market generates procyclical fluctuations in the carbon price. As emissions are influenced by economic activity, they tend to increase during economic booms and decrease during downturns. Since the carbon price should reflect the social cost of emissions, it should rise during booms when emissions are higher. In Section 2, as documented in our analysis, we find that the price of allowances in Europe is indeed positively correlated with the business cycle, aligning with theoretical predictions.

How does this impact the returns of green and brown firms? A procyclical price of carbon implies that the cost borne by brown companies to purchase permits increases during periods of economic strength. Consequently, a procyclical price of carbon helps smooth the profits and dividends of brown companies, as their costs rise in good times and decrease during recessions.

From a risk perspective, carbon pricing acts as an insurance mechanism by reducing the procyclicality of dividends in the brown sector. With a carbon market in place, the decline in dividends received by an investor holding a brown portfolio during economic downturns, characterized by low consumption and high marginal utility, is attenuated. Consequently, compared to investing in green companies unaffected by such regulation, this effect should reduce the compensation demanded by investors to hold brown stocks.

We formalize this mechanism by developing a two-sector neoclassical growth model comprising a green and a brown sector. Drawing from insights gained in finance over the past decades, we analyze this question within a framework that generates realistic asset pricing implications. Our baseline two-sector economy generates an aggregate equity premium exceeding 6%, while simultaneously replicating the low mean risk-free rate observed in recent years as well as the volatility of aggregate profits.

In comparison to a standard two-sector neoclassical growth model, our key innovation is the introduction of a market for carbon, where the price and quantity of emissions permits are endogenously determined. The distinguishing factor between the two sectors is that brown firms, responsible for emissions, are constrained by the quantity of permits they can purchase.

We demonstrate that the magnitude of the green equity premium predicted by the model depends on the strictness of environmental regulations. By setting an average price of 12 euros per ton, the model broadly reproduces the disparity in returns between green and brown firms observed in the data. Without a price on carbon emissions, the two sectors exhibit symmetry, and the green premium generated by the model disappears. Therefore, optimal carbon pricing implemented through a decentralized market for emissions allowances provides a potential solution to the puzzle.

Furthermore, our framework allows for the examination of how investors' preferences for green and brown assets influence relative returns. Consistent with standard theory, introducing aversion to brown equities reduces the green premium generated by the benchmark model. Depending on the value of this aversion parameter, we find that this stigma effect can eliminate the green premium resulting from carbon pricing.

Considering that aversion towards brown equities is captured by a preference parameter, quantifying the exact effect is challenging. However, since stigma reduces the market value of the brown sector, an indirect measure can be utilized. Specifically, we can calculate the reduction in market value induced by stigma that is required to eliminate the green premium predicted by the benchmark model, which reproduces a set of ten macro-finance facts.

Starting from the benchmark scenario, we find that the stigma effect would need to decrease the market value of the brown portfolio by around 26% to reduce the green premium from 2% to 0%. Our framework could also generate a 1% carbon premium if we assume that investors' aversion for brown equities reduces the market value of that sector by 38%. Nevertheless, to our knowledge, there is no evidence suggesting that an aversion for brown stocks could have led to such a dramatic reduction in the market value of companies operating in that sector.

Since our model incorporates leverage, we further analyze how a shift in preference from brown to green bonds affects this return differential. We are motivated by the recent decision by the European Central Bank to shift its portfolio of corporate bonds away from brown companies. Starting from the benchmark model, which assumes similar financial structures across sectors, we find that a preference for green bonds increases the green premium by stimulating the issuance of green bonds relative to brown bonds. While a distaste for brown companies reduces the green premium, a preference for green bonds therefore has the opposite effect.

In summary, introducing a preference for green financial assets, whether in the form of a stigma towards brown stocks or a tilt away from brown bonds, is unlikely to overturn the effect of carbon pricing. Our findings suggest that stigma has a limited impact on returns, as an implausibly large decline in the market value of the brown sector is necessary to generate a carbon premium.

Furthermore, an explanation based solely on preferences for assets cannot exclude bonds from the analysis. Since leverage increases risk premiums, introducing a preference for green bonds counterbalances the potential stigma effect from holding stocks of high-emission firms. In a model encompassing both equities and bonds, introducing an aversion for brown financial assets thus generates offsetting effects on relative returns, making an explanation solely based on preferences an unlikely candidate. And indeed, actual data on leverage suggests the existence of a brown premium rather than a green premium as brown firms have historically been more leveraged than green firms.

Our work contributes to the ongoing debate on the relative performance of green and brown equities, fueled by the remarkable rise of green finance. One line of research indicates that brown companies tend to generate higher returns compared to green companies. For instance, in an examination of stock returns in the United States, Bolton and Kacperczyk (2021) discovered that investors already demand compensation for holding stocks of firms that emit carbon dioxide (CO2) into the atmosphere. This phenomenon, referred to as a "carbon premium", implies that investors are already pricing in carbon risk. In a subsequent study, Bolton and Kacperczyk (2023) extend their analysis to include a sample of 77 countries and 14,400 companies worldwide. They find that the carbon premium is more pronounced in countries with stricter climate policies.

A related study conducted by Hsu et al. (2023) provides evidence of a pollution premium based on data from the Toxic Release Inventory database, which tracks toxic emissions. The authors develop a model that formalizes the mechanism highlighted in their empirical analysis. According to their findings, the primary source of risk stems from environmental regulations, which decrease toxic emissions and subsequently impact firms' profitability.

Pástor et al. (2022) provide an explanation for the recent disparity between green and brown stocks by introducing a green factor that incorporates environmental concerns. This risk factor accounts for why green stocks outperform brown stocks during periods of heightened climate concerns. However, in line with the model developed by Pástor et al. (2021), green assets have lower expected returns in the long run. This difference in average returns can be attributed to investors' preferences for green assets, as supported by empirical studies linking stock returns to investor preferences. In particular, Hong and Kacperczyk (2009) demonstrate that firms operating in sectors perceived as unethical are less frequently held by institutions subject to norms, such as pension funds. The higher expected returns of these stocks align with the notion that investors are hesitant to include such companies in their portfolios.

Bauer et al. (2022) find that green stocks generally exhibit higher realized returns compared to brown stocks. The authors provide international evidence by analyzing data from the G7 countries spanning from 2010 to 2021. It is worth noting that the differences in findings between Bauer et al. (2022) and Bolton and Kacperczyk (2021, 2023) could potentially be attributed to variations in sample selection across the studies. Additionally, the classification of firms as green or brown, whether based on CO2 emissions level or emission intensity, significantly impacts the results. Nonetheless, Bauer et al. (2022) found a green equity premium when incorporating both measures of greenness.

Studying the returns of green and brown assets in theoretical models presents challenges that may not be immediately apparent. One major obstacle is replicating returns of a realistic magnitude, a well-known challenge initially documented by Mehra and Prescott (1985). In contrast to the approaches presented by Bansal and Yaron (2004), Campbell and Cochrane (1999) or Constantinides (1990), we investigate equity returns in an economy with production, building upon the seminal work of Jermann (1998).

Relative to Jermann (1998), or Campanale et al. (2010), who resolve the puzzle by combining Chew-Dekel preferences with capital adjustment costs, we develop a two-sector economy in which labor supply in both sectors is endogenously determined. By incorporating adjustment costs alongside slow-moving composite habits (e.g. Jaccard (2014)), we are able to generate a sizeable equity premium, a low mean risk-free rate, and significantly reduce fluctuations in the risk-free rate, in contrast to what is typically obtained in models with consumption habits (e.g. Jermann (1998); Boldrin et al. (2001)). The volatility of the risk-free rate produced by our mechanism falls within the range of values documented by Jordà et al. (2019), who provide international evidence on financial returns using long data samples.

It is also worth mentioning that our framework can generate large and volatile movements in stock prices. As documented by Croce (2014), who combines long-run risk (e.g. Bansal and Yaron (2004)) with Epstein-Zin-Weil preferences (e.g. Epstein and Zin (1989); Weil (1989); Weil (1990)), resolving the volatility puzzle within this class of models remains a formidable challenge. Generating this high volatility of stock prices is also not an issue for models that rely on consumption habits (e.g. Jermann (1998); Boldrin et al. (2001)). We also develop a version of the model with Epstein-Zin-Weil preferences and match it to the data. While the model is able to resolve the volatility puzzle, the fit to the data is less good compared to the model with composite habits.

We depart from the existing literature by investigating relative returns within a model that not only captures the volatility of aggregate dividends but also the slope of the term structure of interest rates. The equity premium is composed of two components: a term premium, which reflects the impact of interest rate risk on the valuation of risky assets, and a risk premium that is influenced by the cyclicality of dividends (e.g. Jermann (1998); Abel (1999)). As the difference in relative returns is primarily driven by the latter component, it is crucial to examine green and brown returns within a framework that can account for this decomposition.

This article also contributes to the growing literature on the interaction between business cycles and environmental policy, which is comprehensively surveyed in Annicchiarico et al. (2021). One of the pioneering studies in this field is the work of Fischer and Springborn (2011), who analyze the impact of emission caps on the business cycle. They demonstrate that caps mitigate the effects of productivity shocks on macroeconomic variables. Building upon Fischer and Springborn's real business cycle model, Annicchiarico and Di Dio (2015) incorporate nominal rigidities and the role of monetary policy. They find that emission caps reduce the volatility of business cycle shocks, with their New Keynesian model highlighting the amplification of this effect through nominal rigidities.

In another line of research, Heutel (2012) introduces emissions into a real business cycle model and derives the optimal Ramsey policy. Assuming that emissions' damage affects the production side of the economy, the optimal tax is shown to be procyclical. Benmir et al. (2022) find similar results in a model where the environmental externality enters the utility function, indicating that the optimal tax should also be procyclical. The distinction from Heutel (2012) lies in the fact that this procyclicality is driven by the stochastic discount factor (SDF) used by agents to price assets.

Furthermore, Känzig (2023) utilizes the EU ETS market to construct a measure of carbon policy shocks, revealing their substantial impact on macroeconomic variables and distributional implications. This underscores the significance of environmental policies not only in the long term but also at business cycle frequency. While Kanzig's study employs a two-agent model, we endogenize the price of emissions allowances through the introduction of a decentralized carbon market.

To our knowledge, our study represents the first attempt to integrate the asset pricing and environmental literatures within a business cycle model, enabling us to examine the dynamics of green and brown returns.

2 Green vs. brown returns

This section presents empirical evidence on the relative performance of green and brown stocks in the euro area. We do so for two purposes: firstly, to inform the calibration of our model, and secondly, to provide a rationale for the mechanism analyzed in our model.

2.1 Data and filters

In our empirical analysis, we utilize five databases, with the first three databases providing firm-level data. The first database we rely on is Thomson Reuters Refinitiv, which offers comprehensive data on firm-level market capitalization, net sales, and equity prices. The second database we utilize is ISS-ESG, which provides carbon emissions data. Compared to other sources, this database demonstrates superior coverage (see Meinerding et al. (2023)). The third one, OBRIS, offers firm-level data on leverage measured via the ratio of total liabilities to assets.[1]

The fourth database we incorporate is dedicated to capturing the price of emissions allowances traded on the EU ETS. Lastly, the fifth database, provided by Eurostat, supplies data on real GDP specific to the euro area.

In our analysis, we utilize the original levels of the firm-level market capitalization, net sales, the total-liabilities-to-assets ratio, and carbon emissions data in their original levels. For the remaining variables, we calculate four-quarter growth rates. This approach allows us to eliminate higher frequencies that are inherent in quarterly fluctuations, following a similar methodology employed in the study by Stock and Watson (2005).

By employing this technique, we ensure that our empirical analysis primarily captures fluctuations at business cycle frequencies. This alignment with the business cycle is essential for effectively matching the empirical data with our model and maintaining coherence between the two.

There are various approaches available to extract fluctuations at business cycle frequencies. One common method is the use of band-pass filters, which isolate movements within the desired frequency range. However, when dealing with relatively short time spans, such as the one in our analysis of brown and green portfolios, there may be end-point issues that can affect the analysis of correlations (see e.g., Wolf et al. (2024)).

Another option is the utilization of the Hamilton filter, which involves an 8-quarter ahead forecast error to capture business cycle movements. However, this technique necessitates a two-year data

[1] Total liabilities refer to non-current liabilities (e.g. long-term debt) and current liabilities, including loans, creditors, and other current liabilities.

span for filter initialization, further reducing the number of observations in our already limited sample. Additionally, as pointed out by Schüler (2024), the varying filter weights across different time series can distort the analysis of co-movements between the series.

Given these considerations, we opt for a simpler approach using four-quarter growth rates. While this idea is related to Hamilton's concept, it employs constant filter weights and requires fewer time-series observations for initialization. This approach allows us to capture the desired business cycle fluctuations while mitigating potential issues associated with the other filtering methods mentioned.

2.2 Construction of green-minus-brown portfolios

We form long-short portfolios by sorting firms based on their carbon emissions, taking inspiration from the approach outlined in Pástor et al. (2022), Meinerding et al. (2023), and Bauer et al. (2022). Specifically, we focus on Scope 1 emissions, which pertain to the direct CO2 emissions originating from a firm's owned or controlled resources. Scope 1 emissions are considered the most accurate measurement in comparison to Scope 2 emissions (indirect emissions from purchased energy) and Scope 3 emissions (all other emissions not covered by Scope 1 and 2, including those arising from value chains), which we exclude from our analysis.

It is worth noting that we do not consider Scope 1 emissions intensities, which are defined as the ratio of direct emissions to total revenues. This exclusion is justified because total revenues can serve as a proxy for business cycles. Consequently, emission intensities might exhibit correlations with the business cycle purely due to this standardization.[2]

Using the Scope 1 emissions data, we construct decile portfolios. The first decile portfolio represents the green portfolio, consisting of firms with the lowest carbon emissions, while the tenth decile portfolio represents the brown portfolio, consisting of firms with the highest carbon emissions.

To compare the relative performance of green and brown stocks, our empirical approach takes publication lags into account. Specifically, there is a lag in the reporting of firms' carbon emissions,

[2]One may raise the concern that focusing on Scope 1 emissions instead of Scope 1 emission intensities may mean that our green portfolio returns are particularly driven by small firms' returns, as these firms tend to have lower emissions. However, we form value-weighted portfolios, meaning that larger companies receive a higher weight. Furthermore, we check that our green portfolio also includes larger firms, such as EDF.

which in turn affects the sorting of portfolios by investors. The emissions data for a given year is typically published with a delay of up to six months.

To address this issue, we initiate our portfolio construction in July 2013. This starting point allows us to utilize emissions data from the previous year, taking into consideration the publication lag. We conclude our sample at the end of June 2022, incorporating emissions data up to the year 2020. This approach enables us to construct new portfolios each July, utilizing emissions data from the preceding year.

It is important to note that our sample size varies over time due to changes in the inclusion of firms and the availability of emissions data. At the beginning of the sample, we have a total of 1,597 firms included, and this number increases to 2,649 by the end of the sample period.

To calculate the aggregated returns of the green and brown portfolios, we adopt a value-weighted approach. This involves assigning weights to individual firms' returns based on their market capitalization. Specifically, each firm's return is divided by the sum of market capitalizations of all the firms within the respective portfolio, resulting in a normalized weight.

Given that our model operates at a quarterly frequency, we need to convert the daily data into quarterly data. To accomplish this, we use the last available value of each quarter as a representative value. This allows us to align the data with the quarterly nature of our analysis and facilitates consistent comparisons and calculations.

Finally, we derive the remaining statistics, including market capitalization, net sales, and the leverage ratio of the green and the brown portfolio by applying the above methodology similarly. Specifically, identical firm-level weights are employed across each portfolio to compute an aggregated figure for these measures.

2.3 Empirical facts

Table 1 presents the key statistics for our model. It provides valuable insights into the relative performance and characteristics of the green and brown portfolios. Notably, the table demonstrates that the green portfolio has outperformed the brown portfolio throughout our sample period. Specifically, the four-quarter return for the green portfolio is 5.52%, while for the brown portfolio it is 3.52%. This implies a green premium of the green-minus-brown portfolio of 2.01%,

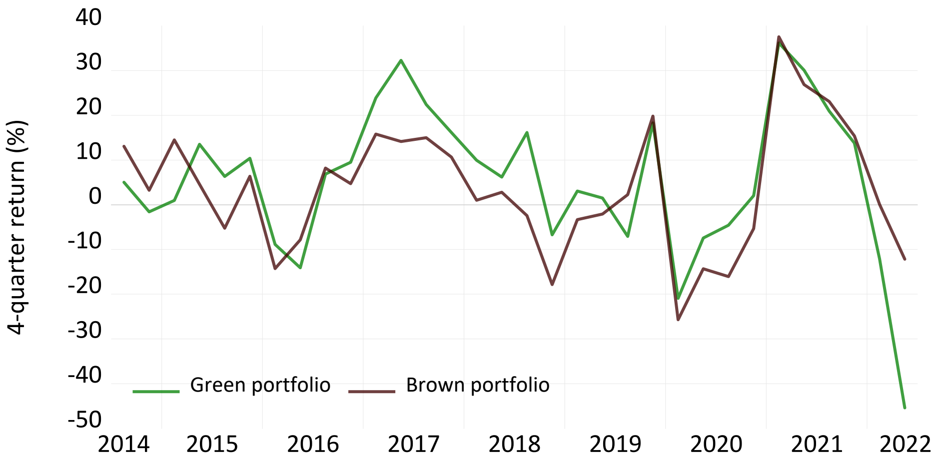

Figure 1: The 4-quarter return of green and brown companies over the last decade.

highlighting the superior performance of green stocks.

This finding is in line with previous studies that have employed a similar approach, such as Bauer et al. (2022) and references therein, although those studies have focused on different geographical regions. Thus, our analysis provides initial evidence supporting the existence of a green premium within the euro area.

Furthermore, the standard deviation of the green portfolio has been higher than that of the brown portfolio, with values of 16.65% and 14.12% respectively. These figures indicate that the green portfolio exhibits greater volatility compared to the brown portfolio. To visualize the performance and correlation between the two portfolios, we provide Figure 1, which displays their respective time series. The figure illustrates a high degree of correlation between the two portfolios, with a correlation coefficient of 0.80.

Examining the performance patterns, it is evident that the green portfolio outperformed the brown portfolio, particularly in the first half of the sample period. However, the onset of the COVID-19 pandemic and the geopolitical events, such as the war in Ukraine and its impact on energy prices, resulted in a more equal or even poorer performance of the green portfolio in subsequent periods. These external factors contributed to the fluctuations in performance observed between the two portfolios over time.

Table 1: Green and brown portfolio characteristics, 2013Q3-2022Q2

	Green	Brown	Difference/Ratio
4-quarter returns (avg.; %)	5.52	3.52	2.01 (diff.)
Std.dev. of 4-quarter returns (%)	16.65	14.12	1.18 (ratio)
Stock market cap. (avg. level; mio euros)	14412.86	50444.96	0.29 (ratio)
Value added/Net sales (avg. level; mio euros)	11.32	51.83	0.22 (ratio)
Total liabilities-to-assets ratio (%)	0.49	0.63	0.77 (ratio)
Std.dev. of total liabilities-to-assets ratio (%)	0.11	0.01	8.77 (ratio)

Notes: The sample period of the net sales data is shorter. Therefore, we can only show net sales for the sample spanning from 2013Q3-2021Q2. "Std.dev." denotes standard deviation.

In addition to the returns and volatility, we also consider the market capitalization and net sales of the stocks to calibrate our model. The comparison reveals that the brown portfolio has significantly larger market capitalization and net sales compared to the green portfolio. Specifically, the market capitalization and net sales of the brown portfolio are substantially higher, with the green portfolio accounting for less than 30% of the market capitalization and net sales of the brown portfolio. Furthermore, the data reveals that brown companies have historically exhibited higher leverage ratios than green companies (average: 0.63 vs. 0.49; standard deviation: 0.01 vs. 0.11, respectively). We use this observation to enrich the discussion on alternative mechanisms that may compete with or complement our proposed model (see Section 7).

In order to provide empirical support for our model mechanism, we proceed by presenting correlation coefficients of the four-quarter growth rates in Table 2. These correlations shed light on the relationship between changes in the price of carbon and the performance of the green-minus-brown portfolio at business cycle frequencies. Our model mechanism is built on two key assumptions: (i) changes in the price of carbon are procyclical, meaning they tend to move in the same direction as the business cycle, and (ii) changes in the price of carbon and the green-minus-brown portfolio are positively related.

The table confirms these assumptions by displaying the correlation coefficients. Firstly, it demonstrates that the growth rate of the price of carbon is indeed procyclical, showing a positive correlation with the business cycle. Secondly, it reveals a positive correlation between the growth rate of the price of carbon and the growth rate of the green-minus-brown portfolio, indicating that a higher price of carbon is associated with a relatively worse performance of the brown firm compared to the green firm.

Table 2: Correlations of four-quarter growth rates, 2013Q3-2022Q2 (2013Q3-2021Q1)

	log(GDP)	Green-minus-brown portfolio
log(ETS)	0.27 (0.25)	0.07 (0.18)

Importantly, the table also highlights the impact of specific events on these correlations. In particular, the Russian invasion of Ukraine and the subsequent energy price shock have influenced the relationships. When excluding these events, the positive correlation between the growth rate of the price of carbon and the growth rate of the green-minus-brown portfolio becomes even stronger, as indicated by the statistics in brackets.

It is important to acknowledge that several factors need to be considered when interpreting these results. One crucial factor is the stage of development of the EU ETS during our sample period. The EU ETS has undergone various phases, each characterized by different allowances supplies and market dynamics.

In the initial years of our sample period, particularly in 2013, allowances were abundant, and their supply decreased linearly each year until the fourth phase of the EU ETS in 2020. These changing dynamics and evolving market mechanisms can introduce complexities and uncertainties into the relationship between the price of carbon and the performance of green and brown portfolios.

Given this context, the observed correlations presented in Table 2 are consistent with our model mechanism. While only imperfectly correlated, the evidence still supports our assumption that changes in the price of carbon are procyclical and positively related to the green-minus-brown portfolio at business cycle frequencies.

3 The model

The key innovation of our model is the incorporation of a market for emissions allowances within a general equilibrium framework, where the price and quantity of these allowances are endogenously determined. In our model, the brown sector, which includes intermediate good producers, needs to acquire emissions allowances, which they purchase from the representative agent.

To maintain simplicity, we develop a baseline model that focuses on the regulatory differences between the green and brown sectors. While there may be other sources of asymmetry between these sectors, we discuss them as potential extensions to our model and explore their impact on returns.

Both sectors consist of final and intermediate good producers. The intermediate good sector is characterized by imperfect competition, with each producer having monopoly power over the variety they produce.

To facilitate the analysis of business cycle dynamics, we begin by discussing the detrended economy, where quantities grow at a rate γ. Our goal is to examine the business cycle effects, and thus we introduce the necessary adjustments to ensure the existence of a balanced growth path.

In contrast to existing environmental business cycle models, which typically incorporate the environmental externality into production or utility functions, we assume that the externality takes the form of a resource cost. In our baseline model, we assume that the price of carbon, determined by the market for emissions allowances, fully internalizes the environmental externality. This implies that the price of emissions allowances is optimal in the sense that a social planner could not improve the allocation achieved in the decentralized equilibrium.

We solve our model using pruned third order perturbation[3] in order to capture asymmetries and non-linearities which are particularly important for the asset pricing part.

3.1 Final brown good producers

The relation between the different varieties of the intermediate brown good y_{Bj} and the final brown good y_B can be expressed using the Dixit-Stiglitz aggregator. The final brown good y_{Bt} is produced by a final good sector that combines the different varieties of the intermediate brown good y_{Bj}. The aggregator is given as follows:

$$\left(\int_0^1 y_{Bjt}^{\frac{\kappa_B - 1}{\kappa_B}} dj \right)^{\frac{\kappa_B}{\kappa_B - 1}} \geq y_{Bt} \tag{1}$$

[3]We use the MacroModelling.jl package set out in Kockerols (2023) in Julia for our computations. To the best of our knowledge the efficient algorithms implemented in the package and in Julia, while not new, are the only publicly available codes allowing us to match the moments of our medium-sized model in reasonable time and with standard hardware.

where the parameter κ_B denotes the elasticity of substitution between each variety. The demand for each variety is obtained by choosing the quantity y_{Bj} that solves the following cost minimization problem:

$$\min_{y_{Bjt}} \int_0^1 z_{Bjt} y_{Bjt} dj$$

where z_{Bjt} is the relative price of the intermediate brown good, subject to the Dixit-Stiglitz aggregator given by equation (1).

The outcome of this cost minimization problem gives rise to the usual demand schedule for variety j:

$$z_{Bjt} = z_{Bt} \left(\frac{y_{Bt}}{y_{Bjt}} \right)^{\frac{1}{\kappa_B}} \tag{2}$$

3.2 Intermediate brown good producers

The dividend distributed by intermediate good producer j in period t is given as follows:

$$d_{Bjt} = z_{Bjt} y_{Bjt} - w_t n_{Bjt} - r_{Bt} k_{Bjt} - p_{Pt}(\gamma s_{Pjt+1} - s_{Pjt}) + p^B_{Bt} \gamma b^B_{jt+1} - b^B_{jt} - \gamma \iota b^B_{jt+1} + \rho^d_B d_{Bjt-1} \tag{3}$$

Dividends firstly depend on the revenue from selling their intermediate good to the final good sector described above. The intermediate good is produced via a Cobb-Douglas production function:

$$y_{Bjt} = k^{\alpha_B}_{Bjt} n^{1-\alpha_B}_{Bjt} \tag{4}$$

where k_B and n_B denote the number of hours worked and the capital stock, respectively. The parameter α_B is the share of capital in total valued added. Wages as well as the rental rate of capital are denoted by w_t and r, respectively. Since in equilibrium, input prices are common across all firms, we omit the index j.

What differentiates a brown intermediate good producer from its green counterpart is the need to purchase polluting permits. Production in the brown sector leads to carbon emissions, the quantity of which is regulated by the government. The quantity of permits purchased by firm j is denoted s_{Pj}. The price of carbon allowances faced by producer j is denoted by p_P, and is similar for all firms in the brown sector.

Each period, firms in the brown intermediate good sector issue short-term bonds b^B_{jt+1}, which are sold to households at the price p_B^B. At the same time, a coupon payment, which is denoted by b_{jt}^B, needs to be paid to service the cost of debt. Issuing new bonds also entails costs, which are captured by the parameter ι. The cost of issuing new debt is proportional to the amount of new issuance and is therefore given by ιb_{jt+1}^B.

A large strand of literature has documented the role of corporate policy on the dynamics of dividends. In particular, the fact that managers tend to smooth dividend payments is empirically well-established. To account for the effect of corporate policies, a dividend smoothing term, which we denote by $\rho_B^d d_{Bjt-1}$, is introduced into the analysis. The persistence of the dividend smoothing, which is given by the parameter ρ_B^d, aims to capture this well-documented phenomenon. This additional degree of freedom is necessary to ensure that the introduction of leverage does not generate excessive dividend volatility (e.g. Jermann (1998)). Since the smoothing motive is internalized by managers, this modification of the baseline neoclassical model implies that dividends are endogenously chosen by managers.

The production process of intermediate good producers in the brown sector generates emissions. We follow the literature and model the link between production and emissions by assuming that emissions are determined by the output of the brown sector:

$$e_{jt} = z_{Bjt} y_{Bjt} \tag{5}$$

where e_j denotes emissions of firm j.

The role of regulation is captured by introducing a constraint that relates the quantity of output produced by intermediate good producer j to the number of permits held by this firm at the beginning of the period. This constraint implies that the quantity of emissions produced by brown firms must be inferior or equal to the number of permits held at the end of period t:

$$e_{jt} \leq \gamma s_{Pjt+1} \tag{6}$$

Each period, a manager in intermediate good producer j chooses the dividend to distribute d_{Bjt}, the optimal number of hours worked n_{Bjt} to hire, the amount of capital k_{Bjt} to rent, the quantity of emissions e_{jt}, the quantity of output to produce y_{Bj}, as well as the number of polluting permits

s_{Pjt} to maximize the discounted sum of dividends d_{Bj} :

$$\max_{n_{Bjt}, d_{Bjt}, k_{Bjt}, e_{jt}, s_{Pjt+1}} E_0 \sum_{t=0}^{\infty} \widehat{\beta}^t \frac{\lambda_t}{\lambda_0} d_{Bjt} \tag{7}$$

subject to the set of constraints given by equations (2), (3), (4), (5), and (6). Future expected dividends are discounted using the SDF of the representative agent which is the owner of the firm. The SDF is denoted by $\widehat{\beta}\lambda_t/\lambda_t$, where λ is the representative agent's marginal utility and $\widehat{\beta}$ the modified discount factor, which is adjusted for growth (e.g. Kocherlakota (1990)).

3.3 The green sector

The only difference between the brown and green sectors is that the latter does not face any regulatory constraints. Apart from the environmental block, the two sectors are completely symmetric, and we use the subscript G instead of B to denote variables corresponding to the green sector. In the green sector, the production process is decarbonized, allowing green firms the freedom to choose any level of production. Since the problem is similar to that of brown firms, we will provide a brief overview of the green intermediate good producers.

Dividends of green intermediate producer j are given as follows:

$$d_{Gjt} = z_{Gjt} y_{Gjt} - w_t n_{Gjt} - r_t k_{Gjt} + p^G_{Bt} \gamma b^G_{jt+1} - b^G_{jt} - \gamma \iota b^G_{jt+1} + \rho^d_G d_{Gjt-1} \tag{8}$$

The production function of the green intermediate good is given by a Cobb-Douglas specification:

$$y_{Gjt} = k^{\alpha_G}_{Gjt} n^{1-\alpha_G}_{Gjt} \tag{9}$$

Since each green intermediate good producer produces a unique variety, they enjoy some monopoly power over their respective products. As a result, each green intermediate good producer takes the demand from the final good producer as given. The demand schedule for the green final good producer, derived from their problem, is as follows:

$$z_{Gjt} = z_{Gt} \left(\frac{y_{Gt}}{y_{Gjt}} \right)^{\frac{1}{\kappa_G}} \tag{10}$$

Green intermediate good producers then choose their dividend policy d_{Gj}, the optimal number of hours worked to hire n_{Gj}, the amount of capital k_{Gj} as well as the optimal quantity of output to produce y_{Gj}, and the quantity of bonds to issue b_j^G that maximize the dividends that they distribute to shareholder, which is given by equation (8), subject to constraints (9) and (10).

3.4 Final good sector

Firms in the final good sector produce the final output good y_T by combining the intermediate goods produced by the green and brown sectors. Profits of the final output good producer are given as follows:

$$\pi_t = y_{Tt} - z_{Bt}y_{Bt} - z_{Gt}y_{Gt}$$

where z_B and z_G denote the relative price of the brown and green final goods, respectively. The final output good is produced via a Cobb-Douglas production function:

$$y_{Tt} = a_{Tt}y_{Bt}^{\varrho}y_{Gt}^{1-\varrho}$$

where the share of the brown sector in total production is given by the parameter ϱ. The only source of exogenous shocks stems from fluctuations in total factor productivity in the final output good sector, the level of which is denoted by a_{Tt}. The shock process is given as follows:

$$\log(a_{Tt}) = \rho_a \log(a_{Tt-1}) + \varepsilon_{at}$$

and where ε_{at} is a random variable that is independently and identically distributed with mean 0 and standard deviation $std(\varepsilon_{at})$.

3.5 Households

The period t budget constraint of households is given as follows:

$$\begin{aligned} & p_{St}^G(\gamma s_{Gt+1} - s_{Gt}) + p_{St}^B(\gamma s_{Bt+1} - s_{Bt}) + p_{Bt}^G\gamma b_{t+1}^G + p_{Bt}^B\gamma b_{t+1}^B \\ & +c_t + i_{Bt} + i_{Gt} + \frac{\xi}{2}\gamma s_{Pt+1}^2 \\ = \quad & p_{Pt}(\gamma s_{Pt+1} - s_{Pt}) + s_{Gt}\int_0^1 d_{Gjt}dj + s_{Bt}\int_0^1 d_{Bjt}dj \\ & +r_t(k_{Gt} + k_{Bt}) + w_t(n_{Bt} + n_{Gt}) + b_t^B + b_t^G \end{aligned} \tag{11}$$

On the expenditure side, which corresponds to the left hand-side of equation (11), agents first decide how to allocate their wealth between the green and brown sectors. Instead of purchasing individual stocks, they invests in a diversified portfolio composed of green or brown intermediate good producers. The price of the green and the brown portfolio is denoted by p^G_{St}, while the price of brown portfolio is denoted p^B_{St}. Here, the subscript S stands for stock prices. The quantity of stocks issued by each sector and purchased by the representative agent is denoted by s_G and s_B. For the sake of parsimony, and since the allocation between green and brown stocks is not the main focus of the paper, we assume that the quantity of stocks in each sector is fixed and normalized to 1.

We also introduce leverage by allowing agents to invest in short-term risk-free bonds issued by the green and brown sectors. The price of the risk-free asset issued by the green sector is denoted by p^G_{Bt}, whereas the quantity of green bonds purchased by the agent in period t is b^G_{t+1}. The quantity of brown bonds purchased in period t is b^B_{t+1}, whereas the price of the brown risk-free asset is denoted by p^B_{Bt}.

The main component of aggregate demand consists of consumption, which is denoted by c. The agent also owns and accumulate the capital stocks used in the production process, which is then rented to the brown and green sectors. The amount invested in brown and green capital is denoted by i_B and i_G, respectively.

Following Jermann (1998), investment is subject to adjustment costs. This implies the following laws of motion for the accumulation of brown and green capital, k_B and k_G:

$$\gamma k_{Bt+1} = (1-\delta)k_{Bt} + \left(\frac{\theta^B_1}{1-\epsilon_B}\left(\frac{i_{Bt}}{k_{Bt}}\right)^{1-\epsilon_B} + \theta^B_2\right)k_{Bt} \quad (12)$$

$$\gamma k_{Gt+1} = (1-\delta)k_{Gt} + \left(\frac{\theta^G_1}{1-\epsilon_G}\left(\frac{i_{Gt}}{k_{Gt}}\right)^{1-\epsilon_G} + \theta^G_2\right)k_{Gt} \quad (13)$$

where $\theta^B_1, \theta^B_2, \theta^G_1$ and θ^G_2 are parameters that are calibrated to ensure that adjustment costs have no effect on the deterministic steady state of the model. With this specification, the parameters ϵ_B and ϵ_G measure the sensitivity of the investment to capital ratio to a change in Tobin's Q. Consequently, a higher value for this parameter value implies a higher degree of capital adjustment costs. The depreciation rate of physical capital δ is identical across sectors.

The revenue stemming from regulation of the brown sector, as well as the associated costs, are directly transferred to the representative agents. Emitting polluting permits gives rise to a social cost, as it implies larger quantities of greenhouse gases that are emitted into the atmosphere. The cost of emissions on the economy is captured by introducing a quadratic adjustment cost that takes the form of a resource cost. This resource cost aims to capture the adverse effects of emissions on the economy, as emissions lead to a higher frequency of natural disasters, the occurence of which consumes resources. This cost is denoted by $\frac{\xi}{2}\gamma s_{Pt+1}^2$, where ξ is a parameter that measures the economic cost induced by the environmental externality, and where s_P denotes the quantity of emissions allowances that are issued.[4] Since in equilibrium the number of permits in circulation determines the quantity of emissions (see equation (6)), agents internalize the effects of the environmental externality on the economy when issuing those allowances.

On the aggregate income side, which is shown by the right hand side of equation (11), households firstly receive a revenue from emitting emissions allowances that are purchased by brown firms. The price of emissions allowances is denoted by p_P. The aggregate dividend received from owning a portfolio of green and brown firms is denoted by $\int_0^1 d_{Gjt}dj$ and $\int_0^1 d_{Bjt}dj$, respectively.

The representative agent receives a capital income from renting physical capital to firms $r_t(k_{Gt} + k_{Bt})$, where the rental rate of capital is denoted by r. Agents also allocate their time between leisure activities, denoted as l_t, and hours worked in the brown and green sectors:

$$1 = l_t + n_{Bt} + n_{Gt} \tag{14}$$

[4] In the growing economy, the resource cost of emissions is given as follows:

$$\Psi(S_{Pt}) = \frac{\widehat{\xi}}{\Gamma_t}\frac{S_{Pt+1}^2}{2}$$

where Γ_t is the economy's level of labor augmenting technological progress, the growth rate of which is given by γ. To be consistent with balanced growth, we assume that technological progress reduces the damage caused by emissions over time. Consequently, in the detrended economy, we have:

$$\psi(s_{Pt}) = \frac{\Psi(S_{Pt})}{\Gamma_t} = \frac{\xi}{2}\gamma s_{Pt+1}^2$$

where $\xi = \widetilde{\xi}\gamma$.

where the total time endowment is normalized to 1. The labor income that they receive is denoted by $w_t(n_{Bt} + n_{Gt})$, where w is the hourly wage rate. Since production inputs are perfectly mobile across sectors, wages and the rental rate of capital are equalized across sectors.

Households also receive a financial income from their holding of green and brown bonds, which is denoted by b_t^B and b_t^G, respectively. To determine the allocation of green and brown bonds, and since it matters for the equity premium, we assume that agents have a preference for safe assets. To keep the analysis tractable, this preference for safety takes the form of a constraint implying that agents always keep a minimum amount of their wealth invested in the respective safe assets. This threshold, which is exogenously determined, in turn pins down the equilibrium quantity of green and brown bonds available in the economy. For the brown asset, this constraint is given as follows:

$$\gamma b_{t+1}^B \geq \varkappa_B$$

whereas for the green bond we have that:

$$\gamma b_{t+1}^G \geq \varkappa_G$$

where $\varkappa_B$ and $\varkappa_G$ are two financial structure parameters.

Since the objective of this work is to explain financial returns, we need to introduce a non-standard preference specification. One parsimonious way to generate realistic asset pricing implications is to assume that habits depend on utility rather than consumption alone. This specification implies a law of motion for the habit stock that is given as follows:

$$\gamma h_{t+1} = m h_t + (1 - m) c_t \left(\psi + l_t^{\upsilon}\right) \tag{15}$$

where m is a memory parameter that controls the persistence of habits, which takes values between 0 and 1. This specification considerably improves the asset pricing implications of models in which labor supply is endogenously determined and only introduces one additional parameter to calibrate (e.g. Jaccard (2014)). ψ and υ are two labor supply parameters that determine the steady state allocation of time between hours worked and leisure as well as the

Frisch elasticity of labor supply, respectively. To miminize the number of degrees of freedom, the sensitivity of the habit stock to changes in utility is given by $1 - m$. This specification therefore implies that the habit stock is equal to zero when m is set to 1. In contrast to the standard specification, the intensity of habits therefore increases as the value of m declines below 1.

Each period, the representative agent optimally chooses consumption c, investment as well as the capital stock in both sectors, i_G, i_B, k_G and k_B, the number of hours worked in each sector n_B and n_G, the quantity of green and brown financial assets to purchase s_G, s_B, b^G and b^B, while internalizing the impact of these decisions on the evolution of the habit stock h:[5]

$$\max_{c_t, i_{Gt}, i_{Bt}, k_{Bt+1}, k_{Gt+1}, s_{Bt+1}, s_{Gt+1}, b^B_{t+1}, b^G_{t+1} h_{t+1}} E_0 \sum_{t=0}^{\infty} \widehat{\beta}^t \frac{\left(c_t \left(\psi + l_t^{\upsilon}\right) - h_t\right)^{1-\sigma}}{1-\sigma}$$

subject to constraints (11), (12), (13), (14) and (15).

3.6 Market clearing

The economy's consolidated budget constraint is given as follows:

$$y_{Tt} = c_t + i_{Bt} + i_{Gt} + \frac{\xi}{2} \gamma s_{Pt+1}^2 \tag{16}$$

The environmental externality $\psi(s_{Pt+1}) = (\xi/2)\gamma s_{Pt+1}^2$ affects the economy by consuming resources. Total output is therefore divided between consumption, investment and the damage arising from emissions.

The number of shares issued by each sector is fixed and normalized to 1:

$$s_{Gt} = s_{Bt} = 1$$

3.7 The price and quantity of emissions allowances

The difference between the green and brown firms is that the latter need to obtain pollution permits in order to produce. For the brown sector, the value of a permit depends on the marginal increase in production made possible by owning those emissions allowances. In our setting, this

[5] See Appendix B for the model derivations and results using Epstein-Zin-Weil preferences.

increase in production is determined by the Lagrange multiplier associated with the regulatory constraint (6), and which is denoted by χ. The corresponding asset pricing formula can be derived from the optimality condition with respect to the quantity of permits purchased by the brown sector:

$$p_{Pt} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} p_{Pt+1} + \chi_t \tag{17}$$

where $\beta\frac{\lambda_{t+1}}{\lambda_t}$ is the representative agent's SDF, who owns all firms in the economy.[6]

This formula illustrates that the price of an emission allowance can be interpreted as the expected discounted sum of present and future values of χ. A tightening of regulation, due for example to a scarcity of pollution permits, therefore raises the price of carbon.

In the baseline version of the model, we assume that the supply of permits is optimally determined. This implies that agents are able to fully internalize the cost associated to issuance.[7] Given our reduced form specification of environmental damage, the optimality condition with respect to issuance is given as follows:

$$p_{Pt} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} p_{Pt+1} + \xi s_{Pt+1} \tag{18}$$

Keeping everything else equal, an increase in the marginal cost of emissions leads to a higher price of carbon. Additionally, in the absence of any social cost of emissions ($\xi = 0$), the price of carbon emissions would be zero.

Introducing a decentralized market for carbon not only produces a carbon price that is optimal but also allows us to characterize the corresponding supply of permits, which in turn determines the quantity of emissions.

Proposition 1 *The optimal supply of permits is derived by equalizing the marginal social damage of emissions with the tightness of the regulatory constraint, which determines the marginal benefit from issuing emissions allowances:*

$$\xi s_{Pt+1} = \chi_t$$

[6]Where $\beta = \widehat{\beta}\gamma^{-\sigma}$.

[7]In reality, issuance of carbon allowances is delegated to a government entity. In the decentralized equilibrium, adding an additional sector in charge of issuing permits would not change the optimality conditions in this representative agent economy.

Proof. Combine equations (17) and (18). ■

4 Parameter selection

Following standard practice in the asset pricing literature, the main model parameters are selected to maximize the model's ability to replicate key features of the data. In our case, we are particularly interested in explaining the observed green equity premium. Given the significance of the EU ETS market in Europe, we begin by examining whether this disparity in returns between the green and brown sectors can be attributed to carbon pricing. In our setting, the effect of the market for carbon allowances on relative returns can be conveniently captured by one single parameter, the social damage parameter ξ. Indeed, when ξ is set to zero, the two sectors are symmetric, as the price of carbon allowances falls to zero in this case. Since the price of carbon allowances p_P only affects the profitability of the brown sector, regulation is a possible source of return asymmetry.

The facts that we document in Section 2 could also be explained by differences in economic and financial structures. Such differences can be captured by adopting different calibration for the brown and green sectors which are otherwise symmetric. A main challenge, however, is to find theoretical moments that can be exploited to identify differences in parameter values across sectors. Although financial market data on green and brown returns are becoming increasingly available, it remains very difficult to find this information by sector for macroeconomic aggregates such as investment or output. Information about financial structure is also very scarce and currently difficult to exploit.

Given this lack of data availability, we restrict the possible source of cross-sector heterogeneity by imposing the following restrictions: (i) the production functions of the green and brown sectors are symmetric, i.e. $\alpha_G = \alpha_B = \alpha$; (ii) there are no differences in capital adjustment costs, i.e. $\epsilon_G = \epsilon_B = \epsilon$, (iii) markups in the green and brown sectors are identical, i.e. $\kappa_G = \kappa_B = \kappa$; (iv) firms in the two sector have similar dividend policies, i.e. $\rho_G^d = \rho_B^d = \rho^d$; and (v) financial structures across sectors are symmetric. Given that the two sectors can differ in size, we set the two financial structure parameters $\varkappa_G$ and $\varkappa_B$ to ensure that leverage ratios are identical across sectors, i.e. $lev_{Bt} = lev_{Gt}$, where for sector I, the aggregate market leverage ratio is given as follows:

$$lev_{It} = \frac{p^I_{Bt} b_{It}}{q_{It} k_{It+1}}$$

This restriction eliminates one degree of freedom by pinning down the relative supply of green and brown bonds. A sensitivity analysis with respect to the financial structure parameters is conducted in Section 7.

Which moments to target?

On the business cycle side, the objective is to study financial returns in a model that matches the standard deviation of the growth rates of total output, consumption and total investment, which we denote by $std(\Delta y_T)$, $std(\Delta c)$, and $std(\Delta i_T)$, respectively.[8] Since the volatility of aggregate demand components is not independent of the model steady state, we also add the investment to output ratio $E(i_T/y_T)$ to the list of business cycle moments to match.

In order to study relative returns, the main challenge is to provide a solution to the equity premium puzzle within a two-sector model with investment and endogenous labor supply. A main test of the model is therefore whether it can match the aggregate equity premium $E(r^T - r^F)$, i.e. the difference between total returns on the green and brown portfolios, and the risk-free rate. Using the EUROSTOXX index as a proxy for total equity returns, we obtain an aggregate equity premium for the period from 1995 to 2022 of 6.7%, where the risk-free rate is the 3-month EURIBOR rate.

As documented in Jermann (1998), whereas introducing leverage helps to increase risk premiums, the challenge is that this improvement may come at the cost of generating excessive fluctuations in dividends. To avoid this potential issue, we also add a measure of volatility of aggregate profits into the list of asset pricing moments to match. In the absence of information on dividends by sectors, we use a measure of aggregate profits, which therefore correspond to the sum of profits in the green and brown sectors. Using net operating surplus as a proxy for profits, which in our model is equivalent to the dividends paid to households, we find a standard deviation for dividends, denoted $std(\Delta d_T)$, that is around 4 times more volatile compared to the standard deviation of output.

[8] Both in the model and in the data, growth rates are defined as the four quarter log difference multiplied by 100. For output, we thus have that:

$$\Delta y_T = (\log(y_T) - \log(y_{T-4})) \cdot 100$$

On the asset pricing side, the challenge is to replicate an equity premium in excess of 6% while at the same time reproducing the low mean risk-free rate (e.g. Weil (1989)). Using the 3-month EURIBOR rate as well as the consumer price index to compute inflation, we obtain an average mean risk-free real rate, which we denote by $E(r^F)$, for the period from 1995 to 2022 of 0%.

Relative to the set of moments typically studied in the equity premium literature, we also target a measure of long-term interest rates. Indeed, in contrast to the prediction of the vast majority of macroeconomic models, the term structure is upward sloping, which implies a positive spread between long- and short-term rates. This information can therefore be exploited to identify the main parameter values. Using the 10-year government bond as a proxy for long-term rates, we find an average value for $E(r^{10Y})$ of 1.6%.

Provided that it is able to broadly reproduce this set of macro-finance facts, this framework can then be used to study the determinants of green and brown returns. In particular, our objective is to reproduce the 2% green equity premium denoted $E(r^G - r^B)$ that we document in Section 2.

Given that parameters determine both the steady state and all moments of the model, we also need a moment that can be exploited to calibrate the relative size of the two sectors. This is achieved by including the market capitalization of the green and brown portfolios, i.e. p_S^G and p_S^B, respectively, into the set of targeted moments, where this average ratio is denoted $E(p_S^G/p_S^B)$.

Relation between structural parameters and targeted moments

In this analysis, we focus on a set of 10 moments of interest. To align the model more closely with the data, we select certain parameters to be adjusted accordingly. The standard deviation of aggregate technology shocks, σ_a, is pivotal, as it influences the standard deviations of all business cycle-related variables in the model. The habit parameter m and the persistence of technology shocks ρ_a are crucial for adding persistence to the model, impacting all variable moments significantly.

The capital adjustment cost ϵ plays a significant role, especially for asset pricing variables, compared to its impact on the business cycle. The depreciation rate of capital, δ, affects both capital stocks and investments, and indirectly influences all variables, albeit less than other parameters.

As explained by Abel (1999), the equity premium can be decomposed into two components. First, the term premium measures the effect of interest rate risk on the valuation of risky assets. Even if an asset provides a constant dividend, it can still be considered risky if its price decreases during economic downturns due to a rise in real interest rates. The second component is a risk premium that is determined by the dynamics of dividends. Indeed, part of the risk associated with an investment in equity can be attributed to the decline in remuneration paid by firms during recessions, which are periods of low consumption and hence high marginal utility.

Introducing leverage amplifies the risk premium by increasing dividend volatility and making it more procyclical. The financial structure parameter $\varkappa_B$ affects firm equity returns and, albeit to a lesser extent, the equity premium, while the dividend smoothing parameter ρ^d mainly influences dividend volatility.

The discount factor β is crucial for all model variables, particularly for interest rates, aiding in parameter identification. The environmental damage coefficient ξ distinguishes between the green and brown sectors by creating a return differential and a nonzero carbon price, affecting the green equity premium and relative sectoral market values.

Lastly, the share of the brown sector in the aggregate output good y_T, denoted by ϱ, predominantly impacts green finance variables.

Calibrated parameters

In selecting the remaining parameter values, we rely on available information and well-established values from the literature. The parameters ψ and υ determine the labor supply behavior in the model. The parameter ψ determines the steady-state share of time allocated to work activities, while υ represents the Frisch elasticity of labor supply. To align with empirical evidence, we set ψ such that agents, on average, allocate around 20% of their time to work activities in the different sectors. This value is informed by data from the Time of Use Survey. The value of υ is chosen to yield a Frisch elasticity of labor supply of approximately 0.7. This value is commonly used in the real business cycle literature and supported by studies such as Hall (2009) and Chetty et al. (2011).

In models with production, increasing the curvature coefficient σ somehow raises the equity premium but at cost of seriously distorting the model's business cycle predictions. Since with internal habits, risk aversion is determined by σ, but is independent from the habit parameter (e.g. Constantinides (1990); Jermann (1998); Boldrin et al. (2001)), we set the curvature parameter σ to 1, as in Boldrin et al. (2001).

Given the decline in trend growth observed since the Great Financial Crisis, we set the economy deterministic growth rate γ to 1.002. This value corresponds to an average growth rate of 0.8% per year, as observed in the post-crisis sample.

For each sector, the demand elasticity of the variety produced by intermediate good producers in both sectors is given by the parameter κ, a coefficient assumed to be identical across sectors. In the business cycle literature, this parameter is calibrated to produce an average price markup of 12.5% (e.g. Galí (2015)). Although we abstract from price rigidities, in our two-sector environment imperfect competition still creates a wedge between the relative price of intermediate good producer j and its marginal cost of production. Given our structure, setting κ to 17 allows us to generate a price markup of 1.125 in both sectors.[9]

The last parameter to calibrate is the issuance cost ι, which is common to both sectors. This parameter drives a wedge between the return on corporate bonds in both sectors and the risk-free rate. Given the lack of empirical evidence on the impact of issuance cost on corporate bond yields, we set ι to 0.001, which implies a spread of 0.4% per annum.

Methodological and Computational Approach

We solve the model using pruned third order perturbation[10] because this method allows us to capture the asymmetries and non-linearities which are important for the asset pricing part of the model. In order to match the first two moments of the model to the data we need to compute

[9] For example, in the green sector, profit maximization implies the following expression for the marginal cost of production of intermediate good producer j :

$$z_{Gt}\left(y_{Gt}\right)^{\frac{1}{\kappa}}\left(1-\frac{1}{\kappa}\right)y_{Gjt}^{-\frac{1}{\kappa}}=\Xi_{Gjt}$$

where Ξ_{Gjt} denotes the marginal cost of intermediate good producer j in the green sector. Given the demand schedule for the intermediate good of green producer j (e.g. Equation (10)) the price markup is given as follows:

$$\frac{z_{Gjt}}{\Xi_{Gjt}}=\frac{1}{1-\frac{1}{\kappa}}$$

[10] See Kim et al. (2008) and Andreasen et al. (2017).

covariance matrices, for which there exist closed form solutions[11], but they are computationally challenging. The problem involves solving linear systems of equations which grow in size by $O(k^6)$ with k being the number of states. To achieve this at all and within reasonable time we leverage efficient implementations of algorithms in MacroModelling.jl.[12]

5 Comparing the model with the data

Given the set of parameters that are calibrated and therefore fixed to the values discussed above, is the model able to reproduce these empirical facts? The set of parameter values that minimizes the distance between the model and the data is reported in Table 3. Table 4 reports the estimated theoretical moments and compares them with the corresponding moment generated by the model.

As illustrated by Table 4, the four business cycle facts can be reproduced. The ability of real business cycle models to account for these facts is well-documented (e.g. King and Rebelo (1999)). The main challenge is to simultaneously reproduce the asset market facts reported in the middle part of Table 4. In this respect, the fact that the model matches the high equity premium $E(r^T - r^F)$ and the low mean risk-free real rate $E(r^F)$ is noteworthy.

Relative to some of the solutions proposed in the literature, a main difference is that the model also matches the volatility of aggregate dividends $std(\Delta d_T)$ as well as the mean long-term rate $E(r^{10Y})$.

Matching the volatility of dividends as well as the difference between short- and long-term rates addresses Abel's critique (e.g. Jermann (1998); Abel (1999)). Indeed, the fact that the model reproduces the slope of the term structure of interest rates ensures that the term premia component of the equity premium is not overstated. Moreover, since the model can also account for the volatility of dividends, the large equity premium that we obtain is not due to excessive dividend volatility.

In comparison with the macro-finance literature, the model also accounts for the differences between the green and brown sectors that are reported in the lower part of Table 4. Indeed,

[11]See Mutschler (2018) and Andreasen et al. (2017).
[12]See Appendix A for more details on the algorithm enabling efficient method of moments matching.

Table 3: Outcome of moment matching procedure

σ_a	ρ_a	m	ϵ	δ	$\varkappa$	ρ_d	β	ξ	ϱ
0.0137	0.99253	0.986	3.7049	0.01111	0.444	0.9672	0.9907	0.5996	0.5157

Table 4: Data vs. model (pruned 3rd order)

Business cycle	Data	Model
$std(\Delta y_T)$	2.8	2.79
$std(\Delta c)$	2.2	2.16
$std(\Delta i_T)$	5.0	4.97
$E(i_T/y_T)$	0.21	0.21
Asset pricing	**Data**	**Model**
$E(r^T - r^F)$	6.7	6.76
$std(\Delta d_T)$	11.2	11.19
$E(r^F)$	0.0	0.01
$E(r^{10Y})$	1.6	1.41
Green finance	**Data**	**Model**
$E(r^G - r^B)$	2.0	1.97
$E(p_S^G/p_S^B)$	0.29	0.29

a green equity premium can be reproduced while jointly matching the relative size of the two sectors, as proxied by the relative market capitalizations, i.e. $E(p_S^G/p_S^B)$.

Parameter elasticities

To further explore the model's responsiveness to parameter variations, we examine the elasticities of key variables to a 1% change in each parameter. This analysis provides insights into how sensitive the model's outcomes are to changes in specific parameters, shedding light on the robustness and flexibility of the model in replicating empirical facts. When look at how sensitive the empirical model moments are to changes in its parameters, a few key findings stand out (see Table 5).

Firstly, the discount factor β and the habit parameter m emerge as having widespread effects across various aspects of the model. For instance, a small 1% increase (decrease) in β (m) drastically reduces the risk-free real rate ($E(r^F)$), indicating β's (m's) critical role in shaping investment and savings behavior in the model. Similarly, changing m by just 1% leads to a 27.81% increase in the standard deviation of consumption ($std(\Delta c)$), showing how composite habits can significantly influence economic fluctuations.

Looking at business cycle variables, the persistence of technology shocks ρ_a is particularly influential, affecting the standard deviation of output ($std(\Delta y_T)$) and consumption volatility. This highlights the importance of technology shocks in driving economic cycles in the model.

Regarding asset pricing, the habit parameter m again stands out, especially affecting the equity premium ($E(r^T - r^F)$) and making it a crucial factor in explaining the high returns on risky assets compared to risk-free assets. The parameter ρ_d, indicating dividend smoothing, also significantly impacts dividend volatility ($std(\Delta d_T)$) and the green-brown equity premium ($E(r^G - r^B)$).

Lastly, when focusing on green finance, both the environmental damage coefficient ξ and the financial structure parameter $\varkappa$ show notable effects. For example, a small change in ξ can alter the green-brown equity premium.

In summary, β, m, and ρ_a have broad-reaching impacts across the model, influencing everything from the business cycle to asset pricing and green finance. The remaining parameters exhibit less large elasticities but nonetheless are vital in matching the model to the data and understanding the importance of the underlying economic mechanisms.

Table 5: Parameter elasticities: Percentage change in variables for a 1% change in parameters

Variable	Value	σ_a	m	ϵ	$\varkappa$	ρ_d	ρ_a	ϱ	δ	β	ξ
$std(\Delta y_T)$	2.7909	1.01	−2.40	−0.13	0.0	−1.28	6.45	−0.13	0.049	−2.53	−0.07
$std(\Delta c)$	2.1552	0.94	27.81	0.03	0.0	−1.64	22.45	−0.20	0.089	−39.02	−0.10
$std(\Delta i_T)$	4.9828	1.11	−48.63	−0.38	0.0	−0.54	−20.02	−0.09	−0.17	26.06	−0.05
$E(i_T/y_T)$	0.2066	0.16	−10.31	−0.03	0.0	−0.02	2.24	−0.10	0.31	52.16	−0.05
$E(r^T - r^F)$	6.7690	2.00	−111.07	0.73	0.33	−1.91	−31.64	−0.58	0.029	14.05	−0.11
$std(\Delta d_T)$	11.1942	1.80	−49.32	0.19	1.27	−27.34	−4.84	−1.35	0.35	−28.06	0.04
$E(r^F)$	0.0029	−2594.71	140403.0	−794.51	0.0	58.07	10419.72	95.87	30.99	−173730.18	59.07
$E(r^{10Y})$	1.4101	−3.41	172.58	−0.75	0.0	0.25	−33.34	0.16	−0.11	−359.56	0.096
$E(r^G - r^B)$	1.9705	2.00	−97.87	0.41	2.24	−17.60	12.72	−1.79	0.54	−243.09	0.53
$E(p_S^G/p_S^B)$	0.2895	−0.26	12.75	−0.049	−0.77	−0.24	−0.59	−1.998	−0.024	109.18	−0.73
Parameter values		0.0137	0.986	3.7049	0.444	0.9672	0.99253	0.5157	0.01111	0.9907	0.5996

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Table 6: Non-targeted moments

	Estimated moment	99% CI	Model
$std(r^F)$	2.9	[2.5, 3.5]	4.18
$std(r^G)/std(r^B)$	1.2	[0.8, 1.7]	1.55

Additional implications

Whereas the model is able to reproduce the moment reported in Table 4, a legitimate question to ask is how it performs on other dimensions. A well-known limitation of models with habits is their tendency to generate excessive risk-free rate fluctuations. Table 6 reports the risk-free rate volatility produced by model, which is compared to the data. Whereas the model with slow-moving composite habits generates fluctuations that are higher than those observed in recent years, we obtain a risk-free rate standard deviation that stands below 5%, which is well within the range of values obtained when using long data samples (e.g. Jordà et al. (2019); Mehra and Prescott (1985)).

As shown by the second line in Table 6, it is also possible to account for the relative standard deviation of returns across sectors. Our data suggest that green returns are about 1.2 times more volatile than brown ones, a fact that the model can broadly capture if we take uncertainty into account.

6 Carbon pricing and the green equity premium

Given that our model is able to reproduce risk premiums of a realistic magnitude, it can be used to study relative returns. Since the presence of a market for carbon is the only source of asymmetry across sectors, we next ask whether the facts that we document could be explained by carbon pricing.

As discussed in Section 3.7, the average carbon price p_P depends on the value of the social damage parameter ξ. Under the assumption that the supply of emissions allowances is optimally determined, the price of carbon reflects the social damage caused by emissions. Consequently, the average value of emissions allowances increases with ξ, as a more stringent regulation of emissions is necessary if the damage to the economy is more severe.

A higher average price of carbon in turn generates differences in returns across sectors, as brown

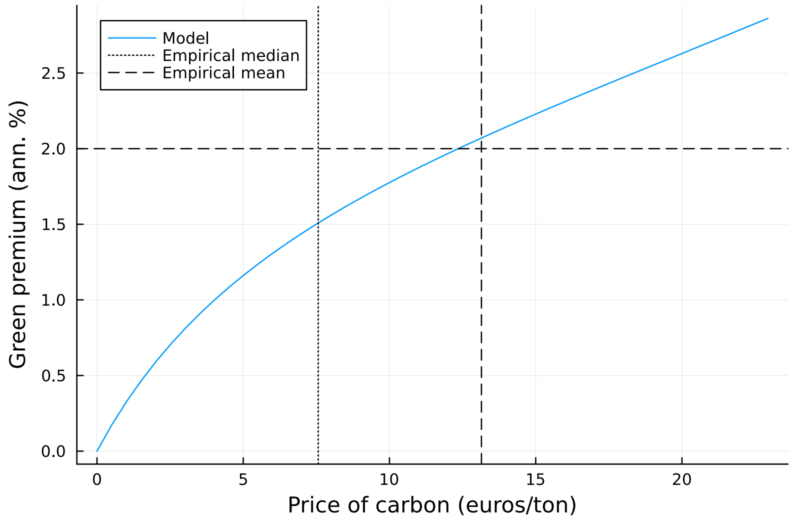

Figure 2: x-axis: Price of carbon in euros per ton. y-axis: Green equity premium in annualized percent. Empirical mean and median refer to the series on the respective axis over the data sample (2013Q3 until 2021Q1).

firms need to allocate a larger fraction of their resources to the purchase emissions allowances. In contrast, if the average price of carbon is sufficiently low, fluctuations in p_P only have a marginal effects on the profitability of the brown sector, making carbon pricing irrelevant for the dynamics of returns.

If the social damage of emissions is sufficiently high, our main finding is that carbon pricing can rationalize the green equity premium that we document in Section 2. Indeed, as in the data, our framework generates procyclical fluctuations in the price of carbon. A procyclical price of carbon in turn implies that the cost borne by brown companies to purchase permits increases in good times when economic activity is strong. A procyclical price of carbon therefore contributes to smooth profits, and hence dividends, of brown companies since the cost faced by these firms increases in good times and declines during recessions.

The introduction of carbon pricing reduces the equity premium of the brown sector by lowering the risk premium component. The reason is that carbon pricing attenuates the decline in dividends that an investor holding a brown portfolio will receive in economic downturns. During booms, the increase in dividends is more muted as brown firms need to purchase permits to increase production. From a consumption smoothing perspective, the more stable dividend income generated by carbon pricing therefore reduces the risk associated with an investment in brown stocks. Consequently, relative to an investment in green companies that are not subject to regulation, this effect reduces the compensation demanded by investors for holding brown stocks.

Figure 2 reports the average value of the green equity premium $E(r^G - r^B)$ produced by the model as a function of the price of carbon per ton, i.e. $E(p_P)$. If the social damage parameter ξ is set to zero, the price of carbon falls to zero and returns are identical across sectors, as the model reduces to a symmetric two-sector economy in this limiting case.

An higher cost of emissions raises the average price of carbon, which in turn increases the value of the green equity premium. For the baseline calibration that reproduces a 2% green equity premium, the model generates an average price of carbon of around 12 euros per ton. As reported in Table 3, an average price of carbon of 12 euros per ton corresponds to a value for the coefficient ξ of 0.6. A value for ξ of 0.6 in turn implies an average cost of emissions, as measured by the ratio $E\left(\frac{\xi}{2}\gamma s^2_{Pt+1}/y_{Tt}\right)$, that represents around 2% of output.

7 The preference for green financial assets

Whereas studies that use recent data find evidence of a green premium, other studies find that brown firms outperform green ones. Using longer data samples, Bolton and Kacperczyk (2021) and Bolton and Kacperczyk (2023), for example, document the existence of a carbon premium for many different countries. In our production economy, a carbon premium can arise if we introduce an aversion for brown firms into the utility function.

To account for the possible disutility caused by holding stocks from brown sectors, we introduce a "sin stock" (e.g. Hong and Kacperczyk (2009)) parameter, which is denoted by ω, where $\omega \geq 0$. This parameter captures the stigma associated with holding stocks from brown sectors and the utility function, which now contains an additional term, is given as follows:

$$\max_{c_t, i_{Gt}, i_{Bt}, k_{Bt+1}, k_{Gt+1}, s_{Bt+1}, s_{Gt+1}, h_{t+1}} E_0 \sum_{t=0}^{\infty} \widehat{\beta}^t \frac{\left(\frac{c_t}{s^{\omega}_{Bt}}\left(\psi + l^{\upsilon}_t\right) - h_t\right)^{1-\sigma}}{1-\sigma} \tag{19}$$

Relative to the benchmark model studied in the previous version, the introduction of stigma gives rise to the following modified asset pricing formula:

$$p_{St}^B \lambda_t = \beta E_t \lambda_{t+1} \left(p_{St+1}^B + d_{Bt+1} - \omega \frac{c_{t+1}}{s_{Bt+1}} \right)$$

In equilibrium, the presence of stigma therefore increases the return of the brown portfolio, as a higher expected return is necessary to compensate investors' aversion for brown stocks, which in the above formula is captured by the term $\omega c_{t+1}/s_{Bt+1}$. In contrast to the effect of carbon pricing, which relies on the comovement between dividends and marginal utility, the effect of stigma on relative returns is independent of uncertainty and is therefore not a compensation for risk.

Given that this is a preference parameter, one main challenge is to identify a range of plausible values to calibrate the coefficient ω. Since the value of ω also impacts equity prices in the brown sector, an indirect approach is to measure the effect of stigma on the market value of the brown sector p_S^B. Indeed, in equilibrium, an increase in ω increases the expected return of brown companies but reduces their market value.

To evaluate the effect of stigma on our results, Figure 3 shows how an increase in ω affects the average green equity premium generated by the model. If ω is set to zero, there is no loss in market value caused by stigma, as the utility specification in equation (19) reduces to the baseline model that generates a green equity premium of 1.9%. As shown in Figure 3, increasing the value of ω decreases the green equity premium while at the same time reducing the market value of the brown sector, the percentage decline of which is shown on the horizontal axis.

Relative to the benchmark calibration that generates a 2% green premium, increasing ω from 0 to 0.29 reduces the green equity premium from 2% to essentially zero. How to interpret such an increase in this preference parameter? As illustrated in Figure 3, relative to the baseline model, eliminating the green equity premium implied by carbon pricing by setting ω to 0.29 leads to a reduction in the market value of the brown sector stemming from stigma of around 26%.

This sensitivity analysis also shows that a negative green premium, or carbon premium, could be generated despite the effect of carbon pricing on relative returns. At the same time, starting from the baseline model that reproduces a 2% green premium, stigma would have to reduce the market value of brown firms by around 38% to generate a 1% carbon premium. To our knowledge, there no evidence suggesting that stigma could have led to such a dramatic reduction in the market value of brown companies.

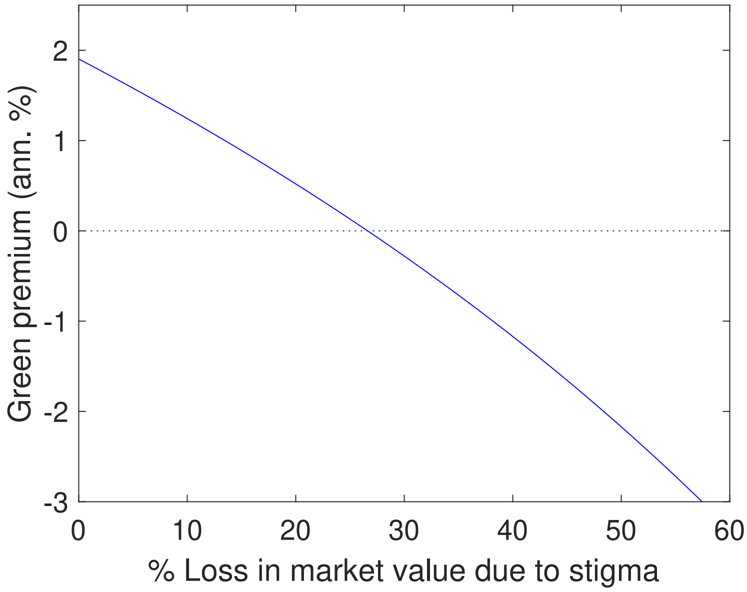

Figure 3: x-axis: Loss in market value due to stigma. y-axis: Green equity premium in annualized percent.

Preference for green bonds

As illustrated by the recent decision of the European Central Bank to tilt its portfolio of corporate bonds away from brown companies, investors' preference could also affect the bond market. In our environment, a shift in the demand for bonds can be analyzed by changing the two preference for safety parameters, $\varkappa_B$ and $\varkappa_G$. In equilibrium, these parameters determine the quantity of bonds that can be issued in both sectors.

As discussed in Section 4, the benchmark calibration assumes no difference in financial structure, which implies that leverage ratios across sectors are identical, i.e. $lev_{Bt} = lev_{Gt}$. Relaxing the assumption of symmetric financial structures, due for example to a shift in demand, is another possible source of return differential. Indeed, most studies rely on leverage to provide a solution to the equity premium puzzle.

To illustrate this point, we introduce a tilting parameter in the aggregate portfolio of the representative agent that increases the weight of green bonds at the expense of holdings of brown bonds. Starting from the benchmark calibration with symmetric financial structures, which generates a 2% green premium, Figure 4 shows the outcome of this experiment. On the horizontal axis, we report the relative leverage ratios, i.e. $E(lev_{Gt}/lev_{Bt})$, whereas the vertical axis depicts the magnitude of the green premium predicted by the model. The origin corresponds to the

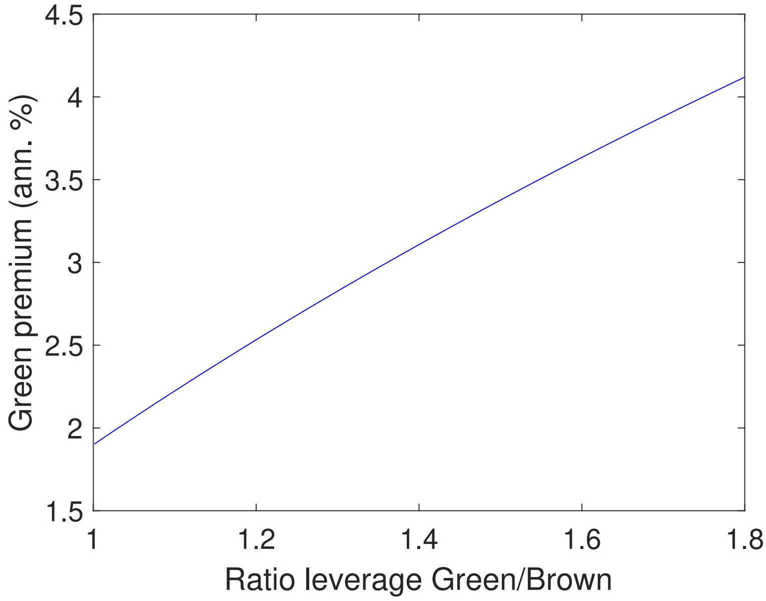

Figure 4: x-axis: Relative leverage ratios between green and brown firms. y-axis: Green equity premium in annualized percent.

benchmark calibration that assumes symmetric financial structures.

Tilting the portfolio of corporate bonds increases the leverage of the green sector while reducing that of brown firms. The effect of tilting on relative returns is sizeable, as a rise in relative leverage from 1 to 1.5 increases the green premium from 2% to 3.3%. Whereas an aversion for brown stocks increases the relative return of brown with respect to green companies, a preference for green bonds therefore has the opposite effect.

Interestingly, as evidenced by Table 1, the data reveals that brown companies have consistently exhibited higher levels of leverage compared to their green counterparts. This observation suggests that, if leverage were the sole determining factor, one might expect to witness a brown premium rather than a green premium. Taken together, a preference for green bonds – driven by both stigma and the tilting of the portfolio – therefore appears unlikely to account for the green premium observed in the data.

8 Conclusion

How does carbon pricing affect financial returns? This article addresses this question by examining the impact of a decentralized market for carbon, such as the EU ETS market, on the risk premiums of green and brown companies. When the carbon price is optimally determined, our findings suggest that green firms should earn higher returns than their brown counterparts, which aligns with the novel empirical evidence presented in our study.

Our result holds generally, as it primarily depends on the dynamics of carbon pricing, assuming an optimal design that exhibits procyclicality. While the EU ETS market is not perfect, it has produced, in recent years, a positive correlation between the price of carbon and economic activity, which in our model is what generates a green equity premium.

Our framework offers flexibility and can be easily utilized to quantitatively assess the impact of alternative mechanisms on green and brown returns. For instance, based on the quantitative predictions derived from our general equilibrium framework, we concluded that incorporating a preference for green assets in the utility function is unlikely to fully offset the influence of carbon pricing on relative returns.

Future research could employ and extend our flexible framework to conduct comparative analyses across various jurisdictions. For instance, is the green premium observed within the EU replicable under varying stages of carbon market developments (take, for instance, the US with regional initiatives such as the California Cap-and-Trade Program and the Regional Greenhouse Gas Initiative) or, more generally, under alternative regulatory frameworks? Expanding our analysis in this way would provide comprehensive insights into the diverse drivers influencing the returns of green and brown firms.

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Online appendix

A Efficient computation of pruned third-order theoretical moments

Given the importance of non-linearities and asymmetries in asset pricing, our model employs a pruned third-order perturbation method. This approach allows for capturing the complex dynamics crucial for accurately pricing assets in fluctuating markets. Key references include Kim et al. (2008) and Andreasen et al. (2017), which provide foundational methodologies for our computational framework.

A.1 Computational challenges and solutions

A.1.1 Computational bottleneck

The core computational challenge lies in computing the covariance matrices of model variables, a process integral to matching the model's first two moments with empirical data. For any pruned higher-order solution, the covariance matrix of the model variables, Σ_y, and the covariance matrix of the augmented state vector, Σ_z, are defined as follows:

$$\Sigma_y = C\Sigma_z C' + D \tag{20}$$

$$\Sigma_z = A\Sigma_z A' + B \tag{21}$$

where $z = \begin{bmatrix} x^{1^{st}} & x^{2^{nd}} & x^{1^{st}} \otimes x^{1^{st}} & x^{3^{rd}} & x^{1^{st}} \otimes x^{2^{nd}} & x^{1^{st}} \otimes x^{1^{st}} \otimes x^{1^{st}} \end{bmatrix}$ represents an augmented state vector, with $x^{1^{st}}$ being the first, $x^{2^{nd}}$ the second, and $x^{3^{rd}}$ the third-order approximation of the states. The elements of the state vector capture interactions among states, leading to a dimensionality of $3n + 2n^2 + n^3$ for n states. Solving the discrete Lyapunov equation (21) for Σ_z becomes increasingly difficult the larger the number of states, due to the exponential increase in augmented states.

As Table 7 illustrates, the computational and memory requirements quickly become prohibitive for the purpose of moment matching with an increasing number of states, necessitating efficient computational strategies.

Table 7: Matrix size for different number of states

States	Size of Σ_z	Memory in MB
3	2 916	0.02
5	36 100	0.29
10	1 512 900	12.10
15	14 976 900	119.82
20	78 499 600	628.00
25	287 302 500	2 298.42
30	834 632 100	6 677.06

A.1.2 Exploiting independent subsets of states

A key innovation in our approach involves identifying and exploiting independent subsets of the state vector for calculating the required parts of the augmented states covariance matrix. By selecting the smallest necessary subset for each variable of interest, we can significantly reduce the computational complexity.

The procedure can be described as follows:

1. **Initial dependency identification:** the initial dependencies, denoted by the binary vector $\mathbf{d}$, among the states, denoted by $\mathbf{s}$, are those for which the partial derivatives of the part of the solution function for variable i, the one for which we establish the relevant subset of states, with regards to the states, $\frac{\partial f_i(\mathbf{s})}{\partial \mathbf{s}}$, is different from zero:

$$\mathbf{d} = \mathbf{1}_{\left|\frac{\partial f_i(\mathbf{s})}{\partial \mathbf{s}}\right| > \tau}$$

2. **Iterative refinement:** Following the initial identification, the dependencies are iteratively refined by passing the dependencies vector to the model solution, $f(\mathbf{d})$, and setting those states which are different from zero as being connected to variable i:

$$\text{while } \mathbf{d} \neq \mathbf{d} \vee \mathbf{1}_{\{|f(\mathbf{d})| > \tau\}} \quad \mathbf{d} = \mathbf{d} \vee \mathbf{1}_{\{|f(\mathbf{d})| > \tau\}}$$

   This iterative process continues until a stable set of dependencies is achieved.

This strategy improves computational efficiency and thereby also enables analysis of more complex models. The methodology for determining the smallest necessary subset of states has been implemented in the MacroModelling.jl toolbox. This Julia-based framework facilitates the efficient computation of dynamic stochastic general equilibrium models, and allows method of moment computations on a conventional retail computer in a reasonable amount of time.

B Model with Epstein-Zin-Weil preferences

$$U_t = \frac{[c_t\,(\psi + l_t^{\upsilon})]^{1-\sigma}}{1-\sigma} + \beta E_t\left[\left(U_{t+1}^{(1-\theta_{EZ})}\right)^{\frac{1}{1-\theta_{EZ}}}\right]$$

$$\lambda_t = (\psi L + l_t^{\nu})^{\frac{1-\sigma}{\theta_{EZ}}}\,(1-\beta)v_{F,t}^{\frac{\theta_{EZ}-(1-\sigma)}{\theta_{EZ}}}\,c_t^{\frac{1-\sigma}{\theta_{EZ}}-1}$$

$$v_{F,t} = \left((1-\beta)c_t^{\frac{1-\sigma}{\theta_{EZ}}}\,(\psi L + l_t^{\nu})^{\frac{1-\sigma}{\theta_{EZ}}} + \beta\gamma^{\frac{1-\sigma}{\theta_{EZ}}}\,v_{F,t+1}^{\frac{1-\sigma}{\theta_{EZ}}}\right)^{\frac{\theta_{EZ}}{1-\sigma}}$$

$$c_{e,t} = v_{F,t+1}^{1-\sigma}$$

$$\Delta\lambda_t = \gamma^{\left(\frac{1-\sigma}{\theta_{EZ}}-1\right)}\left(\frac{v_{F,t}^{(1-\sigma)}}{c_{e,t-1}}\right)^{\left(1-\frac{1}{\theta_{EZ}}\right)}\frac{c_t^{\left(\frac{1-\sigma}{\theta_{EZ}}-1\right)}\,(\psi L + l_t^{\nu})^{\left(\frac{1-\sigma}{\theta_{EZ}}\right)}}{c_{t-1}^{\left(\frac{1-\sigma}{\theta_{EZ}}-1\right)}\,(\psi L + l_{t-1}^{\nu})^{\left(\frac{1-\sigma}{\theta_{EZ}}\right)}}\frac{c_{t-1}}{c_t}$$

Table 8: Outcome of moment matching procedure

σ_a	ρ_a	θ_{EZ}	ϵ	δ	$\varkappa$	ρ_d	β	ξ	ϱ	σ
0.0267	0.9844	0.8593	1.654	0.0403	0.0285	0.9452	0.9998	5.4415	0.2234	9.8190

Table 9: Data vs. model (pruned 3rd order)

Business cycle	Data	Model
$std(\Delta y_T)$	2.8	3.24
$std(\Delta c)$	2.2	2.64
$std(\Delta i_T)$	5.0	4.96
$E(i_T/y_T)$	0.21	0.21
Asset pricing	Data	Model
$E(r^T - r^F)$	6.7	6.92
$std(\Delta d_T)$	11.2	11.22
$E(r^F)$	0.0	0.004
$E(r^{10Y})$	1.6	1.59
Green finance	Data	Model
$E(r^G - r^B)$	2.0	1.82
$E(p_S^G/p_S^B)$	0.29	0.25

C Model equations

C.1 The competitive equilibrium

Consumers

$$E_0 \sum_{t=0}^{\infty} \widehat{\beta}^t \frac{\left[c_t s_{Bt}^{-\omega} \left(\psi + l_t^{\upsilon}\right) - h_t\right]^{1-\sigma}}{1-\sigma}$$

subject to:[13]

$$\begin{aligned} & c_t + i_{Bt} + i_{Gt} + p_{St}^G(\gamma s_{Gt+1} - s_{Gt}) + p_{St}^B(\gamma s_{Bt+1} - s_{Bt}) \\ & + p_{Bt}^B \gamma b_{t+1}^B + p_{Bt}^G \gamma b_{t+1}^G + \frac{\xi}{2}\gamma s_{Pt+1}^2 \\ = \; & s_{Gt} \int_0^1 d_{Gjt} dj + s_{Bt} \int_0^1 d_{Bjt} dj + w_t(n_{Bt} + n_{Gt}) + r_t(k_{Gt} + k_{Bt}) \\ & + p_{Pt}(\gamma s_{Pt+1} - s_{Pt}) + b_t^B + b_t^G \end{aligned}$$

$$\gamma k_{Bt+1} = (1-\delta_B) k_{Bt} + \left(\frac{\theta_1^B}{1-\epsilon_B} \left(\frac{i_{Bt}}{k_{Bt}} \right)^{1-\epsilon_B} + \theta_2^B \right) k_{Bt}$$

[13] where $\widehat{\beta} = \widetilde{\beta}\gamma^{1-\sigma}$

$$\gamma k_{Gt+1} = (1-\delta)k_{Gt} + \left(\frac{\theta_1^G}{1-\epsilon_G}\left(\frac{i_{Gt}}{k_{Gt}}\right)^{1-\epsilon_G} + \theta_2^G\right)k_{Gt}$$

$$\gamma h_{t+1} = mh_t + (1-m)c_t s_{Bt}^{-\omega}\left(\psi + l_t^{\upsilon}\right)$$

$$l_t = 1 - n_{Bt} - n_{Gt}$$

$$\gamma b_{t+1}^B \geq \widetilde{\varkappa}_{Bt}$$

$$\gamma b_{t+1}^G \geq \widetilde{\varkappa}_{Gt}$$

First-order conditions:

$$p_{Bt}^G \lambda_t = \beta E_t \lambda_{t+1} + \eta_{Gt}$$

$$p_{Bt}^B \lambda_t = \beta E_t \lambda_{t+1} + \eta_{Bt}$$

$$p_{Pt}\lambda_t = \beta E_t \lambda_{t+1} p_{Pt+1} + \lambda_t \xi s_{Pt+1}$$

$$\varphi_t = m\beta E_t \varphi_{t+1} + \beta E_t\left[c_{t+1}s_{Bt+1}^{-\omega}\left(\psi + l_{t+1}^{\upsilon}\right) - h_{t+1}\right]^{-\sigma}$$

$$\gamma h_{t+1} - mh_t - (1-m)c_t s_{Bt}^{-\omega}\left(\psi + l_t^{\upsilon}\right) = 0$$

$$\left[\left(c_t s_{Bt}^{-\omega}\left(\psi + l_t^{\upsilon}\right) - h_t\right)^{-\sigma} - \varphi_t(1-m)\right] s_{Bt}^{-\omega}\left(\psi + l_t^{\upsilon}\right) = \lambda_t$$

$$\left[\left(c_t s_{Bt}^{-\omega}\left(\psi + l_t^{\upsilon}\right) - h_t\right)^{-\sigma} - \varphi_t(1-m)\right] c_t s_{Bt}^{-\omega} \upsilon l_t^{\upsilon-1} = \lambda_t w_t$$

$$p_{Gt}\lambda_t = \beta E_t \lambda_{t+1}\left[p_{Gt+1} + d_{Gt+1}\right]$$

$$\begin{aligned} p_{Bt}\lambda_t &= \beta E_t \lambda_{t+1}\left[p_{Bt+1} + d_{Bt+1}\right] \\ &\quad -\beta E_t \left[\left(c_{t+1} s_{Bt+1}^{-\omega}\left(\psi + l_{t+1}^{\upsilon}\right) - h_{t+1}\right)^{-\sigma} - \varphi_{t+1}(1-m)\right] \omega c_{t+1} s_{Bt+1}^{-\omega-1}\left(\psi + l_{t+1}^{\upsilon}\right) \end{aligned}$$

$$\lambda_t q_{Bt} = \beta E_t \lambda_{t+1} q_{Bt+1}\left[(1-\delta) + \frac{\theta_1^B}{1-\epsilon_B}\left(\frac{i_{Bt+1}}{k_{Bt+1}}\right)^{1-\epsilon_B} + \theta_2^B - \theta_1^B\left(\frac{i_{Bt+1}}{k_{Bt+1}}\right)^{1-\epsilon_B}\right] + \beta E_t \lambda_{t+1} r_{Bt+1}$$

$$1 = q_{Bt}\theta_1^B\left(\frac{i_{Bt}}{k_{Bt}}\right)^{-\epsilon_B}$$

$$\lambda_t q_{Gt} = \beta E_t \lambda_{t+1} q_{Gt+1}\left[(1-\delta) + \frac{\theta_1^G}{1-\epsilon_G}\left(\frac{i_{Gt+1}}{k_{Gt+1}}\right)^{1-\epsilon_G} + \theta_2^G - \theta_1^G\left(\frac{i_{Gt+1}}{k_{Gt+1}}\right)^{1-\epsilon_G}\right] + \beta E_t \lambda_{t+1} r_{Gt+1}$$

$$1 = q_{Gt}\theta_1^G\left(\frac{i_{Gt}}{k_{Gt}}\right)^{-\epsilon_G}$$

where $\lambda_t, \eta_{Gt}, \eta_{Bt}, \varphi_t$ are Lagrange multipliers. q_{Bt} and q_{Gt} denote Tobin's Q in the brown and green sectors, respectively.

Intermediate green sector

Maximize dividends:

$$E_0 \sum_{t=0}^{\infty} \widehat{\beta}^t \frac{\lambda_t}{\lambda_0} d_{Gt}$$

subject to:

$$d_{Gjt} = \rho_G d_{Gjt-1} + z_{Gjt} y_{Gjt} - w_t n_{Gjt} - r_{Gt} k_{Gjt} + p_{Bt}^G \gamma b_{jt+1}^G - b_{jt}^G - \gamma \iota b_{jt+1}^G$$

$$y_{Gjt} = a_{Gjt} k_{Gjt}^{\alpha_G} n_{Gjt}^{1-\alpha_G}$$

First-order conditions:

$$1 + \rho^G \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \Omega_{Gjt+1} = \Omega_{Gjt}$$

$$\Omega_{Gjt} \left(1 - \frac{1}{\kappa_G}\right) z_{Gt} \left(y_{Gt}\right)^{\frac{1}{\kappa_G}} y_{Gjt}^{-\frac{1}{\kappa_G}} = \Xi_{Gjt}$$

$$p_{Bt}^G \Omega_{Gjt} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \Omega_{Gjt+1} + \Omega_{Gjt} \iota$$

$$\Omega_{Gjt} w_t = \Xi_{Gjt} (1 - \alpha_G) \frac{y_{Gjt}}{n_{Gjt}}$$

$$\Omega_{Gjt} r_{Gt} = \Xi_{Gjt} \alpha_G \frac{y_{Gjt}}{k_{Gjt}}$$

$$\pi_{Gjt} = \frac{1}{\kappa_G} z_{Gjt} y_{Gjt} + p_{Bt}^G \gamma b_{jt+1}^G - b_{jt}^G - \iota \gamma b_{jt+1}^G$$

where Ω_{Gjt} is a Lagrange multiplier and Ξ_{Gjt} denotes marginal costs.

Intermediate brown sector

Maximize dividends:

$$E_0 \sum_{t=0}^{\infty} \widehat{\beta}^t \frac{\lambda_t}{\lambda_0} d_{Bjt}$$

subject to:

$$d_{Bjt} = \rho^B d_{Bjt-1} + z_{Bjt} y_{Bjt} - w_t n_{Bjt} - r_{Bt} k_{Bjt} - p_{Pt}(\gamma s_{jPt+1} - s_{jPt}) + p^B_{Bt} \gamma b^B_{jt+1} - b^B_{jt} - \gamma \iota b^B_{jt+1}$$

$$a_{Bjt} k^{\alpha_B}_{Bjt} n^{1-\alpha_B}_{Bjt} = y_{Bjt}$$

$$e_{jt} = \mu z_{Bjt} y_{Bjt}$$

$$\mu z_{Bjt} y_{Bjt} \leq \gamma s_{Pjt+1}$$

$$z_{Bjt} = z_{Bt} \left(\frac{y_{Bt}}{y_{Bjt}} \right)^{\frac{1}{\kappa_B}}$$

First-order conditions:

$$1 + \rho^B \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \Omega_{Bjt+1} = \Omega_{Bjt}$$

$$\left(1-\frac{1}{\kappa_B}\right) z_{Bt} y_{Bt}^{\frac{1}{\kappa_B}} y_{Bjt}^{-\frac{1}{\kappa_B}} \left[\Omega_{Bjt}-\mu\chi_{jt}\right]=\Xi_{Bjt}$$

$$\Omega_{Bjt} p^{B}_{Bt}=\beta E_t \frac{\lambda_{t+1}}{\lambda_t}\Omega_{Bjt+1}+\iota\Omega_{Bjt}$$

$$\Omega_{Bjt} p_{Pt}=\beta E_t \frac{\lambda_{t+1}}{\lambda_t}\Omega_{Bjt+1} p_{Pt+1}+\chi_{jt}$$

$$\Omega_{Bjt} w_t=\Xi_{Bjt}(1-\alpha_B)\frac{y_{Bjt}}{n_{Bjt}}$$

$$\Omega_{Bjt} r_{Bt}=\Xi_{Bjt}\alpha_B\frac{y_{Bjt}}{k_{Bjt}}$$

where Ω_{Bjt} is a Lagrange multiplier and Ξ_{Bjt} denotes marginal costs.

Final good producers

$$d_{Tt}=y_{Bt}^{\varrho} y_{Gt}^{1-\varrho}-z_{Bt}y_{Bt}-z_{Gt}y_{Gt}$$

$$y_{Tt}=y_{Bt}^{\varrho} y_{Gt}^{1-\varrho}$$

First-order conditions:

$$z_{Bt}=\varrho\frac{y_{Tt}}{y_{Bt}}$$

$$z_{Gt}=(1-\varrho)\frac{y_{Tt}}{y_{Gt}}$$

Market clearing

$$y_{Tt} = c_t + i_{Bt} + i_{Gt} + \frac{\xi}{2}\gamma s_{Pt+1}^2$$

C.2 The dynamic system

Consumers

$$\left[\left(c_t s_{Bt}^{-\omega}\left(\psi + l_t^{\upsilon}\right) - h_t\right)^{-\sigma} - \varphi_t(1-m)\right] s_{Bt}^{-\omega}\left(\psi + l_t^{\upsilon}\right) = \lambda_t$$

$$\left[\left(c_t s_{Bt}^{-\omega}\left(\psi + l_t^{\upsilon}\right) - h_t\right)^{-\sigma} - \varphi_t(1-m)\right] c_t s_{Bt}^{-\omega}\upsilon l_t^{\upsilon-1} = \lambda_t(1-\alpha_G)\left(1 - \frac{1}{\kappa_G}\right)\frac{z_{Gt}y_{Gt}}{n_{Gt}}$$

$$(1-\alpha_G)\left(1 - \frac{1}{\kappa_G}\right)\frac{z_{Gt}y_{Gt}}{n_{Gt}} = (1-\alpha_B)\left(1 - \frac{1}{\kappa_B}\right)\frac{z_{Bt}y_{Bt}}{n_{Bt}}\frac{[\Omega_{Bt} - \mu\chi_t]}{\Omega_{Bt}}$$

$$l_t = 1 - n_{Bt} - n_{Gt}$$

Dividends

$$1 = \rho^G \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\Omega_{Gjt+1}$$

$$1 = \rho^B \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\Omega_{Bjt+1}$$

$$d_{Bt} = \left\{1 - \frac{\left(1 - \frac{1}{\kappa_B}\right)[\Omega_{Bt} - \mu\chi_t]}{\Omega_{Bt}}\right\} z_{Bt}y_{Bt} - p_{Pt}(\gamma s_{Pt+1} - s_{Pt}) + p_{Bt}^B\gamma b_{t+1}^B - b_t^B - \gamma\iota b_{t+1}^B$$

$$d_{Gt} = \frac{1}{\kappa_G} z_{Gt}y_{Gt} + p_{Bt}^G\gamma b_{jt+1}^G - b_{jt}^G - \iota\gamma b_{jt+1}^G$$

Habits

$$\varphi_t = m\beta E_t \varphi_{t+1} + \beta E_t \left[c_{t+1} s_{Bt+1}^{-\omega} \left(\psi + l_{t+1}^{v}\right) - h_{t+1}\right]^{-\sigma}$$

$$\gamma h_{t+1} - m h_t - (1-m) c_t s_{Bt}^{-\omega} \left(\psi + l_t^{v}\right) = 0$$

Market for carbon

$$\gamma s_{Pt+1} = \mu z_{Bt} y_{Bt}$$

$$\Omega_{Bj} p_{Pt} \lambda_t = \beta E_t \lambda_{t+1} \Omega_{Bt+1} p_{Pt+1} + \lambda_t \chi_t$$

$$p_{Pt} \lambda_t = \beta E_t \lambda_{t+1} p_{Pt+1} (1-\varrho) + \lambda_t \xi s_{Pt+1}$$

Brown sector

$$\begin{aligned} \lambda_t q_{Bt} &= \beta E_t \lambda_{t+1} q_{Bt+1} \left[(1-\delta_B) + \frac{\theta_1^B}{1-\epsilon_B} \left(\frac{i_{Bt+1}}{k_{Bt+1}}\right)^{1-\epsilon_B} + \theta_2^B - \theta_1^B \left(\frac{i_{Bt+1}}{k_{Bt+1}}\right)^{1-\epsilon_B}\right] \\ &\quad + \beta E_t \lambda_{t+1} \alpha_B \left(1 - \frac{1}{\kappa_B}\right) \frac{z_{Bt+1} y_{Bt+1}}{k_{Bt+1}} \frac{[\Omega_{Bt+1} - \mu \chi_{t+1}]}{\Omega_{Bt+1}} \end{aligned}$$

$$1 = q_{Bt} \theta_1^B \left(\frac{i_{Bt}}{k_{Bt}}\right)^{-\epsilon_B}$$

$$\gamma k_{Bt+1} = (1-\delta_B) k_{Bt} + \left(\frac{\theta_1^B}{1-\epsilon_B} \left(\frac{i_{Bt}}{k_{Bt}}\right)^{1-\epsilon_B} + \theta_2^B\right) k_{Bt}$$

Green sector

$$\lambda_t q_{Gt} = \beta E_t \lambda_{t+1} q_{Gt+1}\left[(1-\delta_G) + \frac{\theta_1^G}{1-\epsilon_G}\left(\frac{i_{Gt+1}}{k_{Gt+1}}\right)^{1-\epsilon_G} + \theta_2^G - \theta_1^G\left(\frac{i_{Gt+1}}{k_{Gt+1}}\right)^{1-\epsilon_G}\right]$$
$$+\beta E_t \lambda_{t+1}\alpha_G\left(1-\frac{1}{\kappa_G}\right)\frac{z_{Gt+1}y_{Gt+1}}{k_{Gt+1}}$$

$$1 = q_{Gt}\theta_1^G\left(\frac{i_{Gt}}{k_{Gt}}\right)^{-\epsilon_G}$$

$$\gamma k_{Gt+1} = (1-\delta_G)k_{Gt} + \left(\frac{\theta_1^G}{1-\epsilon_G}\left(\frac{i_{Gt}}{k_{Gt}}\right)^{1-\epsilon_G} + \theta_2^G\right)k_{Gt}$$

Resource constraint

$$y_{Tt} = c_t + i_{Gt} + i_{Bt} + \frac{\xi}{2}\gamma s_{Pt+1}^2$$

Production

$$y_{Bt} = a_{Bt}k_{Bt}^{\alpha_B}n_{Bt}^{1-\alpha_B}$$

$$y_{Gt} = a_{Gt}k_{Gt}^{\alpha_G}n_{Gt}^{1-\alpha_G}$$

$$y_{Tt} = y_{Bt}^{\varrho}y_{Gt}^{1-\varrho}$$

$$z_{Bt} = \varrho\frac{y_{Tt}}{y_{Bt}}$$

$$z_{Gt} = (1-\varrho)\frac{y_{Tt}}{y_{Gt}}$$

Leverage

$$p^G_{Bt}\lambda_t = \beta E_t \lambda_{t+1} + \eta_{Gt}$$

$$p^B_{Bt}\lambda_t = \beta E_t \lambda_{t+1} + \eta_{Bt}$$

$$p^G_{Bt}\lambda_t \Omega_{Gt} = \beta E_t \lambda_{t+1}\Omega_{Gt+1} + \lambda_t \Omega_{Gt}\iota$$

$$p^B_{Bt}\lambda_t \Omega_{Bt} = \beta E_t \lambda_{t+1}\Omega_{Bt+1} + \lambda_t \Omega_{Bt}\iota$$

Asset pricing

$$p_{Gt}\lambda_t = \beta E_t \lambda_{t+1}\left[p_{Gt+1} + d_{Gt+1}\right]$$

$$\begin{aligned} p_{Bt}\lambda_t &= \beta E_t \lambda_{t+1}\left[p_{Bt+1} + d_{Bt+1}\right] \\ &\quad -\beta E_t \left[\left(c_{t+1} s^{-\omega}_{Bt+1}\left(\psi + l^{v}_{t+1}\right) - h_{t+1}\right)^{-\sigma} - \varphi_{t+1}(1-m)\right] \omega c_{t+1} s^{-\omega-1}_{Bt+1}\left(\psi + l^{v}_{t+1}\right) \end{aligned}$$

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