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Abstract

We study optimal monetary policy in a flexible state-dependent pricing framework, in which monopolistic competition and stochastic menu costs are the only distortions. We show analytically that it is optimal to commit to zero inflation in the long run. Moreover, our numerical simulations indicate that the optimal stabilization policy is "price stability". These findings represent a generalization to a state-dependent framework of the same results found for the simple Calvo model with exogenous timing of price adjustment.

Keywords: optimal monetary policy, price stability, stochastic menu costs, state-dependent pricing

JEL Codes: E31

Non-Technical Summary

A key normative question in monetary economics is one of the optimal design of monetary policy in the presence of price stickiness. An extensive literature has studied this question under the assumption that the timing of price changes is exogenously given. While this is a useful first step, it is nonetheless subject to the criticism that the timing of price changes in principle should not be treated as independent of policy.

We extend the analysis of optimal monetary policy to a model of "state-dependent" pricing by monopolistically competitive firms. In our setting, firms face random lump sum costs of price adjustment, and as a result the timing price changes depends endogenously on the state of the economy. A feature of our framework is that it nests a variety of alternative pricing specifications, including the Calvo model and the fixed menu cost model as extreme limiting cases. Other than that, we maintain the basic setup of Benigno and Woodford (2005) who study optimal monetary policy in the Calvo model. Namely, the monetary authority sets the nominal interest rate optimally, with money's role being only that of a unit of account. Importantly, we do not assume that the permanent distortion due to monopolistic competition is offset through a production subsidy. There are thus two sources of distortion: price stickiness and monopolistic competition.

Our central finding is that the optimal monetary policy in this environment is practically identical to the one derived in the much simpler Calvo model. In particular, we demonstrate analytically that committing to zero inflation in the long run is optimal for a general specification of preferences and of the price-setting strategies followed by firms. Thus, the presence of a permanent monopolistic distortion does not in itself justify either a positive or a negative rate of inflation in the long run, and the optimal policy involves a commitment to eventually bringing down inflation to zero. Moreover, we find that the optimal short-run stabilization policy can be characterized as "price stability". The responses to aggregate productivity and to government expenditure shocks are virtually the same as those under Calvo pricing.

1 Introduction

A key normative question in monetary economics concerns the optimal design of monetary policy in the presence of nominal price rigidity. An extensive literature has studied this question under the assumption that the timing of price changes is given exogenously, as in the Calvo (1983) model with Poisson arrival of price adjustment opportunities.[1] Undoubtedly a useful first step, this literature is nonetheless subject to the Lucas (1976) critique in the sense that the timing of price changes in principle should not be treated as independent of policy.

We extend the analysis of optimal monetary policy to a model of state-dependent pricing by monopolistically competitive firms. Our price-setting framework assumes the presence of stochastic lump sum costs of adjustment *a la* Dotsey, King and Wolman (1999), and is similar to the generalized Ss frameworks adopted by Caballero and Engel (2007) and Costain and Nakov (2008). A feature of these frameworks is that they nest a variety of alternative pricing specifications, including the Calvo model and the fixed menu cost model as extreme limiting cases. Apart for pricing being state-dependent, we maintain the basic setup of Benigno and Woodford (2005) who study optimal monetary policy in the Calvo model. Namely, the monetary authority sets the nominal interest rate optimally, with money's role being only that of a unit of account.[2] Unlike Clarida et. al. (1999), Woodford (2002), or Yun (2005), we do not assume that the static distortion due to monopolistic competition is offset through a production subsidy. There are thus two sources of distortion: price stickiness and monopolistic competition.

Our central finding is that the optimal monetary policy in this environment is practically identical to the one derived in the much simpler Calvo model. In particular, in section 3 we demonstrate analytically that committing to zero inflation in the long run is optimal for a general specification of preferences and of the distribution of menu costs. This generalizes the result of Benigno and Woodford (2005) regarding the optimality of zero steady-state inflation in the Calvo model. Then, in section 4, we assume functional forms for preferences and the distribution of menu costs, and calibrate our model economy. We find that the optimal stabilization policy around the zero inflation steady-state can be characterized as price stability. The impulse-responses to aggregate productivity and government expenditure shocks are virtually the same (to a second-order approximation) as those under Calvo pricing. Moreover, a first-order approximation delivers responses which are basically identical to the ones obtained with a second-order approximation.

[1] E.g. Clarida et. al. (1999), Woodford (2002), Yun (2005), Benigno and Woodford (2005).

[2] As in Woodford (2003), the plan is optimal from a "timeless perspective", that is, it ignores the policymakers' incentives to behave in a special way in the initial few periods, exploiting the fact that private sector expectations had been set prior to the plan's starting date. In the long-run, the "timeless perspective" plan converges to the standard Ramsey optimal policy under commitment.

Our results contrast with recent work by Lie (2009) who also studies optimal monetary policy under state-dependent pricing. Specifically, Lie finds that under state-dependent pricing it is desirable to let inflation vary more than under Calvo pricing. The reason for this discrepancy stems from the fact that Lie considers in addition a monetary distortion, which implies a negative long-run rate of inflation, whereas the optimal long-run rate of inflation is zero in our "cashless" economy. We thus conclude that, although a difference between exogenous-timing and state-dependent pricing models may arise in the presence of distortions implying a non-zero long run rate of inflation, at least in the cashless-limit case the main normative results of exogenous timing models carry over to an economy in which pricing is state-dependent.

2 Model

There are three types of agents: households, firms, and a monetary authority.

2.1 Households

The representative household maximizes the expected flow of period utility $u(C_t) - x(N_t)$, discounted by β, subject to

$$C_t = \left(\int_0^1 C_{it}^{(\epsilon-1)/\epsilon} di \right)^{\epsilon/(\epsilon-1)}, \quad \text{and}$$

$$\int_0^1 P_{it} C_{it} di + R_t^{-1} B_t = W_t N_t + B_{t-1} + \Pi_t,$$

where C_t is basket of differentiated goods $i \in [0,1]$, of quantity C_{it} and price P_{it}; N_t denotes hours worked and W_t the nominal wage rate; B_t are nominally riskless bonds with price R_t^{-1}, and Π_t are the profits of firms owned by the household, net of lump-sum taxes.

The first order conditions are

$$u'(C_t) w_t = x'(N_t), \tag{1}$$

$$R_t^{-1} = \beta E_t \frac{u'(C_{t+1})}{\pi_{t+1} u'(C_t)}, \tag{2}$$

where $w_t \equiv W_t / P_t$ is the real wage, $\pi_t \equiv P_t / P_{t-1}$ is gross inflation, and the aggregate price index is given by

$$P_t \equiv \left(\int_0^1 P_{it}^{1-\epsilon} di \right)^{1/(1-\epsilon)}.$$

2.2 Firms

The firm's production function is

$$y_{it} = z_t n_{it},$$

where z_t is an exogenous aggregate productivity process. The firm's labor demand thus equals $n_{it} = y_{it}/z_t$ and its real cost function is $w_t y_{it}/z_t$. The real marginal cost common to all firms is therefore w_t/z_t. Optimal allocation of expenditure among product varieties by households implies that each individual firm faces a downward sloping demand schedule for its good, given by $y_{it} = (P_{it}/P_t)^{-\epsilon} y_t$.

Following Dotsey et. al. (1999), we assume that firms face random physical costs of adjusting prices ("menu costs"), distributed *i.i.d.* across firms and over time. Let $G(\kappa)$ and $g(\kappa)$ denote, respectively, the cumulative distribution function and the probability density function of the stochastic menu cost. We allow for the possibility that a positive random fraction of firms draws a zero menu cost, so that $G(0) > 0$. Assuming that κ is measured in units of labor time, the total cost paid by a firm changing its price is $w_t \kappa$.[3]

Let v_{0t} denote the value of a firm that adjusts its price in period t before subtracting the menu cost. Let $v_{jt}(P)$ denote the value of a firm which has kept its nominal price unchanged at the level P in the last j periods. This firm will change its nominal price only if the value of adjustment, $v_{0t} - w_t \kappa$, exceeds the value of continuing with the current price, $v_{jt}(P)$. Therefore, from each vintage $j = 1, ..., J-1$ only those firms with a menu cost draw $\kappa \leq (v_{0t} - v_{jt}(P))/w_t$ will choose to change their price. The real value of an adjusting firm is given by

$$\begin{aligned} v_{0t} &= \max_P \{\Pi_t(P) + \beta E_t \frac{u'(C_{t+1})}{u'(C_t)} \left[G\left(\frac{v_{0,t+1} - v_{1,t+1}(P)}{w_{t+1}} \right) v_{0,t+1} - \Xi_{1,t+1}(P) \right] \\ &\quad + \beta E_t \frac{u'(C_{t+1})}{u'(C_t)} \left[1 - G\left(\frac{v_{0,t+1} - v_{1,t+1}(P)}{w_{t+1}} \right) \right] v_{1,t+1}(P) \}, \end{aligned}$$

where

$$\Pi_t(P) \equiv \left(\frac{P}{P_t} - \frac{w_t}{z_t} \right) \left(\frac{P}{P_t} \right)^{-\epsilon} Y_t$$

is the firm's real profit as a function of its nominal price P, and

$$\Xi_{j+1,t+1}(P) \equiv w_{t+1} \int_0^{(v_{0,t+1} - v_{j+1,t+1}(P))/w_{t+1}} \kappa g(\kappa)\, dk$$

[3] Alternatively, one can assume that κ is measured in terms of the basket of final goods, in which case the total cost paid by a firm changing its price is simply κ. The results are not dependent on this assumption.

is next period's expected adjustment cost for a vintage-j firm.

The real value of a firm in vintage $j = 1, ..., J-2$, as a function of its current nominal price P, is given by

$$\begin{aligned} v_{jt}(P) &= \Pi_t(P) + \beta E_t \frac{u'(C_{t+1})}{u'(C_t)} \left[G\left(\frac{v_{0,t+1} - v_{j+1,t+1}(P)}{w_{t+1}} \right) v_{0,t+1} - \Xi_{j+1,t+1}(P) \right] \\ &\quad + \beta E_t \frac{u'(C_{t+1})}{u'(C_t)} \left[1 - G\left(\frac{v_{0,t+1} - v_{j+1,t+1}(P)}{w_{t+1}} \right) \right] v_{j+1,t+1}(P). \end{aligned} \tag{3}$$

We make two technical assumptions to ensure the existence and uniqueness of a stationary equilibrium defined over a finite state space. First, we assume that J periods after the last price adjustment, firms draw a zero menu cost. This means that J is the oldest vintage so that firms in vintage $J-1$ expect that in the following period they will adjust their price with probability one. Second, we assume that with some probability $1-\delta > 0$ the owner of a vintage-$(J-1)$ firm sells all his shares in the firm (for instance due to retirement) and obtains a perpetuity value equal to $Y_t / [\epsilon(1-\beta)] \equiv \bar{v}_t$. This ensures a unique steady-state solution for the Lagrange multipliers of the optimal monetary policy problem.

With these assumptions, the value of a vintage-$(J-1)$ firm is

$$v_{J-1,t}(P) = \Pi_t(P) + \beta E_t \frac{u'(C_{t+1})}{u'(C_t)} \left[\delta v_{0,t+1} + (1-\delta)\bar{v}_{t+1} \right]. \tag{4}$$

The optimal price setting decision is given by

$$0 = \Pi_t'(P_t^*) + \beta E_t \frac{u'(C_{t+1})}{u'(C_t)} \left[1 - G\left(\frac{v_{0,t+1} - v_{1,t+1}(P_t^*)}{w_{t+1}} \right) \right] v_{1,t+1}'(P_t^*), \tag{5}$$

where

$$\Pi_t'(P) = \left[\epsilon \frac{w_t}{z_t} - (\epsilon - 1) \frac{P}{P_t} \right] (P)^{-\epsilon-1} P_t^{\epsilon} Y_t$$

and we have used the fact that, by Leibniz's rule,

$$\Xi_{1,t+1}'(P_t^*) = -v_{1,t+1}'(P_t^*) \frac{v_{0,t+1} - v_{1,t+1}(P_t^*)}{w_{t+1}} g\left(\frac{v_{0,t+1} - v_{1,t+1}(P_t^*)}{w_{t+1}} \right).$$

Iterating (5) forward, and using the implications of (3) and (4) for the terms $v_{j,t+j}'(P_t^*)$, $j = 1, ..., J-1$, we can express the pricing decision as

$$P_t^* = \frac{\epsilon}{\epsilon - 1} \frac{\sum_{j=0}^{J-1} \beta^j E_t u'(C_{t+j}) \prod_{k=1}^{j} (1 - \lambda_{k,t+k}) P_{t+j}^{\epsilon} Y_{t+j} (w_{t+j}/z_{t+j})}{\sum_{j=0}^{J-1} \beta^j E_t u'(C_{t+j}) \prod_{k=1}^{j} (1 - \lambda_{k,t+k}) P_{t+j}^{\epsilon-1} Y_{t+j}},$$

where

$$\lambda_{jt} \equiv G\left(\frac{v_{0t} - v_{jt}}{w_t} \right) \tag{6}$$

denotes the period t adjustment probability of firms in vintage $j = 1, ..., J-1$, and we define

$v_{jt} \equiv v_{jt}(P^*_{t-j})$ for short. This pricing decision is analogous to the one in the Calvo model, with the term $\prod_{k=1}^{j}(1-\lambda_{k,t+k})$ replacing $\left(1-\lambda^C\right)^j$ where λ^C is the constant adjustment probability in the Calvo framework. We can rewrite the price decision in terms of stationary variables as

$$p^*_t = \frac{\epsilon}{\epsilon-1}\frac{\sum_{j=0}^{J-1}\beta^j E_t \prod_{k=1}^{j}(1-\lambda_{k,t+k})\left(\prod_{k=1}^{j}\pi_{t+k}\right)^{\epsilon} u'(C_{t+j})\, Y_{t+j}\,(w_{t+j}/z_{t+j})}{\sum_{j=0}^{J-1}\beta^j E_t \prod_{k=1}^{j}(1-\lambda_{k,t+k})\left(\prod_{k=1}^{j}\pi_{t+k}\right)^{\epsilon-1} u'(C_{t+j})\, Y_{t+j}}, \tag{7}$$

where $p^*_t \equiv P^*_t/P_t$ is the optimal relative price.

2.3 Market clearing

Labor input is required both for the production of goods and for the process of changing prices. Labor demand for production by firm i is $n_{it} = y_{it}/z_t = (P_{it}/P_t)^{-\epsilon} y_t/z_t$. Thus, total labor demand for production purposes equals $\Delta_t y_t/z_t$, where $\Delta_t \equiv \int_0^1 (P_{it}/P_t)^{-\epsilon}\, di$ denotes relative price dispersion. At the same time, the total amount of labor used by vintage-j firms for pricing purposes equals $\psi_{jt}\int_0^{(v_{0t}-v_{jt})/w_t}\kappa g(\kappa)\, dk$, where ψ_{jt} is the measure of firms in vintage j.

Equilibrium in the labor market therefore implies,

$$N_t = \frac{Y_t\Delta_t}{z_t} + \sum_{j=1}^{J-1}\psi_{jt}\int_0^{(v_{0t}-v_{jt})/w_t}\kappa g(\kappa)\, dk. \tag{8}$$

And equilibrium in the goods market requires that

$$Y_t = C_t + G_t, \tag{9}$$

where government consumption G_t is assumed to follow an exogenous AR(1) process.

2.4 Inflation, price dispersion, and price distribution dynamics

Absent firm-level shocks, all firms adjusting at time t choose the same nominal price, P^*_t. Under the assumption that no nominal price survives for longer than J periods, the finite set of beginning-of-period prices at any time t is $\left\{P^*_{t-1}, P^*_{t-2}, ..., P^*_{t-J}\right\}$. Let ψ_{jt} denote the time-t fraction of firms with beginning-of-period nominal price P^*_{t-j}, for $j = 1, 2, ..., J$, such that $\sum_{j=1}^{J}\psi_{jt} = 1$. The price level evolves according to

$$P_t^{1-\epsilon} = (P^*_t)^{1-\epsilon}\sum_{j=1}^{J}\lambda_{jt}\psi_{jt} + \sum_{j=1}^{J-1}\left(P^*_{t-j}\right)^{1-\epsilon}(1-\lambda_{jt})\,\psi_{jt},$$

where adjustment probabilities λ_{jt}, $j = 1, ..., J-1$, are given by (6), and where $\lambda_{J,t} = 1$. Rescaling both sides of the above equation by P_t, we obtain

$$1 = (p^*_t)^{1-\epsilon}\sum_{j=1}^{J}\lambda_{jt}\psi_{jt} + \sum_{j=1}^{J-1}\left(\frac{p^*_{t-j}}{\prod_{k=0}^{j-1}\pi_{t-k}}\right)^{1-\epsilon}(1-\lambda_{jt})\,\psi_{jt}. \tag{10}$$

This equation determines π_t, given $\left\{p^*_{t-j}\right\}_{j=0}^{J-1}$ and $\{\pi_{t-j}\}_{j=1}^{J-2}$. Similarly, price dispersion follows

$$\Delta_t = (p^*_t)^{-\epsilon} \sum_{j=1}^{J} \lambda_{jt}\psi_{jt} + \sum_{j=1}^{J-1} \left(\frac{p^*_{t-j}}{\prod_{k=0}^{j-1} \pi_{t-k}} \right)^{-\epsilon} (1-\lambda_{jt})\,\psi_{jt}, \tag{11}$$

where again $\lambda_{J,t} = 1$. The distribution of beginning-of-period prices evolves according to

$$\psi_{j,t} = (1-\lambda_{j-1,t-1})\,\psi_{j-1,t-1} \tag{12}$$

for $j = 2, ..., J$, and

$$\psi_{1t} = 1 - \sum_{j=2}^{J} \psi_{j,t} = \sum_{j=1}^{J} \lambda_{j,t-1}\psi_{j,t-1} = \lambda_{1,t-1}\psi_{1,t-1} + \lambda_{2,t-1}\psi_{2,t-1} + ... + \psi_{J,t-1}. \tag{13}$$

2.5 Equilibrium

There are $8 + 2J + (J-1) = 7 + 3J$ stationary endogenous variables: C_t, N_t, Y_t, R_t, π_t, p^*_t, w_t, Δ_t, $\left\{\psi_{jt}\right\}_{j=1}^{J}$, $\{v_{jt}\}_{j=0}^{J-1}$ and $\{\lambda_{jt}\}_{j=1}^{J-1}$. The equilibrium conditions are (1), (2), the $J-1$ equations (6), (7) to (11), the J laws of motion (12) and (13), the value functions

$$\begin{aligned} v_{jt} \;\; = \;\; & \left(\frac{p^*_{t-j}}{\prod_{k=0}^{j-1} \pi_{t-k}} - \frac{w_t}{z_t} \right) \left(\frac{p^*_{t-j}}{\prod_{k=0}^{j-1} \pi_{t-k}} \right)^{-\epsilon} Y_t \\ & + \beta E_t \frac{u'(C_{t+1})}{u'(C_t)} \left[\lambda_{j+1,t+1} v_{0,t+1} + (1-\lambda_{j+1,t+1})\, v_{j+1,t+1} - w_{t+1} \int_0^{(v_{0,t+1}-v_{j+1,t+1})/w_{t+1}} \kappa dG(\kappa) \right] \end{aligned}$$

for $j = 0, 1, ..., J-2$, and

$$v_{J-1,t} = \left(\frac{p^*_{t-(J-1)}}{\prod_{k=0}^{(J-1)-1} \pi_{t-k}} - \frac{w_t}{z_t} \right) \left(\frac{p^*_{t-(J-1)}}{\prod_{k=0}^{(J-1)-1} \pi_{t-k}} \right)^{-\epsilon} Y_t + \beta E_t \frac{u'(C_{t+1})}{u'(C_t)} \left[\delta v_{0,t+1} + (1-\delta)\, \bar{v}_{t+1} \right];$$

plus a specification of monetary policy. If we were to close the model with a Taylor rule, this would give us a total of $2 + (J-1) + 5 + J + J + 1 = 7 + 3J$ equations. We will, however, focus on optimal policy, which will essentially double the number of equations and variables.

2.5.1 Flexible price equilibrium

In the flexible price equilibrium, menu costs are zero and all firms choose the same nominal price $P^*_t = \frac{\epsilon}{\epsilon-1} \frac{w_t}{z_t} P_t$ in each period t. All relative prices are one: $p^*_t = P^*_t/P_t = 1$. The equilibrium conditions simplify to

$$
\begin{aligned}
u'\left(C_t^{fp}\right) w_t^{fp} &= x'\left(N_t^{fp}\right), \\
z_t N_t^{fp} &= Y_t^{fp}, \\
Y_t^{fp} &= C_t^{fp} + G_t, \\
z_t &= \frac{\epsilon}{\epsilon - 1} w_t^{fp},
\end{aligned}
$$

and so we obtain the classical decoupling of real and nominal variables. The flexible-price output Y_t^{fp} derived above is used in defining the output gap as the difference between actual output and its flexible price counterpart.

3 Optimal monetary policy problem

For the purpose of deriving the optimality conditions of the Ramsey plan, it is useful to define

$$
\pi_{jt}^{acc} \equiv \prod_{k=0}^{j-1} \pi_{t-k} = \frac{P_t}{P_{t-j}}, \quad j = 1, ..., J-1,
$$

that is, the accumulated inflation between periods $t-j$ and t. This implies $\prod_{k=1}^{j} \pi_{t+k} = \pi_{j,t+j}^{acc}$. We also define

$$
\theta_{jt}^{acc} \equiv \prod_{k=0}^{j-1} \left(1 - \lambda_{j-k,t-k}\right), \quad j = 1, ..., J-1,
$$

which in turn implies $\prod_{k=1}^{j} \left(1 - \lambda_{k,t+k}\right) = \theta_{j,t+j}^{acc}$. These definitions allow us to express the optimal pricing decision in a more compact form,

$$
p_t^* = \frac{\epsilon}{\epsilon - 1} \frac{\sum_{j=0}^{J-1} \beta^j E_t \theta_{j,t+j}^{acc} \left(\pi_{j,t+j}^{acc}\right)^{\epsilon} u'\left(C_{t+j}\right) Y_{t+j} \left(w_{t+j}/z_{t+j}\right)}{\sum_{j=0}^{J-1} \beta^j E_t \theta_{j,t+j}^{acc} \left(\pi_{j,t+j}^{acc}\right)^{\epsilon-1} u'\left(C_{t+j}\right) Y_{t+j}}.
$$

Similarly, we replace $\prod_{k=0}^{j-1} \pi_{t-k}$ by π_{jt}^{acc} in the laws of motion of inflation and price dispersion, and in the firms' value functions. It is useful to express the variables π_{jt}^{acc} and θ_{jt}^{acc} recursively,

$$
\pi_{jt}^{acc} = \pi_t \pi_{j-1,t-1}^{acc}, \quad j = 1, ..., J-1,
$$

$$
\theta_{jt}^{acc} = \left(1 - \lambda_{jt}\right) \theta_{j-1,t-1}^{acc}, \quad j = 1, ..., J-1,
$$

where the recursions start with $\pi_{0,t-1}^{acc} = 1$ and $\theta_{0,t-1}^{acc} = 1$, respectively. In addition, we use the constraint $Y_t = C_t + G_t$ to substitute for C_t in the equilibrium conditions. At time 0, the central bank chooses the state-contingent path for all endogenous variables which maximizes the following Lagrangian,

$$\begin{aligned}
&\mathcal{L}_0 = E_0 \sum_{t=0}^{\infty} \beta^t \{ u\left(Y_t - G_t\right) - x\left(N_t\right) \qquad (14)\\
&+\phi_t^w \left[u'\left(Y_t - G_t\right) w_t - x'\left(N_t\right)\right]\\
&+\phi_t^{p^*} p_t^* \sum_{j=0}^{J-1} \beta^j \theta_{j,t+j}^{acc} \left(\pi_{j,t+j}^{acc}\right)^{\epsilon-1} u'\left(Y_{t+j} - G_{t+j}\right) Y_{t+j}\\
&-\phi_t^{p^*} \left[\epsilon/(\epsilon-1)\right] \sum_{j=0}^{J-1} \beta^j \theta_{j,t+j}^{acc} \left(\pi_{j,t+j}^{acc}\right)^{\epsilon} u'\left(Y_{t+j} - G_{t+j}\right) Y_{t+j} w_{t+j}/z_{t+j}\\
&+\phi_t^N \left[N_t - Y_t \Delta_t / z_t - \sum_{j=1}^{J-1} \psi_{jt} \int_0^{(v_{0t}-v_{jt})/w_t} \kappa g\left(\kappa\right) dk\right]\\
&+\phi_t^{\pi} \left[\left(p_t^*\right)^{1-\epsilon} \sum_{j=1}^{J} \lambda_{jt} \psi_{jt} + \sum_{j=1}^{J-1} \left(p_{t-j}^* / \pi_{jt}^{acc}\right)^{1-\epsilon} \left(1 - \lambda_{jt}\right) \psi_{jt}\right]\\
&+\phi_t^{\Delta} \left[\left(p_t^*\right)^{-\epsilon} \sum_{j=1}^{J} \lambda_{jt} \psi_{jt} + \sum_{j=1}^{J-1} \left(p_{t-j}^* / \pi_{jt}^{acc}\right)^{-\epsilon} \left(1 - \lambda_{jt}\right) \psi_{jt} - \Delta_t\right]\\
&+\sum_{j=1}^{J-1} \phi_t^{\lambda,j} \left[\lambda_{jt} - G\left((v_{0t} - v_{jt})/w_t\right)\right]\\
&+\sum_{j=2}^{J} \phi_t^{\psi,j} \left[\psi_{j,t} - \left(1 - \lambda_{j-1,t-1}\right) \psi_{j-1,t-1}\right] + \phi_t^{\psi,1} \left[\psi_{1t} + \sum_{j=2}^{J} \psi_{j,t}\right]\\
&+\sum_{j=0}^{J-2} \phi_t^{v,j} \left[\left(p_{t-j}^* / \pi_{jt}^{acc} - w_t / z_t\right) \left(p_{t-j}^* / \pi_{jt}^{acc}\right)^{-\epsilon} Y_t u'\left(Y_t - G_t\right) - v_{jt} u'\left(Y_t - G_t\right)\right]\\
&+\sum_{j=0}^{J-2} \phi_t^{v,j} \beta u'\left(Y_{t+1} - G_{t+1}\right) \left(\lambda_{j+1,t+1} v_{0,t+1} + \left(1 - \lambda_{j+1,t+1}\right) v_{j+1,t+1}\right)\\
&-\sum_{j=0}^{J-2} \phi_t^{v,j} \beta u'\left(Y_{t+1} - G_{t+1}\right) w_{t+1} \int_0^{(v_{0,t+1} - v_{j+1,t+1})/w_{t+1}} \kappa g\left(\kappa\right) dk\\
&+\phi_t^{v,J-1} \left[\left(p_{t-(J-1)}^* / \pi_{(J-1),t}^{acc} - w_t / z_t\right) \left(p_{t-(J-1)}^* / \pi_{(J-1),t}^{acc}\right)^{-\epsilon} Y_t u'\left(Y_t - G_t\right) - v_{J-1,t} u'\left(Y_t - G_t\right)\right]\\
&+\phi_t^{v,J-1} \beta u'\left(Y_{t+1} - G_{t+1}\right) \left(\delta v_{0,t+1} + (1 - \delta) \bar{v}_{t+1}\right)\\
&+\phi_t^{\pi acc,1} \left[\pi_{1t}^{acc} - \pi_t\right] + \sum_{j=2}^{J-1} \phi_t^{\pi acc,j} \left[\pi_{jt}^{acc} - \pi_t \pi_{j-1,t-1}^{acc}\right]\\
&+\phi_t^{\theta acc,1} \left[\theta_{1t}^{acc} - \left(1 - \lambda_{1t}\right)\right] + \sum_{j=2}^{J-1} \phi_t^{\theta acc,j} \left[\theta_{jt}^{acc} - \left(1 - \lambda_{jt}\right) \theta_{j-1,t-1}^{acc}\right] \}.
\end{aligned}$$

Since the nominal interest rate only appears in the consumption Euler equation, the latter is excluded from the set of constraints on the Ramsey problem. Instead, this equation is used residually to back out the nominal interest rate path consistent with the optimal allocation. The first order conditions of the above problem are given in Appendix A.

4 Optimal long run goal: zero inflation

In Appendix B we prove that the Ramsey problem has a steady-state in which inflation is zero. This generalizes the result of Benigno and Woodford (2005) obtained for the Calvo model to a state-dependent setting. Namely, the presence of a static monopolistic distortion does not justify either a positive or a negative rate of inflation in the long run, and the optimal policy involves a commitment to eventually eliminating any inefficient price dispersion due to nominal price rigidity. The key insight of the Calvo framework, about the desirability of zero long-run inflation, therefore, survives in the more general case of state-dependent pricing.

To better understand this result, let us consider the different welfare effects of inflation in steady state. First, in the presence of price stickiness, inflation increases the extent of price dispersion in the economy. This is inefficient as it increases the amount of labor effort needed to produce any given amount of output, and hence it lowers welfare. Second, a commitment to positive inflation raises the inflation expectations of price-setters. The latter shifts the Phillips curve upwards, worsening the short-run trade-off between output and inflation. Third, holding constant future inflation expectations, a rise in current inflation raises output towards its socially efficient level, thus reducing the monopolistic distortion and improving welfare. It turns out that at zero inflation, the marginal welfare cost of a small increase in inflation exactly offsets the marginal welfare benefit. As a result, it is optimal to commit to eventually reaching zero inflation in the absence of aggregate shocks.

Indeed, the welfare effects of steady state inflation are analogous to the ones in exogenous-timing models of price adjustment, such as the Calvo or the Taylor model. In our state-dependent pricing framework, trend inflation affects the value functions of firms in each vintage, by affecting their steady-state relative price and hence their profits. One might think that this would influence the steady-state price adjustment probabilities, which in turn would affect both the position of the Phillips curve and the total amount of resources used in pricing activities. However, the fact that price-setting firms choose their prices optimally implies that, at zero inflation, a marginal increase in inflation has no effect on firms' profits and hence on adjustment probabilities. Therefore, the reasons for which zero steady state inflation is optimal in the exogenous-timing models continue to hold in a state-dependent pricing framework.

Importantly, the above result is independent of the specification of preferences, or of the shape of the menu cost distribution. The key assumption is that of a "cashless economy", that is, the absence of a monetary friction which pushes optimal inflation towards the negative of the real interest rate. In this respect our analysis differs from that of Lie (2009) who considers explicitly a monetary distortion.[4]

5 Optimal stabilization policy: price stability

In this section we analyze the optimal stabilization policy in our economy. We illustrate this by showing the optimal dynamic responses of several key variables to two types of shocks: to aggregate productivity and to government consumption. Our main finding is that, under a second-order approximation to the general equilibrium dynamics of the model, these responses are identical to the ones obtained in the Calvo model. Moreover, the responses are essentially the same when approximating to first rather than to second order.

[4]Lie argues that monetary frictions are needed to ensure finiteness of the number of vintages. Indeed, trend deflation induced by monetary frictions together with the assumption of an upper bound on menu costs imply an endogenous finite number of price vintages in steady state. In contrast, under zero inflation firms' prices never drift away from the optimum and therefore the number of "vintages" in principle must grow over time. We circumvent this issue by simply assuming a finite (but arbitrarily large) number of vintages, as a useful approximation to a "true model" with infinite vintages.

5.1 Calibration

To produce impulse responses we must specify functional forms and assign values to the model's parameters. We take most of the parameters from Golosov and Lucas (2007). In particular, $u(C_t) = C_t^{1-\gamma}/(1-\gamma)$ with $\gamma = 2$, and $x(N_t) = \chi N_t^{1+\varphi}/(1+\varphi)$ with $\chi = 6$ and $\varphi = 1$. The discount factor is $\beta = 1.04^{-1/4}$ and the elasticity of substitution among product varieties is $\epsilon = 7$.

We further assume that the cumulative distribution function of menu costs takes the form

$$G(\kappa) = \frac{\xi + \kappa}{\alpha + \kappa},$$

where both ξ and α are positive parameters. Therefore, from equation (6) the fraction of vintage-j firms that adjust their price in a given period equals

$$\lambda_{jt} = G\left(\frac{v_{0t} - v_{jt}}{w_t}\right) = \frac{\xi + (v_{0t} - v_{jt})/w_t}{\alpha + (v_{0t} - v_{jt})/w_t}.$$

As in Costain and Nakov (2008), this function is increasing in the gain from adjustment $v_{0t} - v_{jt}$ and is bounded above by 1. Unlike Costain and Nakov (2008), the function is bounded below not by 0 but by $\xi/\alpha > 0$. We make this technical assumption to ensure a unique stationary distribution of firms over the (finite number of) price vintages even with zero steady-state inflation. We are free to choose any arbitrarily small ξ and so we pick the value $1e - 10$. We then set $\alpha = 0.0006$ so that, under a policy targeting 2% inflation (consistent with the average observed rate in the US since the mid-1980's) the model produces an average frequency of price changes of once every three quarters (consistent with the micro evidence found e.g. by Nakamura and Steinsson, 2008). With these settings, the model implies virtually zero probability of adjustment when the gain from adjustment is zero. Finally, we set the maximum price duration to $J = 24$ quarters, a number which is much greater than any observed price duration in recent US evidence.

Figure 1 shows the adjustment hazard function and the distribution of firms by price vintage with 2% trend inflation. In the left panel, the adjustment probability increases rapidly with price age, reaching 90% after ten quarters. As shown in the right panel, this implies that virtually no price survives for longer than eight quarters.

We focus on two types of shock. One is an aggregate technology shock with persistence $\rho_z = 0.95$ and the other is a government expenditure shock with persistence $\rho_g = 0.9$. Government expenditure is calibrated so that it accounts for roughly 17% of GDP in steady-state, consistent with US postwar experience. In Section 5 on robustness, we discuss the effects of cost-push and idiosyncratic shocks.

5.2 Impulse-responses under the optimal policy

We use a second-order Taylor expansion to approximate the equilibrium dynamics of our model. Figure 2 plots the responses of several variables of interest to two independent shocks: a 1% improvement in aggregate technology, and a 1% increase in the level of government spending. Characteristically, four variables – the optimal reset price, inflation, and price dispersion (shown in the last row of the figure), and the output gap, defined as the difference between actual output and its flexible price counterpart (and shown in the third panel on the top row), remain completely constant in response to each of the shocks. In fact, this is precisely what happens in response to the same shocks in the Calvo model (not shown due to the overlap, but available on request). Moreover, the responses of the interest rate, consumption, hours worked and wages, all coincide (up to a second order approximation) with their counterparts in the Calvo model. While this constitutes no formal proof, it is strongly indicative of the optimality of price stability in our framework.

What is the intuition for this result? There are four potential inefficiencies in the present model, related to: (1) the level and volatility of price dispersion; (2) the volatility of the average markup; (3) the waste of resources due to menu costs; and (4) the level of the average markup due to imperfect competition. Distortions (1) to (3) are directly related to the friction in price-setting, and, absent idiosyncratic shocks, a policy of price stability eliminates all three by eliminating the incentives for price adjustment, thus replicating the flexible price equilibrium. The fourth inefficiency is a static markup distortion due to monopolistic competition. As we have already shown in the previous section, the optimal Ramsey plan does not involve a correction of this inefficiency because it is outweighed by the gains of committing to zero inflation and achieving the minimum possible price dispersion in the long run, independently of the particular Ss price-setting policies followed by firms. Figure 2 shows that the central bank's incentives to deviate from zero inflation so as to reduce monopolistic distortions are virtually inexistent also in response to real shocks.

In passing we note that a first-order accurate solution of the model yields virtually identical impulse-responses, both under Calvo and under stochastic menu costs, at least for small aggregate shocks.[5] We thus find that the simple linear Calvo framework offers a very good approximation to the behavior of a cashless state-dependent pricing economy under the optimal monetary policy rule.

Our finding is in contrast with Lie (2009) who also studies optimal monetary policy with state-dependent pricing. Specifically, Lie finds that in a stochastic menu cost environment it is desirable to let inflation vary more than with Calvo pricing. Since the only substantial difference between our models is the fact that he considers in addition a monetary distortion (implying a negative long-run rate of inflation), we are led to conclude that the discrepancy

[5] We use 24 vintages when approximating the solution to first order, and 8 vintages when approximating it to second order. When plotted, the two sets of impulse-responses are practically indistinguishable to the naked eye.

in our results stems entirely from the fact that we study a "cashless" economy in which the optimal long-run rate of inflation is zero.

6 Robustness

6.1 Cost-push shocks

The two shocks which we analyze in the previous section (to productivity and to government spending) involve virtually no trade-off between stabilizing prices and stabilizing the output gap (the difference between output and its efficient level).[6] However, a number of economists argue that an important source of aggregate fluctuations are the so-called "cost-push" shocks. As a robustness check, we introduce such a shock as a multiplicative disturbance to the reset price p_t^* as defined in equation (7). The disturbance is assumed to follow an exogenous AR(1) process with persistence $\rho_\mu = 0.8$.

Figure 3 plots the responses to this shock in the Calvo and in the stochastic menu cost model. In the Calvo model there is a small transitory rise in inflation accompanied by a temporary fall in consumption and the output gap. Yet, price dispersion remains virtually constant under the optimal policy. We find that the latter is true also in the stochastic menu cost model, namely, price dispersion is unaffected by the cost-push shock. However, there are important quantitative differences with the Calvo model in the responses of other variables. In particular, while inflation still rises and consumption drops on impact, in both cases the initial effect is much smaller, but more persistent, than in the Calvo model. Interestingly, and contrary to the Calvo model, with stochastic menu costs output initially rises above its flexible price level, opening a positive output gap, which impact however is quickly reversed within a couple of quarters, followed by a persistently lower level of output compared to its flexible price counterpart.

6.2 Firm-level shocks

For tractability the above analysis abstracts from the presence of firm-level shocks despite the strong evidence in favor of their existence (e.g. Klenow and Kryvtsov, 2008, Golosov and Lucas, 2007). Yet, we would argue that the mere existence of such shocks does not necessarily imply very different monetary policy recommendations.

With state-dependent pricing, monetary policy has the additional channel of improving real allocations by increasing firms' likelihood of adjusting their prices. On the other hand, the existence of firm level shocks implies that replicating the flexible price equilibrium is no longer feasible. In particular, actual price dispersion would differ from the efficient (non-degenerate) one under any monetary policy that fails to trigger continuous price adjustment by all firms.

[6] In fact they do involve a tiny tradeoff, but it is so small that the deviations of the price level from steady-state are on the order of 1/1000th of a basis point.

Since desired price increases and price decreases due to idiosyncratic factors are estimated to be quite large in practice, and yet firms find it optimal sometimes not to adjust their prices, the inflation impulse necessary to trigger a synchronized adjustment by all firms must be extremely big, e.g. more than 50% per month. Even assuming that such a radical policy would succeed in producing a simultaneous price response by *all* economic agents, it would imply a maximum flow of menu costs per period, which is likely to be suboptimal.

As for small deviations from price stability, in principle the existence of this channel in combination with firm level shocks could shift the balance in minimizing distortions 1 to 4 (listed in the previous section).[7] However, we find no compelling reasons to think that this effect would be quantitatively important. For one thing, increasing the probability of adjustment would still come at the cost of real resources used in price setting, and firms are already taking this margin into account when making their optimal decisions. Therefore, there must be some important externality that individual firms fail to take into account when setting their prices, and which monetary policy is able to alleviate, for it to be optimal to deviate from price stability.

In particular, for small deviations from price stability, large idiosyncratic shocks call for a roughly equal number of price increases and price decreases. By raising inflation somewhat the central bank can induce marginal firms contemplating a price increase to change their price, thereby increasing economic efficiency, but at the same time it will discourage a similar number of marginal firms contemplating a price decrease, which presumably would lower efficiency. While these opposing effects on efficiency might roughly cancel each other out, the inflation shock would result in a persistent price misalignment for all those firms which failed to adjust their nominal price and whose desired relative price change does not happen to exactly equal the negative of the rate of inflation. In reality the majority of firms do not change their prices every month (even in the presence of both idiosyncratic and aggregate shocks), and therefore we conjecture that the latter effect of inefficient price dispersion due to variability of the price level is likely to dwarf any potential gains in efficiency from a marginal increase in the probability of adjustment by firms.[8]

[7] With idiosyncratic productivity shocks the relevant measure of inefficient price dispersion is $\Delta_t \equiv P_t^{\epsilon} \int_0^1 P_{it}^{-\epsilon} A_{it}^{-1} di$, where A_{it} denotes firm i's productivity level. This generalizes the dispersion measure in Yun (2005) to allow for heterogeneous productivity. See Costain and Nakov (2008b)

[8] One simple way of introducing firm-level shocks into our framework is to assume, as in Gertler and Leahy (2008), that such shocks have a constant exogenous probability of arrival. If, in addition, idiosyncratic shocks are *always* much larger in absolute value than aggregate shocks, *and* adjustment to them brings value gains which exceed even the maximum possible menu cost, then firms would adjust their prices immediately in response to an idiosyncratic shock, but only sluggishly in response to aggregate shocks. While this may seem like a "knife-edge" scenario, it is consistent with recent empirical studies by Mackowiak et. al. (2009) and Boivin et. al. (2009) who estimate immediate price adjustment to sectoral shocks, but only sluggish adjustment to aggregate shocks. In the environment described above, idiosyncratic shocks would simply be irrelevant for monetary policy because prices are fully flexible with respect to such shocks. Hence, our previous analysis from sections 4 and 5 would carry over, namely price stability would still be the optimal policy.

7 Conclusion

We have shown that the main lessons for optimal monetary policy derived in the canonical Calvo model carry over to a more general setup in which firms' probability of changing prices depends on the state of the economy. In particular, the optimal long run rate of inflation is zero, and the optimal stabilization policy can be characterized as "price stability". This means that, in the long run, the central bank should not try to offset the static distortion arising from monopolistic competition by varying the rate of inflation.

The above result lends support to more informal statements about the suitability of the Calvo model for optimal monetary policy exercises despite its apparent conflict with the Lucas (1976) critique.

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Appendices

A. First order conditions of the monetary policy problem

Each expression below is a first order condition with respect to the variable in parenthesis on the right, and must be equal to zero:

$$\begin{aligned}
&u'(C_t)+\phi_t^w u''(C_t)\,w_t+[u''(C_t)\,Y_t+u'(C_t)]\textstyle\sum_{j=0}^{J-1}\phi_{t-j}^{p^*}\left(p_{t-j}^*/\pi_{jt}^{acc}-\epsilon/(\epsilon-1)w_t/z_t\right)\theta_{jt}^{acc}\left(\pi_{jt}^{acc}\right)^{\epsilon}\\
&-\phi_t^N\Delta_t/z_t+\textstyle\sum_{j=0}^{J-1}\phi_t^{v,j}\left\{\left(p_{t-j}^*/\pi_{jt}^{acc}-w_t/z_t\right)\left(p_{t-j}^*/\pi_{jt}^{acc}\right)^{-\epsilon}[u''(C_t)\,Y_t+u'(C_t)]-u''(C_t)\,v_{jt}\right\}\\
&\qquad+\textstyle\sum_{j=0}^{J-2}\phi_{t-1}^{v,j}\left(\lambda_{j+1,t}v_{0,t}+(1-\lambda_{j+1,t})\,v_{j+1,t}-w_t\int_0^{L_{j+1,t}}\kappa dG(\kappa)\right)u''(C_t)\\
&\qquad\qquad+\phi_{t-1}^{v,J-1}\left(\delta v_{0,t}+(1-\delta)\,\bar{v}_t\right)u''(C_t)+\phi_{t-1}^{v,J-1}(1-\delta)\,u'(C_t)\,/\,[\epsilon(1-\beta)]\,,\quad (Y_t)
\end{aligned}$$

$$\begin{aligned}
&\phi_t^{p^*}\textstyle\sum_{j=0}^{J-1}\beta^j\theta_{j,t+j}^{acc}\left(\pi_{j,t+j}^{acc}\right)^{\epsilon-1}u'(C_{t+j})\,Y_{t+j}\\
&\quad+\left[\phi_t^{\pi}\textstyle\sum_{j=1}^{J}\lambda_{jt}\psi_{jt}+E_t\sum_{j=1}^{J-1}\beta^j\phi_{t+j}^{\pi}\left(\pi_{j,t+j}^{acc}\right)^{\epsilon-1}(1-\lambda_{j,t+j})\,\psi_{j,t+j}\right](1-\epsilon)\,(p_t^*)^{-\epsilon}\\
&\quad+\left[\phi_t^{\Delta}\textstyle\sum_{j=1}^{J}\lambda_{jt}\psi_{jt}+E_t\sum_{j=1}^{J-1}\beta^j\phi_{t+j}^{\Delta}\left(\pi_{j,t+j}^{acc}\right)^{\epsilon}(1-\lambda_{j,t+j})\,\psi_{j,t+j}\right](-\epsilon)\,(p_t^*)^{-\epsilon-1}\\
&\quad+E_t\textstyle\sum_{j=0}^{J-1}\beta^j\phi_{t+j}^{v,j}\left[\epsilon w_{t+j}/z_{t+j}-(\epsilon-1)\,p_t^*/\pi_{j,t+j}^{acc}\right](p_t^*)^{-\epsilon-1}\left(\pi_{j,t+j}^{acc}\right)^{\epsilon}Y_{t+j}u'(C_{t+j})\,,\quad (p_t^*)
\end{aligned}$$

$$\begin{aligned}
&\phi_t^w u'(C_t)-\frac{\epsilon}{\epsilon-1}\frac{u'(C_t)\,Y_t}{z_t}\textstyle\sum_{j=0}^{J-1}\phi_{t-j}^{p^*}\theta_{jt}^{acc}\left(\pi_{jt}^{acc}\right)^{\epsilon}+\phi_t^N\sum_{j=1}^{J-1}\dfrac{\psi_{jt}L_{jt}^2}{w_t}g(L_{jt})\\
&\qquad+\textstyle\sum_{j=1}^{J-1}\phi_t^{\lambda,j}g(L_{jt})\dfrac{L_{jt}}{w_t}-\sum_{j=0}^{J-1}\phi_t^{v,j}\left(p_{t-j}^*/\pi_{jt}^{acc}\right)^{-\epsilon}\dfrac{u'(C_t)\,Y_t}{z_t}\\
&\qquad\qquad-\textstyle\sum_{j=0}^{J-2}\phi_{t-1}^{v,j}u'(C_t)\left[\int_0^{L_{j+1,t}}\kappa dG(\kappa)-L_{j+1,t}^2 g(L_{j+1,t})\right],\quad (w_t)
\end{aligned}$$

$$\phi_{t-j}^{p^*}\theta_{jt}^{acc}\left(\pi_{jt}^{acc}\right)^{\epsilon-1}\left[(\epsilon-1)\frac{p_{t-j}^*}{\pi_{jt}^{acc}}-\frac{\epsilon^2}{\epsilon-1}\frac{w_t}{z_t}\right]Y_t u'\left(C_t\right)$$
$$+\phi_t^{v,j}\left[(\epsilon-1)\frac{p_{t-j}^*}{\pi_{jt}^{acc}}-\epsilon\frac{w_t}{z_t}\right]\left(p_{t-j}^*\right)^{-\epsilon}\left(\pi_{jt}^{acc}\right)^{\epsilon-1}Y_t u'\left(C_t\right)+\phi_t^{\pi acc,j}-\beta E_t\phi_{t+1}^{\pi acc,j+1}\pi_{t+1}$$
$$+\left[\phi_t^{\pi}(\epsilon-1)\frac{p_{t-j}^*}{\pi_{jt}^{acc}}+\phi_t^{\Delta}\epsilon\right]\left(p_{t-j}^*\right)^{-\epsilon}\left(\pi_{jt}^{acc}\right)^{\epsilon-1}\left(1-\lambda_{jt}\right)\psi_{jt},\quad\left(\pi_{j=1,\ldots,J-2,\,t}^{acc}\right)$$

$$\phi_{t-(J-1)}^{p^*}\theta_{J-1,t}^{acc}\left(\pi_{J-1,t}^{acc}\right)^{\epsilon-1}\left[(\epsilon-1)\frac{p_{t-(J-1)}^*}{\pi_{J-1,t}^{acc}}-\frac{\epsilon^2}{\epsilon-1}\frac{w_t}{z_t}\right]Y_t u'\left(C_t\right)+\phi_t^{\pi acc,J-1}$$
$$+\left[\phi_t^{\pi}(\epsilon-1)\frac{p_{t-(J-1)}^*}{\pi_{J-1,t}^{acc}}+\phi_t^{\Delta}\epsilon\right]\left(p_{t-(J-1)}^*\right)^{-\epsilon}\left(\pi_{J-1,t}^{acc}\right)^{\epsilon-1}\left(1-\lambda_{J-1,t}\right)\psi_{J-1,t}$$
$$+\phi_t^{v,J-1}\left[(\epsilon-1)\frac{p_{t-(J-1)}^*}{\pi_{J-1,t}^{acc}}-\epsilon\frac{w_t}{z_t}\right]\left(p_{t-(J-1)}^*\right)^{-\epsilon}\left(\pi_{J-1,t}^{acc}\right)^{\epsilon-1}Y_t u'\left(C_t\right),\quad\left(\pi_{J-1,t}^{acc}\right)$$

$$\phi_{t-j}^{p^*}\left(\pi_{jt}^{acc}\right)^{\epsilon}\left[\frac{p_{t-j}^*}{\pi_{jt}^{acc}}-\frac{\epsilon}{\epsilon-1}\frac{w_t}{z_t}\right]Y_t u'\left(C_t\right)+\phi_t^{\theta acc,j}-\beta E_t\phi_{t+1}^{\theta acc,j+1}\left(1-\lambda_{j+1,t+1}\right),\quad\left(\theta_{j=1,\ldots,J-2,\,t}^{acc}\right)$$
$$\phi_{t-(J-1)}^{p^*}\left(\pi_{J-1,t}^{acc}\right)^{\epsilon}\left[p_{t-(J-1)}^*/\pi_{J-1,t}^{acc}-\epsilon w_t/(z_t(\epsilon-1))\right]Y_t u'\left(C_t\right)+\phi_t^{\theta acc,J-1},\qquad\left(\theta_{J-1,t}^{acc}\right)$$

$$\phi_t^{\pi}\left[\left(p_t^*\right)^{1-\epsilon}-\left(\frac{p_{t-j}^*}{\pi_{jt}^{acc}}\right)^{1-\epsilon}\right]\psi_{jt}+\phi_t^{\Delta}\left[\left(p_t^*\right)^{-\epsilon}-\left(\frac{p_{t-j}^*}{\pi_{jt}^{acc}}\right)^{-\epsilon}\right]\psi_{jt}+\phi_t^{\lambda,j}+\beta E_t\phi_{t+1}^{\psi,j+1}\psi_{jt}$$
$$+\phi_{t-1}^{v,j-1}u'\left(C_t\right)\left(v_{0t}-v_{jt}\right)+\phi_t^{\theta acc,j}\theta_{j-1,t-1}^{acc},\quad\left(\lambda_{j=1,\ldots,J-1,\,t}\right)$$

$$-\phi_t^N\int_0^{L_{1t}}\kappa dG\left(\kappa\right)+\phi_t^{\pi}\left[\left(p_t^*\right)^{1-\epsilon}\lambda_{1t}+\left(\frac{p_{t-1}^*}{\pi_{1t}^{acc}}\right)^{1-\epsilon}\left(1-\lambda_{1t}\right)\right]$$
$$+\phi_t^{\Delta}\left[\left(p_t^*\right)^{-\epsilon}\lambda_{1t}+\left(\frac{p_{t-1}^*}{\pi_{1t}^{acc}}\right)^{-\epsilon}\left(1-\lambda_{1t}\right)\right]+\phi_t^{\psi,1}-\beta E_t\phi_{t+1}^{\psi,2}\left(1-\lambda_{1t}\right),\quad\left(\psi_{1t}\right)$$

$$-\phi_t^N\int_0^{L_{jt}}\kappa dG\left(\kappa\right)+\phi_t^{\pi}\left[\left(p_t^*\right)^{1-\epsilon}\lambda_{jt}+\left(\frac{p_{t-j}^*}{\pi_{jt}^{acc}}\right)^{1-\epsilon}\left(1-\lambda_{jt}\right)\right]+\phi_t^{\psi,j}+\phi_t^{\psi,1}$$
$$+\phi_t^{\Delta}\left[\left(p_t^*\right)^{-\epsilon}\lambda_{jt}+\left(\frac{p_{t-j}^*}{\pi_{jt}^{acc}}\right)^{-\epsilon}\left(1-\lambda_{jt}\right)\right]-\beta E_t\phi_{t+1}^{\psi,j+1}\left(1-\lambda_{jt}\right),\quad\left(\psi_{j=2,\ldots,J-1,\,t}\right)$$

$$\phi_t^{\pi}\left(p_t^*\right)^{1-\epsilon}+\phi_t^{\Delta}\left(p_t^*\right)^{-\epsilon}+\phi_t^{\psi,J}+\phi_t^{\psi,1},\qquad\left(\psi_{Jt}\right)$$

$$- \phi_t^N \sum_{j=1}^{J-1} \frac{\psi_{jt}}{w_t} L_{jt} g\left(L_{jt}\right) - \sum_{j=1}^{J-1} \phi_t^{\lambda,j} g\left(L_{jt}\right) \frac{1}{w_t} - \phi_t^{v,0} u'\left(C_t\right)$$

$$+ \sum_{j=0}^{J-2} \phi_{t-1}^{v,j} u'\left(C_t\right)\left[\lambda_{j+1,t} - L_{j+1,t} g\left(L_{j+1,t}\right)\right] + \phi_{t-1}^{v,J-1} u'\left(C_t\right)\delta, \quad (v_{0t})$$

$$\left(\phi_t^N \psi_{jt} L_{jt} + \phi_t^{\lambda,j}\right) g\left(L_{jt}\right) w_t^{-1} - \phi_t^{v,j} u'\left(C_t\right) + \phi_{t-1}^{v,j-1} u'\left(C_t\right)\left[1 - \lambda_{jt} + L_{jt} g\left(L_{jt}\right)\right], \quad (v_{j=1,\dots,J-1,\, t})$$

$$-x'\left(N_t\right) - \phi_t^w x''\left(N_t\right) + \phi_t^N, \quad (N_t)$$

$$-\phi_t^N Y_t / z_t - \phi_t^\Delta, \quad (\Delta_t)$$

$$-\phi_t^{\pi acc,1} - \textstyle\sum_{j=2}^{J-1} \phi_t^{\pi acc,j} \pi_{j-1,t-1}^{acc}, \quad (\pi_t)$$

where we have used that $Y_t - G_t = C_t$ and defined $L_{jt} \equiv \left(v_{0t} - v_{jt}\right)/w_t$ for compactness.

B. Optimality of zero long run inflation

We now prove that the optimality conditions of the Ramsey problem are satisfied in a steady state with zero inflation. We start by guessing that $\pi_{ss} = 1$, which implies $p_{ss}^* = \Delta_{ss} = 1$ and $\pi_{j,ss}^{acc} = 1$ for all j. It is easy to show that under zero inflation all price vintages have the same value: $v_{j,ss} = v_{0,ss} = \frac{Y_{ss}/\epsilon}{1-\beta} = \bar{v}_{ss}$ for all j, where have used the fact that the real wage equals $w_{ss} = (\epsilon - 1)/\epsilon$ and therefore real profits are given by $(1 - w_{ss}) Y_{ss} = Y_{ss}/\epsilon$. The adjustment gain is then zero for every vintage, implying $\lambda_{j,ss} = G(0) \equiv \bar{\lambda} > 0$ for all j. From the laws of motion of the vintage distribution, we then have $\psi_{j,ss} = \left(1 - \bar{\lambda}\right)\psi_{j-1,ss} = \left(1 - \bar{\lambda}\right)^{j-1}\psi_{1,ss}$, for $j = 2, ..., J$, which combined with $\sum_{j=1}^{J} \psi_{j,ss} = 1$ implies

$$\psi_{j,ss} = \frac{\left(1 - \bar{\lambda}\right)^{j-1}}{\sum_{j=0}^{J-1}\left(1 - \bar{\lambda}\right)^j} \equiv \bar{\psi}_j,$$

for $j = 1, .., J$. Finally, $\theta_{j,ss}^{acc} = \left(1 - \bar{\lambda}\right)^j$ for each $j = 1, ..., J-1$. This completes the characterization of the steady-state equilibrium of the endogenous variables other than the Lagrange multipliers of the Ramsey problem.

We now need to show that the first order conditions of the Ramsey problem are satisfied too in the steady state with zero inflation. Notice that there are $3 + 5J$ first order conditions but only $2 + 5J$ Lagrange multipliers (see Appendix A). Therefore, we will use $2 + 5J$ first order conditions in order to solve for the steady-state Lagrange multipliers and then check whether the remaining equation holds given the solution for all the other variables. Consider now the first order conditions of the Ramsey problem in the steady state with zero inflation (all expressions are equal to zero),

$$u'(C_{ss}) + \phi_{ss}^{w} u''(C_{ss}) w_{ss} - \phi_{ss}^{N} + (u''(C_{ss}) Y_{ss} + u'(C_{ss}))\epsilon^{-1} \sum_{j=0}^{J-1} \phi_{ss}^{v,j} \tag{15}$$
$$+\phi_{ss}^{v,J-1}(1-\delta) u'(C_{ss})(\epsilon(1-\beta))^{-1},$$

$$-x'(N_{ss}) - \phi_{ss}^{w} x''(N_{ss}) + \phi_{ss}^{N}, \tag{16}$$

$$-\phi_{ss}^{N} Y_{ss} - \phi_{ss}^{\Delta}, \tag{17}$$

$$\phi_{ss}^{p^*} u'(C_{ss}) Y_{ss} \sum_{j=0}^{J-1} \beta^j \left(1-\bar{\lambda}\right)^j \tag{18}$$
$$-\left[\sum_{j=1}^{J-1} \bar{\lambda}\bar{\psi}_j + \bar{\psi}_J + \sum_{j=1}^{J-1} \beta^j \left(1-\bar{\lambda}\right) \bar{\psi}_j\right] \left[(\epsilon-1)\phi_{ss}^{\pi} + \epsilon\phi_{ss}^{\Delta}\right],$$

$$\phi_{ss}^{w} - \epsilon\phi_{ss}^{p^*} Y_{ss} \sum_{j=0}^{J-1} \left(1-\bar{\lambda}\right)^j / (\epsilon-1) - Y_{ss} \sum_{j=0}^{J-1} \phi_{ss}^{v,j}, \tag{19}$$

$$-\phi_{ss}^{\pi acc,1} - \sum_{j=2}^{J-1} \phi_{ss}^{\pi acc,j}, \tag{20}$$

$$-\phi_{ss}^{p^*} \left(1-\bar{\lambda}\right)^j Y_{ss} u'(C_{ss}) + \left[\phi_{ss}^{\pi}(\epsilon-1) + \phi_{ss}^{\Delta}\epsilon\right] \left(1-\bar{\lambda}\right) \bar{\psi}_j \tag{21}$$
$$+\phi_{ss}^{\pi acc,j} - \beta\phi_{ss}^{\pi acc,j+1}, \quad j = 1, ..., J-2,$$

$$-\phi_{ss}^{p^*} \left(1-\bar{\lambda}\right)^{J-1} Y_{ss} u'(C_{ss}) + \left[\phi_{ss}^{\pi}(\epsilon-1) + \phi_{ss}^{\Delta}\epsilon\right] \left(1-\bar{\lambda}\right) \psi_{J-1,ss} + \phi_{ss}^{\pi acc,J-1}, \tag{22}$$

$$\phi_{ss}^{\theta acc,j} - \beta\phi_{ss}^{\theta acc,j+1} \left(1-\bar{\lambda}\right), \quad j = 1, ..., J-2, \tag{23}$$

$$\phi_{ss}^{\theta acc,J-1}, \tag{24}$$

$$\phi_{ss}^{\lambda,j} + \beta\phi_{ss}^{\psi,j+1}\bar{\psi}_j + \phi_{ss}^{\theta acc,j} \left(1-\bar{\lambda}\right)^{j-1}, \quad j = 1, ..., J-1, \tag{25}$$

$$\phi_{ss}^{\pi} + \phi_{ss}^{\Delta} + \phi_{ss}^{\psi,1} - \beta\phi_{ss}^{\psi,2} \left(1-\bar{\lambda}\right), \tag{26}$$

$$\phi_{ss}^{\pi} + \phi_{ss}^{\Delta} + \phi_{ss}^{\psi,j} - \beta\phi_{ss}^{\psi,j+1} \left(1-\bar{\lambda}\right) + \phi_{ss}^{\psi,1}, \quad j = 2, ..., J-1, \tag{27}$$

$$\phi_{ss}^{\pi} + \phi_{ss}^{\Delta} + \phi_{ss}^{\psi,J} + \phi_{ss}^{\psi,1}, \tag{28}$$

$$-\sum_{j=1}^{J-1} \phi_{ss}^{\lambda,j} \frac{g(0)}{w_{ss}} - \phi_{ss}^{v,0} u'(C_{ss}) + \sum_{j=0}^{J-2} \phi_{ss}^{v,j} u'(C_{ss}) \bar{\lambda} + \phi_{ss}^{v,J-1} u'(C_{ss}) \delta, \tag{29}$$

$$\phi_{ss}^{\lambda,j} g(0) / w_{ss} - \phi_{ss}^{v,j} u'(C_{ss}) + \phi_{ss}^{v,j-1} u'(C_{ss}) \left(1-\bar{\lambda}\right), \quad j = 1, ..., J-1. \tag{30}$$

We now use equations (15) to (30), except for (20), in order to solve for the steady state Lagrange multipliers. From (23) and (24), it follows immediately that

$$\phi_{ss}^{\theta acc,j} = 0, \quad j = 1, ..., J-1. \tag{31}$$

Equations (26) to (28) allow us to solve for the $\phi_{ss}^{\psi,j}$ multipliers, obtaining

$$\phi_{ss}^{\psi,1} = -\left(\phi_{ss}^{\pi} + \phi_{ss}^{\Delta}\right),$$

$$\phi_{ss}^{\psi,j} = 0, \quad j = 2, ..., J. \tag{32}$$

Using (31) and (32) in equations (25), we obtain

$$\phi_{ss}^{\lambda,j} = 0, \quad j = 1, ..., J-1. \tag{33}$$

We now solve for the $\phi_{ss}^{\pi acc,j}$ multipliers. From (22), we have

$$\begin{aligned}
\phi_{ss}^{\pi acc,J-1} &= \phi_{ss}^{p^*}\left(1-\bar{\lambda}\right)^{J-1} Y_{ss} u'\left(C_{ss}\right) - \left[\phi_{ss}^{\pi}\left(\epsilon-1\right) + \phi_{ss}^{\Delta}\epsilon\right]\left(1-\bar{\lambda}\right)\bar{\psi}_{J-1} \qquad (34) \\
&= \left[\frac{\sum_{j=1}^{J-1}\bar{\lambda}\bar{\psi}_j + \bar{\psi}_J + \sum_{j=1}^{J-1}\beta^j\left(1-\bar{\lambda}\right)\bar{\psi}_j}{\left(1-\bar{\lambda}\right)^{1-J}\sum_{j=0}^{J-1}\beta^j\left(1-\bar{\lambda}\right)^j} - \left(1-\bar{\lambda}\right)\bar{\psi}_{J-1}\right]\left[\left(\epsilon-1\right)\phi_{ss}^{\pi} + \epsilon\phi_{ss}^{\Delta}\right],
\end{aligned}$$

where in the second equality we have used (18) to substitute for $\phi_{ss}^{p^*} Y_{ss} u'\left(C_{ss}\right)$. It follows that

$$\begin{aligned}
\phi_{ss}^{\pi acc,J-1} &\propto \left(\sum_{j=1}^{J-1}\left[\bar{\lambda}\bar{\psi}_j + \beta^j\left(1-\bar{\lambda}\right)\bar{\psi}_j\right] + \bar{\psi}_J\right)\left(1-\bar{\lambda}\right)^{J-1} - \left(1-\bar{\lambda}\right)\bar{\psi}_{J-1}\sum_{j=0}^{J-1}\beta^j\left(1-\bar{\lambda}\right)^j \\
&= \left(1-\bar{\lambda}\right)^{J-1}\bar{\lambda}\sum_{j=1}^{J-1}\bar{\psi}_j + \left(1-\bar{\lambda}\right)^{J-1}\bar{\psi}_J \\
&\quad -\bar{\psi}_J + \sum_{j=1}^{J-1}\beta^j\left(\left(1-\bar{\lambda}\right)^J\bar{\psi}_j - \left(1-\bar{\lambda}\right)^j\bar{\psi}_J\right), \qquad (35)
\end{aligned}$$

where in the equality we have used that $\left(1-\bar{\lambda}\right)\bar{\psi}_{J-1} = \bar{\psi}_J$. The fact that $\bar{\psi}_J = \left(1-\bar{\lambda}\right)^{J-j}\bar{\psi}_j$ for each $j = 1, ..., J-1$ implies that

$$\sum_{j=1}^{J-1}\bar{\psi}_j = \frac{\bar{\psi}_J}{\left(1-\bar{\lambda}\right)^J}\sum_{j=1}^{J-1}\left(1-\bar{\lambda}\right)^j, \tag{36}$$

The geometric series in (36) can be expressed as

$$\sum_{j=1}^{J-1}\left(1-\bar{\lambda}\right)^j = \frac{1-\left(1-\bar{\lambda}\right)^J}{1-\left(1-\bar{\lambda}\right)} - 1 = \frac{1-\bar{\lambda}-\left(1-\bar{\lambda}\right)^J}{\bar{\lambda}}.$$

Combining the latter with (36), we find that

$$\left(1-\bar{\lambda}\right)^{J-1}\bar{\lambda}\sum_{j=1}^{J-1}\bar{\psi}_j = \bar{\psi}_J\left[1-\left(1-\bar{\lambda}\right)^{J-1}\right].$$

Therefore, the terms in the second line of (35) cancel out. Also, notice that $\left(1-\bar{\lambda}\right)^J\bar{\psi}_j = \left(1-\bar{\lambda}\right)^j\left(1-\bar{\lambda}\right)^{J-j}\bar{\psi}_j = \left(1-\bar{\lambda}\right)^j\bar{\psi}_J$, and therefore the terms in the third line of (35) cancel out too. It follows that $\phi_{ss}^{\pi acc,J-1}=0$. Equation (21) for $j=J-2$ then implies

$$\phi_{ss}^{\pi acc,J-2}=\phi_{ss}^{p^*}\left(1-\bar{\lambda}\right)^{J-2}Y_{ss}u'\left(C_{ss}\right)-\left[\phi_{ss}^{\pi}\left(\epsilon-1\right)+\phi_{ss}^{\Delta}\epsilon\right]\left(1-\bar{\lambda}\right)\bar{\psi}_{J-2}. \tag{37}$$

Multiplying both sides of the latter equation by $\left(1-\bar{\lambda}\right)$, using $\left(1-\bar{\lambda}\right)\bar{\psi}_{J-2}=\bar{\psi}_{J-1}$, and comparing the resulting expression with (34), it follows that $\phi_{ss}^{\pi acc,J-2}=\phi_{ss}^{\pi acc,J-1}/\left(1-\bar{\lambda}\right)=0$. Operating in this fashion, we have that $\phi_{ss}^{\pi acc,j}=0$ also for $j=1,...,J-3$.

Using (33) in equations (29) and (30), the latter can be represented in matrix notation as $B\phi^{v}=0$, where $\phi^{v}=[\phi_{ss}^{v,0},\phi_{ss}^{v,1},...,\phi_{ss}^{v,J-1}]'$ and

$$\mathbf{B}=\begin{bmatrix}\bar{\lambda}-1 & \bar{\lambda} & \bar{\lambda} & ... & \bar{\lambda} & \bar{\lambda} & \delta\\ 1-\bar{\lambda} & -1 & 0 & ... & 0 & 0 & 0\\ 0 & 1-\bar{\lambda} & -1 & ... & 0 & 0 & 0\\ ... & ... & ... & ... & ... & ... & ...\\ 0 & 0 & 0 & ... & 1-\bar{\lambda} & -1 & 0\\ 0 & 0 & 0 & .. & 0 & 1-\bar{\lambda} & -1\end{bmatrix}.$$

For any $\delta<1$, the matrix B has full rank and the unique solution of $B\phi^{v}=0$ is given by $\phi^{v}=0$. In the case of $\delta=1$, the matrix B is rank deficient (the first row is minus the sum of the other rows), which implies an infinity of solutions for the $\phi_{ss}^{v,j}$ multipliers. In the latter case, we focus on one particular solution, namely $\phi^{v}=0$. We can then use equations (15) to (19) above to solve for $\left\{\phi_{ss}^{w},\phi_{ss}^{N},\phi_{ss}^{\Delta},\phi_{ss}^{\pi},\phi_{ss}^{\Delta}\right\}$, obtaining

$$\phi_{ss}^{w}=\frac{u'\left(C_{ss}\right)-x'\left(N_{ss}\right)}{\left(-\right)u''\left(C_{ss}\right)w_{ss}+x''\left(N_{ss}\right)},$$

$$\phi_{ss}^{N}=x'\left(N_{ss}\right)+\phi_{ss}^{w}x''\left(N_{ss}\right),$$

$$\phi_{ss}^{\Delta}=-\phi_{ss}^{N}Y_{ss},$$

$$\phi_{ss}^{\pi}=\frac{1}{\epsilon-1}\frac{\phi_{ss}^{p^*}u'\left(C_{ss}\right)Y_{ss}\sum_{j=0}^{J-1}\beta^{j}\left(1-\bar{\lambda}\right)^{j}}{\sum_{j=1}^{J-1}\bar{\lambda}\bar{\psi}_{j}+\bar{\psi}_{J}+\sum_{j=1}^{J-1}\beta^{j}\left(1-\bar{\lambda}\right)\bar{\psi}_{j}}-\frac{\epsilon}{\epsilon-1}\phi_{ss}^{\Delta},$$

$$\phi_{ss}^{p^*}=\frac{\phi_{ss}^{w}}{\frac{\epsilon}{\epsilon-1}Y_{ss}\sum_{j=0}^{J-1}\left(1-\bar{\lambda}\right)^{j}}.$$

Having solved for the steady state Lagrange multipliers, we finally need to show that the equation that we have left out, equation (20), is satisfied in the zero inflation steady-state. This is obvious, given that we have already found that $\phi_{ss}^{\pi acc,j}=0$ for all $j=1,...,J-1$.

Fig.1: Price adjustment probability and firm distribution by vintage

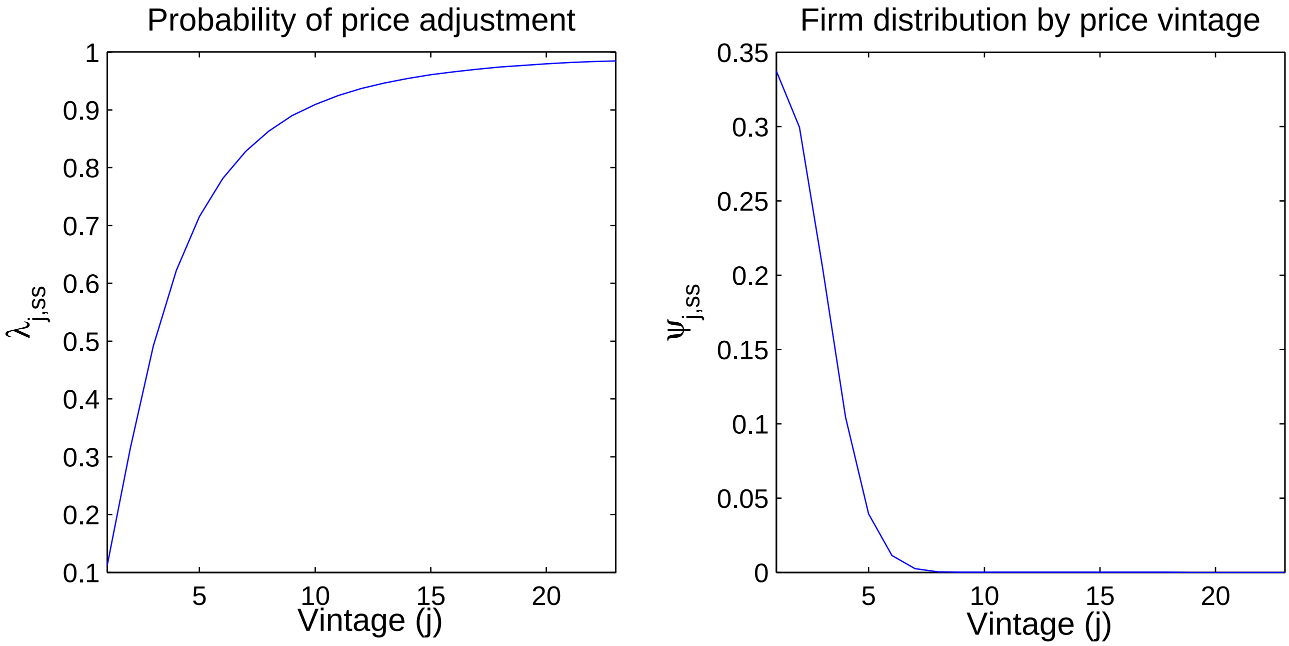

Fig.2: Responses to a technology and a government spending shock

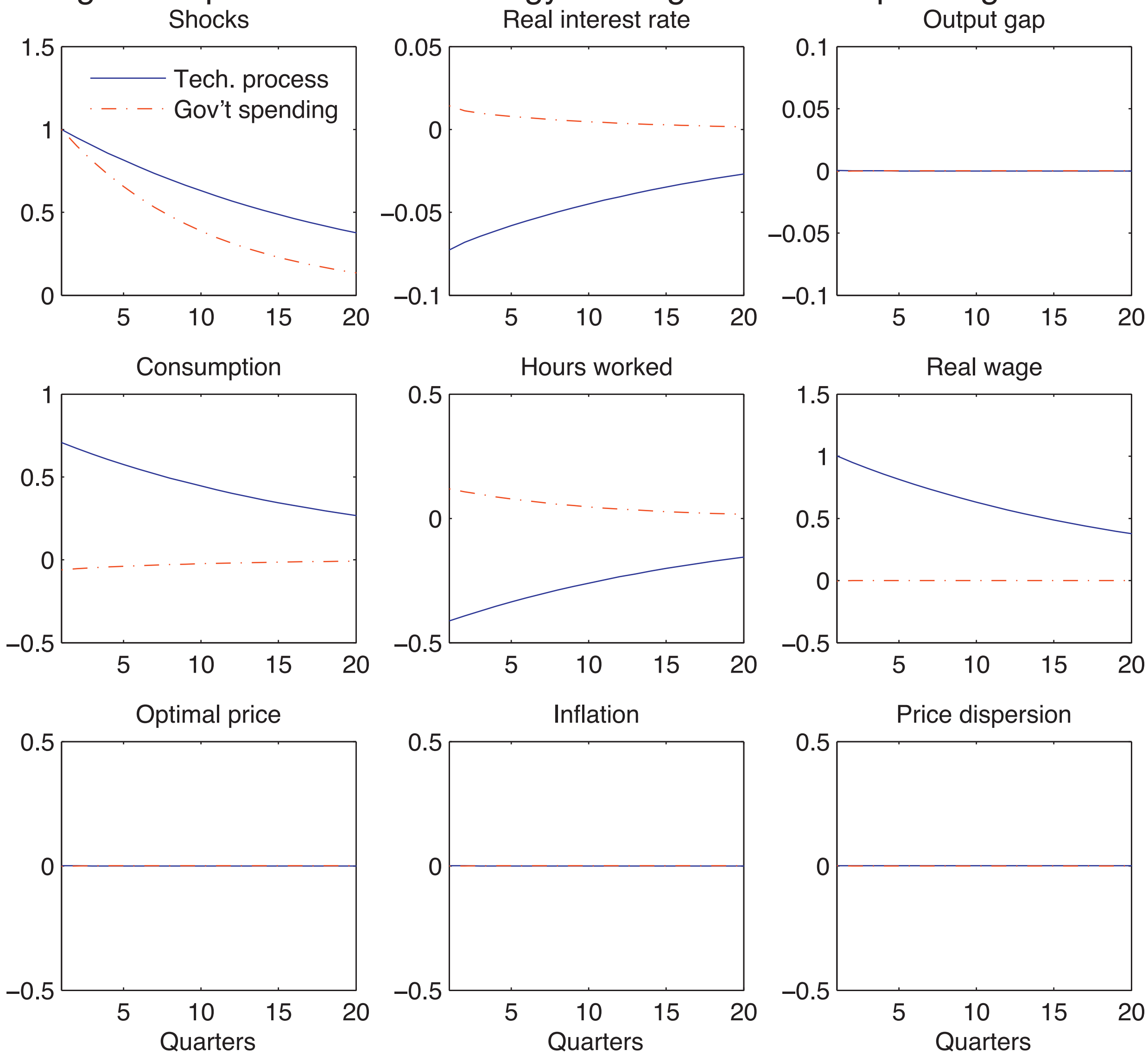

Fig.3: Responses to a cost−push shock