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In 2004 all publications will carry a motif taken from the €100 banknote.

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Abstract

The paper proposes a multi-factor international asset pricing model in which the exchange rate is allowed to be co-determined by a risk factor imperfectly correlated to other priced risks in the economy. The significance of this factor can be established as long as one is able to observe a proxy for the foreign cash order flow. Then, the asset pricing model is decomposed into the standard ICCAPM no-arbitrage setup characterized by a pricing kernel, in which, however, the "autarky" exchange rate is unobserved, and an additional equation that links this autarchic currency price with the FX order flow. The model is put in the state space form. The unobserved variables span the macroeconomic risk factors with an impact on the asset markets and determine the dynamics of the pricing kernel, the autarchic exchange rate and the FX order flow. A comparison of models allowing for an independent OF risk factor with a restricted one, where the forex order flow plays no role, should disclose the existence of a "non-fundamental" source of a systematic divergence of the observed and the autarchic (i.e. fundamental) FX returns. The model is calibrated and tested on the Czech koruna/euro exchange rate in a setting with seven Czech and euro area asset returns.

Key words: exchange rate, pricing kernel, order flow, latent risk, state space
JEL Classification: F31, F41, G12, G15

Non-technical summary

The present paper looks for methods to establish linkages between the uncertainties driving key macroeconomic variables on the one side, and key asset prices, including the exchange rate, on the other, in a stochastic optimizing model of a small open economy.

The paper constructs a risk factor decomposition of the exchange rate in the said model by joining the resources of portfolio optimization theory, factor models of asset prices and micro finance. We do not exclude the existence of liquidity management and other agent-specific determinants of the nominal exchange rate in a financially integrated open economy besides the purely macroeconomic fundamentals, and develop a technique to measure the relative importance of both. In principle, traditional financial economics is able to derive restrictions on the exchange rate dynamics in an open economy from an optimizing investor's actions (even if it does not normally pin the exchange rate value down unambiguously). On the other hand, positivist empirical finance concentrates on decomposing the observed exchange rate into statistically well-defined components without offering much in the way of explaining their economic sources. To address the issue at the point in an imperfectly frictionless market setting, we need a synthesis of both. The "missing link" is found in the New Micro approach (see Lyons, 2001) to exchange rate modeling, owing to its departure from the conventional representative investor (Walrasian) dynamic asset pricing paradigm in the direction of modeling elastic supply and demand environments. Following this line of thought, we model asset markets where the investor faces explicit pricing schedules, and where excess demand is absorbed by exogenous market makers.

Individual investor optimization in the chosen setting leads to a generalization of the international consumption-based asset pricing model (ICCAPM). The well-known price formulae of the latter now turn out to be valid for a set of unobservable "shadow" prices whose deviation from the actual ones dictates the direction of trades in any dynamic equilibrium. Thus, an important methodological contribution of the paper is the extension of the econometric factor asset pricing approach to this environment, having a somewhat richer microeconomic structure compared to the textbook dynamic asset pricing paradigm.

The central notion arising in the said class of models is that of the shadow exchange rate value. For lack of well-established alternatives, we call it the *autarchic exchange rate*. It is the value of the currency that does not induce investors to either buy or sell. In this sense, the autarchic exchange rate is the "balanced" one, although not precisely the "equilibrium" one, since the generic dynamic equilibrium that the model gives rise to, implies non-zero FX transactions. That is, non-zero FX order flow is a summary statistic for all sorts of asymmetries, be it of an inventory , informational or institutional nature that may exist between resident and non-resident investors. Also, the model demonstrates that it is the autarchic and not the actual exchange rate for which the standard uncovered parity properties should hold.

To be able to concentrate on the forex risks, the present paper considers deviations of the actual from the autarchic price for the single asset (FX), assuming balanced pricing for all other assets (i.e. the equality of autarchic and actual prices). This approximation is justified by the application to be studied, since the cross-border FX order flow compared to cross-border order flow in other market segments clearly dominates in the economy whose data we are using in the empirical part. That is why the model version employed by the present paper can be dubbed the *FX Order Flow Gap Model*.

The obtained theoretical results can be cast into the shape of testable state-space model if one assumes the existence of a finite number of (Gaussian) risk factors that jointly determine the dynamics of the (autarchic) asset prices and the FX order flow. These latent factors are assumed to span the uncertainties encountered by a typical investor with an open position in foreign cash. In view of the pursued objective – to detect the forex volatility sources in excess of the standard macroeconomic risks – we let the vector of state variables contain a component imperfectly correlated with the main macro-fundamentals. This residual uncertainty factor may be purely inventory-driven but one cannot *a priori* exclude that it is a reflection of structural asymmetries in the forex. The working hypothesis of our analysis is that this extra state variable shall have a significant correlation with the cross-border FX order flow. To test its validity, estimations are conducted in parallel on our "full" order flow risk-encompassing model and a benchmark (more or less standard ICCAPM with no role for the order flow effects) state space model. The relative performance of the two in explaining the past forex developments would demonstrate the relative importance of "non-fundamental" exchange rate risks.

From the outset, we have intended to apply the theoretical framework and the estimation procedure for the state-space model to the currency of a small open economy closely linked to the euro area. Specifically, we take the example of the Czech koruna exchange rate to the euro. The method is applied to a set of major Czech securities enlarged by the set of their counterparts in the euro area. A Kalman filter procedure conducted on price processes of the Czech and the euro area assets is used to isolate the autarchic CZK/EUR exchange rate and compare it with the actual one.

The main findings of the study are as follows:

- the cross-border FX order flow, reflected in the reported Czech bank spot FX transactions with non-residents, explains a large portion of the koruna/euro rate deviations from the uncovered asset return parity, or, equivalently, the autarchic exchange rate
- the significant risk factors influencing the observed FX order flow include both standard macro fundamentals and idiosyncratic liquidity management-related shocks
- the detected deviations of the actual return from the autarchic return on the Czech koruna cash, as predicted by the model, point at the bubbles in the observed CZK/EUR rate trajectory, the later being associated with episodes of pronounced one-sided cross-border FX order flow.

The analysis presented in the paper could be a useful supporting tool for a fresh EU-member at the phase of ERMII-entry when selecting euro central parity for the national currency. Supposing that the superior performance of the full FX order flow gap model compared to the benchmark one is established, this could suggest the need to augment a purely macroeconomic view of the exchange rate behavior with the analysis of an additional latent component. Since the latter cannot be readily decomposed into the conventional set of macro fundamentals, one is confronted with a poorly classifiable source of nominal exchange rate volatility. Some central banks might choose at times to counteract this by FX interventions. The present paper does not take a stand regarding the chances of such interventions to succeed. Instead, it proposes a methodology helping the central bank to assess the extent of the potential pressure it would have to face in the forex market at the time the binding central parity announcement is to be made.

1. Introduction

The paper investigates the consequences of introducing an exogenous FX market clearing risk in a standard asset price model for a small open economy. Specifically, we assume the existence of a latent risk factor, which is present in the forex order flow along with observable aggregate economic and financial uncertainties. This has consequences for asset pricing in general and for the exchange rate behavior in particular.

The paper discusses a model and reports estimation results for the CZK/EUR rate expectations as a function of Czech macroeconomic fundamentals relative to the euro area and the Czech-EU asset return differences. Beside the theoretical model, we conduct a tentative calibration and partial estimation of the model on selected Czech and euro area assets.

Most models of the exchange rate that use the stochastic optimization approach fall into one of the two categories regarding the treatment of the currency price. They make the choice between

1. considering the exchange rate a redundant price variable – in the Walrasian sense - determined by the remaining prices of goods and assets
2. viewing the exchange rate shocks as independent sources of uncertainty.

Alternative 1 was already implicit in the Lucas, 1982, international consumption-based CAPM. There and in all other models built on the same foundations, the exchange rate satisfies an uncovered parity condition involving any two returns in different currencies. If this alternative were valid in an open economy, then its exchange rate uncertainty would be a derived risk, which could be kept under control by a credible monetary-fiscal policy mix. Empirically, one would need a close correlation between asset prices and macroeconomic fundamentals to justify such a conjecture. This implies a very modest role for direct policy measures in the forex, reducing it, basically, to maintenance of orderly market operation and occasional volatility smoothing. The policy effort should be instead fully dedicated to removal of the prime real and financial shock sources.

The latter alternative means an exchange rate disconnect from other asset markets. It would be supported empirically if one observed that capital flows in the balance of payments constituted a significant source of unexpected exchange rate moves. The assumption of Alternative 2 logically leads to sunspot equilibria for the exchange rate in the model and infers policy recommendations of interventionalist nature, with the aim to impose one out of the many possible equilibrium rates in accordance with the monetary authority preferences. In the case of an accession or fresh member country in the pre-euro period, this means a space for active joint presence of its central bank and the ESCB in the market for national currency.

The present paper assumes a synthetic perspective. It does derive a “shadow” set of asset prices that determine a particular exchange rate value for the hypothetical no forex trade case, in the spirit of Alternative 1. This currency value should obey the standard cross-border asset return parity condition (see Derviz, 2002, for a continuous time model and estimations in the Czech koruna- and the U.S. dollar-euro cases.). However, the model allows for various deviations from the said “Walrasian” outcome, due to heterogeneity of endowments, information or institutional status. We capture the said heterogeneity by a summary statistic of exogenous aggregate FX order flow absorbed by an exogenous market maker. Self-fulfilling beliefs about this aggregate source of uncertainty are able to generate sunspot exchange rate trajectories (see Derviz, 2004, for a model in this vein). That is, Alternative 2 is heeded in our model, although the forex-specific risk factor is not generated by the statistics of the exchange rate itself but by that of the excess FX demand generated by an optimizing investor.[2]

1.1 Methodology and literature review

The distinct feature of the present model is the presence of an independent order flow risk factor not spanned by the asset price in question (FX). Accordingly, the equilibrium can no longer be characterized in terms of risk-neutral expected values. The equilibrium prices, including the exchange rate, become individual preference-dependent. The techniques we use on the way to a testable model, combine the standard individual optimization-based dynamic asset pricing paradigm in discrete time with findings from micro-based FX theory. Altogether, the methodology of the paper has three main sources.

First, we draw on the literature on ***multi-factor models of asset prices and yield curves***. To explore the macroeconomic fundamentals role in asset pricing and allocation, the stochastic intertemporal optimization paradigm has been applied to both multivariate GARCH (Flavin and Wickens, 2003, Wickens and Smith, 2002) and VAR (Ang and Piazzesi, 2003, Bomfim, 2003) asset dynamic specifications. In both variants, one needs to create a model for the pricing kernel in terms of the relevant underlying sources of uncertainty. Sometimes, in the tradition of multifactor yield curve modeling literature (such as Duffie and Kan, 1996), the underlying factors are considered totally unobservable. At the other extreme, the analysis is limited to observed macroeconomic and financial fundamentals only (Flavin and Wickens, 2003, Wickens and Smith, 2002, Bomfim, 2003). We employ a hybrid approach analogous to that of Ang and Piazzesi, 2003, by considering a vector of observed macrofundamentals extended by a latent factor. In our model, this latent factor is responsible for the FX order flow volatility.

Similarly to Bomfim, 2003, we aim at modeling latent factors behind asset returns in terms of financial and economic fundamentals, rather than abstract distributional parameters (be it level, slope and curvature for the term structure models, or processes that define components of volatility matrices, as in many GARCH and stochastic valatility asset pricing models). We also share the view of Bomfim, 2003, that the current and expected stance of monetary policy should be present among the explanatory factors behind asset prices. However, we extend Bomfim's paradigm by including other risk factors beside those directly linked to monetary policy, adding flexibility to the model.

The modeling approach that we take is similar to Ang and Piazzesi, 2003, although we do not orient our state-space estimation on fitting the observed yield curve. Instead, we estimate the pricing kernel parameters that fit the returns of a number of basic infinite- or long-maturity assets. The reason is that we are looking for a possibly direct connection between macroeconomic risk factors, asset prices the exchange rate. To view this connection through the prism of yield curve dynamics would be too circumspect for our purposes, since extraction of business cycle information from the yield curve is a misspecification error-laden process in itself. In contrast, by allowing the model to reflect a one-to-one correspondence between a vector of basic assets and another vector of unobserved factors, we are likely to capture the latent principal components responsible for the economic activity, inflation and monetary policy expectations in both modeled economies in a more direct way. We believe that a model

[2] A microstructurally founded FX trade model in continuous time with the indicated properties is developed in Derviz, 2003.

constructed according to the above outlined principles would contain less noise in the identified business cycle position of the economy than most multi-factor yield curve models in the literature.

Second, the econometric literature that implements ***empirical pricing kernel models*** has provided a framework for linking the observed asset prices to the hidden risk factors in the equations for asset return premia, without the need to use aggregate consumption data.[3]. The basic idea of replacing the theoretical pricing kernel[4] equal to the real marginal rate of intertemporal consumption substitution, by its projection on the space of relevant risk factors, has been exploited in Jackwerth, 2000, Ait-Sahalia and Lo, 2002, or Rosenberg and Engle, 2002, among others. The named authors call the result of the projection (the possibility to replace the original pricing kernel with the projected one is a simple consequence of the law of iterative expectations) *the empirical pricing kernel*. However, the main idea is of an abstract and general nature. So, it can be exploited in different situations irrespective of the chosen empirical context (e.g. in the present paper we apply it to an extended International CCAPM for a small open economy, which has not, to our knowledge, been done yet). Therefore, we prefer to use the term *projected pricing kernel*.

Third, we exploit the ideas of the ***New Micro approach to exchange rate economics***, by assigning the order flow in the FX market a prominent price formation role. The best-known contributions to microstructural FX analysis (such as Evans and Lyons, 2002) rely on real time (i.e. high frequency) trade data to test their models. Nevertheless, many findings within this approach suggest that risks contained in the forex order flow have price relevant consequences in macroeconomically relevant horizons as well (more on this can be found in Lyons, 2001). The idea was exploited in Derviz, 2003, 2004, to explain a part of the exchange risk premium by learning from order flow and model-revision by FX dealers. In this paper, a similar mechanism is implicit. We use its "reduced form" by introducing a deviation from the so-called "autarchic" exchange rate by the actual price in the forex. The said deviation is due to the currency supply side risks and is captured by a sufficient statistic of the investor's dynamically adjusted outgoing FX order flow. This allows us to offer a natural explanation of a seemingly anomalous relation between the spot rate, the forward rate and the order flow, visible in the data. The corresponding equation is a part of a general no-arbitrage asset pricing equation system.

As is usually the case in the international asset pricing literature (see Gourinchas and Tornell, 2002, for a recent example), one needs to model both domestic and foreign resident representative investor state prices to identify the equilibrium exchange rate parameters. Therefore, we start with an optimizing model for an investor resident in the "big country" (euro area in the application) whose investment opportunity set includes assets from the "small country" (Czech Republic in the application) and derive the asset pricing formulae in the big country pricing kernel terms. Then we reverse the perspective to obtain the same pricing formulae for the small country resident in his pricing kernel terms. This is sufficient to obtain the autarchic exchange rate as the difference of the two pricing kernels.

We argue that modeling a representative big country investor as a significant entity in the small country asset markets is justified by the targeted empirical objective. It is a well-

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[3] Applying consumption-based CAPM directly to real financial data seems to be a dead end empirically, see Rosenberg and Engle, 2002.

[4] The standard exposition of the consumption-based CAPM in pricing kernel terms can be found in Campbell et al, 1997, or Cochrane, 2001.

established fact of the Czech financial markets, that non-resident security traders constitute a prominent share of participants in most segments. (Anecdotical evidence on other joining EU-member countries suggests very much the same.) Moreover, the role of non-resident traders in price formation is stable over time. This feature does not contradict the possible saving rate gap effect on investment flows the way international macroeconomics understands it, since it is mainly FDI that close the gap. The latter usually do not pass through publicly traded security markets. There, the balance between resident and non-resident holdings did not change much during the period covered by our data.

Formally, the utilized assumption is that in almost all security markets there is *one domestic* and *one foreign representative* investor and there are no unidirectional transfer of security holdings from one to another. The only exception is the FX market (which is at the center of our interest), where periods of big unidirectional cross-border order flows occur regularly.

We apply the theoretical model to a mixed "Czech- euro area" asset market segment. With the given short time series for the financial variables on the Czech side, the empirical analysis is necessarily a hybrid between estimation and calibration. In particular, there is not enough observations to perform a Kalman filter on a full equation system directly. We make a compromise by carrying over the parameters from one group of estimated equations into another group. Details are explained in Section 4.

Structure of the paper: Section 2 defines the theoretical model and derives the asset pricing formulae to be implemented empirically. Section 3 formulates the statistical assumptions on the underlying risk factors and derives the model to be tested, in state space form. Section 4 presents the estimation results. Section 5 discusses the policy consequences and offers conclusions.

2. The economy

2.1 Definitions

We consider an investor (a financial institution as a typical example) acting in discrete time, for which an investment opportunity set is constituted by domestic and foreign cash, as well as N_1 domestic and N_2 foreign assets. The risk-free domestic and foreign interest rates on cash holdings between dates t and t+1 are denoted by r_{t+1}^0 and r_{t+1}^i. A domestic asset d generates a stochastic sequence $\left(\Gamma_t^d\right)_{t\geq 0}$ of cash incomes (dividends or coupons if fixed income) in the domestic currency (d=1,…,N_1), and a foreign asset f generates a similar sequence $\left(\Gamma_t^f\right)_{t\geq 0}$ in the foreign currency (f=1,…,N_2). One share of domestic asset d costs P_t^d units of domestic currency in period t, and, analogously, P_t^f is the period t-price in foreign currency of one unit of foreign asset f. The end of date t holdings of asset d (f) are denoted by x_t^d (x_t^f), and the cash holdings – by x_t^0 and x_t^i. Finally, the period t-domestic currency price of one foreign currency unit (the nominal exchange rate) will be denoted by S_t. The investor faces the uncertainty associated with asset earnings, Γ^d and Γ^f, and prices, P, P^d, P^f, S (d=1,…,N_1, f=1,…,N_2, P is the domestic consumption price level to be introduced shortly).

The nominal purchase/sale order volume of security d (f) in period t will be denoted by φ_t^d (φ_t^f), and symbol φ_t^i will stand for the foreign cash purchase/sale order volume in period t.

Transaction costs

The asset market transaction-related frictions to be defined below are the key element of the model extending its validity beyond the standard Consumption-based CAPM. Therefore, we proceed by introducing the formal definition and then discussing the reasons behind it.

The investor entering the market for a given security as a market user faces a set of liquidity-providers (market makers in the quote-driven trading mechanism and brokers in the order-driven mechanism). In aggregate, these liquidity providers present the investor with a *pricing schedule*, which is an increasing function his order. That is, a selling investor gets less than the mid-price, the price reduction increasing in the sale volume, and a purchasing investor pays more than the mid-price, the price increase growing with the purchase volume.

Formally, if, at the start of period *t*, the investor decides to purchase φ_t^d, he pays the nominal price P_t^d times a transaction factor $j^d(\varphi_t^d)$. The latter is generated by a strictly increasing and convex function j^d taking the value zero at the origin, with the first derivative at the origin equal to unity. That is, $P_t^d\left(j^d(\varphi_t^d)-\varphi_t^d\right)$ is a "fee", or premium paid (in domestic cash) to an intermediary. This premium is increasing with the order volume φ_t^d. If the amount $-\varphi_t^d>0$ is sold, the investor receives $-P_t^d j^d(\varphi_t^d)$, which is less than $-P_t^d\,\varphi_t^d$, with the intermediation premium being, again, equal to $P_t^d\left(j^d(\varphi_t^d)-\varphi_t^d\right)$. That is, transaction costs are positive for all non-zero trades. Analogously, for transactions in a foreign asset *f*, a similar strictly increasing and convex transaction factor j^f is defined, and a fee $P_t^f\left(j^f(\varphi_t^f)-\varphi_t^f\right)$ must be paid in the foreign currency. The same is true for the transactions between the domestic and foreign cash (the FX market), where the corresponding factor is denoted by *j*. For a net purchase of φ_t^i foreign currency units at date *t*, the investor pays the fee $S_t\left(j(\varphi_t^i)-\varphi_t^i\right)$.

To see the intuition underlying the above definition, let us first consider a quote-driven trading mechanism. For most securities (including FX), such markets have a convention of quoting a bid and an ask for *standard amounts*. Everything beyond the standard is either charged a different price with a premium for the market maker as a routine or has to be negotiated separately, if accepted at all. In any case, a market maker charges a higher price for a higher purchase order (pays a lower price for a higher sale order) as a protection against position risk.

Although economies of scale may exist in terms of the customer base size of a given dealer (i.e. dealers benefit from handling a large number of orders), there is no such thing in terms of the *order size* itself.

In the order-driven market (broker, limit order book), an increase in market order size can have one of the two consequences. The trader either incurs an execution delay and additional costs (the order has to be split with later segments facing a higher price in a changed limit order book) or needs to "walk the book" (i.e. execute outstanding parts of the order at the second-best, third-best, etc. limit price available). Both variants mean execution expenditure growing with volume.

To sum up, one can aggregate the liquidity providers for a given security (market makers and brokers) in an imaginary global limit order book. Then, the resulting order execution costs for a market user (i.e. the investor we are modeling) can be approximated by a convex transaction function of the order size, as defined above. This construction of the market-wide pricing schedule has been used in market microstructure models at least since the classic paper by Kyle, 1985. The property is also sufficiently theoretically supported by microstructure finance literature. For example, a specialist's pricing schedule as an increasing function of order size has been derived from first principles for both a monopolist and a competitive market maker as early as in Glosten, 1989.

The introduced transaction cost functions j summarize all the existing frictions in the asset markets that lead to elastic asset demands by the modeled investor, including the market structure factors. Simple examples of the transaction functions introduced above are quadratic ($j(\varphi)=\varphi+b\varphi^2/2$, b a positive constant) or exponential ($j(\varphi)=(\exp(b\varphi)-1)/b$). More involved examples, including an additional dependence on time, are possible.

Preferences
The investor, of whom we think as residing in the home country, derives utility from the real consumption or dividend, rate c withdrawn from the domestic real cash balances. At every time moment, this utility is influenced by the investor's solvency, expressed in real terms by the liquid balance $l=(x^0+Sx^i)/P$, where P is the domestic price level. The period utility u is a function of two arguments, l and c, smooth, strictly increasing and strictly concave in each of them. To guarantee an internal solution to the investor optimization problem, we also assume $u_c(l,+0)=\frac{\partial u}{\partial c}(l,+0)=+\infty$ for all l. To ensure no cash debt accumulation, we also impose the conditions $\lim_{l\to-\infty} u(l,c)=-\infty$, $u_l(+0,c)=\frac{\partial u}{\partial l}(+0,c)=+\infty$ for all c.

The defined form of utility formally resembles money-in-the-utility models. Alternatively, if one specializes to a functional form $u(l,c)=u^1(c)+\beta u^2(l)$, $\beta>0$, one can think of this definition as the one covering the non-zero liquidation probability of a financial institution. Namely, if u^1 is a standard utility of consumption satisfying the Inada conditions and β is the intensity of a Poisson liquidation event, then the presence of u^2-term expresses the utility derived from the investor's liquid assets in the eventuality of a close-down.

2.2 Investor's optimization problem

The investor has an infinite horizon and maximizes the expected utility ($\delta\in(0,1)$ is the time preference factor)

$$\sum_{\tau\geq 0}\delta^\tau E\left[u\left(\frac{x_\tau^0+S_\tau x_\tau^i}{P_\tau},c_\tau\right)\right] \tag{1}$$

subject to the constraints ($t\geq 0$)

$$x_{t+1}^0=(1+r_{t+1}^0)x_t^0+\sum_{d=1}^{N_1}x_t^d\Gamma_t^d-P_{t+1}c_{t+1}-S_{t+1}j(\varphi_{t+1}^i)-\sum_{d=1}^{N_1}P_{t+1}^d j^d(\varphi_{t+1}^d), \tag{2.1}$$

$$x_{t+1}^i=(1+r_{t+1}^i)x_t^i+\sum_{f=1}^{N_2}x_t^f\Gamma_t^f+\varphi_{t+1}^i-\sum_{f=1}^{N_2}P_{t+1}^f j^f(\varphi_{t+1}^f), \tag{2.2}$$

$$x_{t+1}^d=x_t^d+\varphi_{t+1}^d,\ d=1,\dots,N_1, \tag{2.3}$$

$$x_{t+1}^f = x_t^f + \varphi_{t+1}^f, f{=}1,\ldots,N_2, \tag{2.4}$$

The initial cash and asset holdings x_0^0, x_0^i, x_0^d, x_0^f are given. The decision variables of the investor are the trajectories $t \mapsto c_t$, $t \geq 0$, $t \mapsto \varphi_t^i$, $t \mapsto \varphi_t^d$, $t \mapsto \varphi_t^f$, $t \geq 1$, for all d and f. More specifically, at the end of period t and the beginning of period t+1, the investor knows the realizations of P_{t+1}, r_{t+1}^o, r_{t+1}^i, S_{t+1}, P_{t+1}^d, P_{t+1}^f, Γ_t^d, Γ_t^f (plus the histories of these variables and the distributions of their uncertain values one period ahead) and has to decide about the values of c_t, φ_{t+1}^d, φ_{t+1}^f, φ_{t+1}^i, d=1,…, N_1, f=1,…, N_2.

Alternatively, one can express c_τ by using (2.1), φ_τ^i by using (2.2) and φ_τ^d, φ_τ^f from (2.3), (2.4), and substitute into (1). Then the problem becomes that of unconstrained optimization with respect to the asset holding vector path $t \mapsto x_t = \left[x_t^0, x_t^i, \left(x_t^d\right)_{d=1}^{N_1}, \left(x_t^f\right)_{f=1}^{N_2}\right]^T$, $t \geq 1$.

Observe that due to the liquidity-dependence of the period utility in (1) the transversality conditions on the components of x are not needed in this optimization problem: the Ponzi-like behavior is prohibited by the conditions on the utility function behavior under big negative cash values. Similarly, unlimited short-selling of any asset is prohibited by convex transaction factors j.

Also note that equations (2) allocate the home asset transaction fees to the domestic cash account of the investor and the foreign asset transaction fees to the foreign cash one. For the FX trade, the fee is subtracted from the domestic cash account if the investor resides in the home country (we would have modeled its subtraction from the foreign cash account in the case of a foreign resident). Specifically, (2.1) contains a reduction in domestic cash holdings corresponding to φ_{t+1}^i, namely the term $-S_{t+1} j(\varphi_{t+1}^i)$. The latter can be decomposed as $-S_{t+1}\varphi_{t+1}^i - S_{t+1}\left(j(\varphi_{t+1}^i) - \varphi_{t+1}^i\right)$, the second term in this expression being the transaction fee. Thus, the domestic investor, when purchasing φ_{t+1}^i units of foreign currency, pays the amount in domestic cash determined by the current nominal exchange rate, plus a fee. (Conversely, when foreign cash is sold, the fee is subtracted from the sale revenue.) Similar interpretation can be given to other asset trade expenditures.

To formulate the first order conditions of optimality for the investor's policies, we introduce the *marginal transaction functions* $\varphi \mapsto h^d(\varphi) = \frac{dj^d}{d\varphi}(\varphi)$, $\varphi \mapsto h^f(\varphi) = \frac{dj^f}{d\varphi}(\varphi)$, $\varphi \mapsto h(\varphi) = j'(\varphi)$. They are increasing (since js are convex) and equal to unity at the origin. They give rize to the investor's asset demand schedules (see later).

Further, define the *autarky asset prices* (the reason for the name is that these are the prices that prevail in markets where investors choose to place zero size orders) as follows:

$$X_t^d = P_t^d h^d(\varphi_t^d),\ X_t^f = P_t^f h^f(\varphi_t^f),\ X_t = S_t h(\varphi_t^i).$$

Their role in the model will become clear shortly (see Proposition 1 and the discussion thereafter).

Finally, we introduce the pricing kernel in the usual way:

$$M_t^{t+1} = \frac{\delta P_t u_c(l_{t+1}, c_{t+1})}{P_{t+1} u_c(l_t, c_t)}, \; m_{t+1} = \log M_t^{t+1},$$

and also, for notational covenience, define an auxiliary symbol for the marginal rate of substitution between consumption and liquidity:

$$L_t = \frac{u_l(l_t, c_t)}{u_c(l_t, c_t)}.$$

Now we can formulate the investor's first order conditions of optimality.

Proposition 1 *The first order conditions of optimality for the investor's optimization problem* (1), (2), *are given by a sequence of four equations* (t=0, 1, …)

$$E_t\left[M_t^{t+1}\frac{\Gamma_t^d + X_{t+1}^d}{X_t^d}\right] = 1, \; d=1,\ldots, N_1, \; E_t\left[M_t^{t+1}\frac{X_{t+1}}{X_t}\frac{\Gamma_t^f + X_{t+1}^f}{X_t^f}\right] = 1, \; f=1,\ldots, N_2, \quad (3a)$$

$$E_t\left[M_t^{t+1}\right] = \frac{1 - L_t}{1 + r_{t+1}^0}, \; E_t\left[M_t^{t+1}\frac{X_{t+1}}{X_t}\right] = \frac{1 - L_t / h(\varphi_t^i)}{1 + r_{t+1}^i}. \quad (3b)$$

As usual, E_t *denotes expectation conditional on the information available up to period* t.

The proof is given in the Appendix .

Formulae (3) can be regarded as no-arbitrage pricing conditions. However, differently from the traditional asset pricing model, they are formulated in terms of autarchic prices X instead of the actual prices P and S. Therefore, the price system X can be viewed as the underlying/implicit price process in a Walrasian market, for which the standard pricing kernel relations are valid exactly. Since the actual prices become equal to the implicit ones if and only if there are no transactions in the markets (φ^i=φ^d=φ^f=0), X can be called autarchic prices, as mentioned earlier.

Actual price, autarky price and order flow

An autarchic asset price can be also regarded as the intercept of the investor's current demand schedule for this asset. We'll illustrate this with the FX market example which is in the center of of attention in the present paper.

By rewrting the definition of X as

$$S_t = \frac{X_t}{h(\varphi_t^i)},$$

we can regard the latter equation as the definition of the inverse demand curve in period t. When the current price (exchange rate) is below the autarky value, the investor optimally purchases foreign currency, and vice versa. The price is then reverted back to its autarky value, which is the one that obeys the standard no-arbitrage rules of a hypothetical perfectly frictionless market.

What is the reason for the actual price to deviate from the autarchic one? The general answer is that it must always be the case when the representative investor assumption with instantaneously clearing markets is not justified. We believe that the forex is an archetype of such a market. In this paper, we model a collection of financial market segments that include, beside the spot market for the CZK/EUR pair, a number of assets, both Czech and European, whose prices we believe may be related to the Czech koruna exchange rate. We are not modeling the totality of the euro area security markets, but only its selected elements with a bearing on the financial and real developments in the Czech economy. (For formal reasons, we have also included one asset on the euro area side that should proxy all Czech-unrelated uncertainties.) So, our model is of a partial equilibrium nature. In this section, the case of the big country-resident investor is selected for definiteness. At the estimation phase (Section 4), this perspective will be alternated with the reverse one (a small country-resident investor) in order to identify more of the model parameters, the autarchic FX return $\log\frac{X_{t+1}}{X_t}$ in particular. The investor is allowed to have external couterparties in the koruna/euro market, and we also assume that this investor does not himself make the market in the Czech koruna. Therefore, it is natural to assume the existence of a non-trivial active order flow from this investor outside (directed towards koruna FX dealers and/or Czech monetary authorities). This flow is represented by variable φ^i in the model. As regards the other asset markets under consideration, it will be assumed that the modeled investor is sufficiently representative in the corresponding market segments for the non-Walrasian phenomena to be neglected.

2.3 No-arbitrage asset pricing conditions and the uncovered asset return parity

So far, no assumption has been made about the nature of distributions driving the uncertainty in the model. The risk factors will eventually be defined in terms of normally distributed noises. To prepare for this specialization of the model, it is convenient to rephrase the first order conditions (3) in terms of continuously compounded returns. Specifically, let the *autarchic* one period return on domestic asset d between dates t and t+1 be defined as

$$y_{t+1}^d = \log\frac{\Gamma_t^d + X_{t+1}^d}{X_t^d}.$$

Analogously, the *domestic currency autarchic* one period return on foreign asset f between the same dates is defined as

$$y_{t+1}^f = \log\frac{X_{t+1}(\Gamma_t^f + X_{t+1}^f)}{X_t X_t^f}.$$

Similarly, $\rho_{t+1}^0 = \log(1+r_{t+1}^0)$, $\rho_{t+1}^i = \log(1+r_{t+1}^i)$ will be the continuously compounded one-period risk-free interest rates at home and abroad. Finally, let z_{t+1} be the one period autarchic return on foreign cash: $z_{t+1} = \log\frac{X_{t+1}}{X_t}$.

Conditions (3a) can be now stated uniformly as

$$E_t\left[e^{m_{t+1}+y^j_{t+1}}\right]=1, j=1, \ldots, N=N_1+N_2. \tag{4}$$

The focus on macroeconomic implications of the forex risks suggests we may ignore the non-Walrasian effects in other asset markets. Therefore, (4) can be considered the no-arbitrage pricing equations in terms of actual and not just autarchic returns. In other words, the standard pricing kernel equations will be assumed to hold for all assets except foreign cash. Under such an assumption, (4) contain only one unobservable variable (the pricing kernel logarithm m), whereas the asset returns are observable. On the other hand, (3b) only contain unobservable variables, with the exception of the risk-free rates. Modeling the cash-liquidity marginal substitution rate would mean opening an additional dimension of the analysis with no direct benefit for the objective of this study. Instead, we choose to use only a consequence of (3b) in terms of the pricning kernel and the FX order flow, that can be obtained by substituting L_t away. After having done the said substition we get the following equation:

$$h(\varphi^i_t)\left\{1-E_t\left[e^{\rho^i_{t+1}+m_{t+1}+z_{t+1}}\right]\right\}=1-E_t\left[e^{\rho^0_{t+1}+m_{t+1}}\right]. \tag{5}$$

Note that, under the autrarchic conditions of a Walrasian forex (φ^i=0), when z becomes the actual one period FX return, (5) degenerates to the uncovered interest rate parity condition (UIP) with a time-variable risk premium determined by the pricing kernel. The present paper gives up the UIP in favor of its order flow-based extension (5). In particular, our model does not link the FX order flow to the FX return directly, but to its excess return over the forward premium (the latter is equal to the difference between ρ^0_{t+1} and ρ^i_{t+1}).

Equations (4) and (5) will form the foundation of our empirical model. We shall assume that there are n independent Gaussian risk factors jointly driving the returns on the selected securities and the FX order flow. The dynamics of these unobserved risk factors will be estiamted together with the signal equations to be obtained from (4), (5).

Differently from most hidden factor models that use the pricing kernel, we do not calibrate the pricing kernel equation to fit the observed yield curve (or, in our two-country case, two curves), although the selected method would allow us to do so. Instead, we place the dominant weight on stock returns (even though there are two points on the yield curve(s), that of 1Y and 10Y maturity, that we use along the equities), since our ultimate objective is to find a link between the exchange rate volatility and real economic developments. This, we believe, can be more easily achieved by following the selected stock returns than the yield curve geometry. The existing explanations of the latter in terms of inflation, business cycle and monetary policy expectations are not straightforward enough to enable one a transparent interpretation of the estimation outputs.

On the technical side, since it is not necessary to fit the yield curve, the number of risk factors is not limited by the term structure tractability considerations. Instead of using a small number of unobserved factors to explain a large number of observed term structure parameters, we employ the number of factors comparable to that of the assets (plus one for the FX order flow). Therewith, we return to the original philosophy of stochastic asset pricing: "one asset – one risk factor". The difference from the textbook (international) consumption CAPM is the market incompleteness due to a separate source of forex risk. To keep track of the introduced innovation, we shall analyze two models in parallel. The one is a standard ICCAPM with uncertainties defined by (6)-(8) below, with no special role for the forex microstructure or order flow. The FX market is Walrasian and the first order conditions (3b) must be simplified accordingly. We will refer to this variant as the benchmark model. The other model attributes to the FX order flow an independent source of risk: the investor faces a non-trivial supply uncertainty in the forex, so that the information to be extracted from (3b) (or (5)) is necessary to price all the basic assets. This variant will be called the (full) FX order flow risk model.

3. The model in state space form

The asset pricing formulae following from the investor's first order conditions of optimality (3) (Proposition 1) will now serve as a starting point for representing the model in the state space form with the purpose to estimate it by a Kalman filter technique. The individual equations in (3) will be specialized to observation equations after we will have made specific assumptions about the statistical properties of uncertainty driving the model variables. At the same time, pinning down the structure of uncertainty will be paramount to formulating the state variable dynamics of the model (cf. (7) and (10) below).

The observation variables of our model are the above defined traded asset yields $y^1,\ldots,y^N$, as defined in Subsection 2.3, and the one-period interest rate (as appearing in (5)) differential $f=\rho^0-\rho^i$ (see also (15) below). The state process will be assumed multivariate autoregressive of order one, with Gaussian innovations. Restrictions on observation equation coefficients will follow from the no-arbitrage asset pricing conditions (3) (cf. Proposition 2). As a result, we shall obtain the system of observation equations (12), (15) and the state equation system (10), to be used in estimations. Formal specification and derivation follows.

We start by writing the observation equations for the yields formally as

$$y_{t+1}^j = a_0^j + a_1^j x_t + A^j x_{t+1},\, j=1,\ldots,N. \tag{6}$$

Here, $a_0 = \left[a_0^1,\ldots,a_0^N\right]^T$ is an $N\times 1$-vector of intercepts, a_1 and A are $N\times n$-matrices of coefficients with rows $a_1^j = \left[a_{11}^j,\ldots,a_{1n}^j\right]$ and $A^j = \left[A_1^j,\ldots,A_n^j\right]$ respectively. The n-dimensional vector x of unobserved state residuals follows the VAR(1)-process

$$x_{t+1} = bx_t + B\varepsilon_{t+1}. \tag{7}$$

Coefficient matrices b and B in (7) are of size $n\times n$. Process ε is an n-dimensional vector of mutually independent standard normal errors. In general, $n\neq N$, and, if there is a reason to assume, e.g. cyclical components in the observations, one will need to take $n>N$.

An additional unobserved variable to be used in the sequel is the log of the one period pricing kernel $m_{t+1} = \log M_t^{t+1}$:

$$m_{t+1} = \lambda_0 + \lambda_1 x_t + \Lambda x_{t+1}. \tag{8}$$

Here, λ_0 is a scalar constant, whereas λ_1 and Λ are row vectors of dimension n. The observation equation system (6) together with the state equation system (7), (8) constitute the state-space representation of the present model. However, this is not the definitive representation to be estimated, since one must incorporate the coefficient restrictions following from the no-arbitrage pricing conditions (4), (5).

Proposition 2 *The no-arbitrage pricing conditions* (4) *for the asset return model defined by* (6)-(8) *above are equivalent to the following constraints on the model coefficients*:

$$a_0^j = -\lambda_0 - \frac{\left|(\Lambda + A^j)B\right|^2}{2},\ a_1^j = -\lambda_1 - (\Lambda + A^j)b,\ j=1,\dots,N. \tag{9}$$

The proof is given in the Appendix. Note that, whereas the first equality in (4) is scalar, the second one is for N-dimensional row vectors.

We are able to reduce the number of modeled parameters by simplifying the covariance structure of the state equation through a change of variables. Specifically, we may consider a non-singular matrix B (otherwise, there would be too many states) and put $x_t=Bu_t$ for all t. Then

$$u_{t+1} = \Phi u_t + \varepsilon_{t+1},\ \Phi = B^{-1}bB. \tag{10}$$

The log-pricing kernel equation is now given by

$$m_{t+1} = c_0 + c_1 u_t + Cu_{t+1} \tag{11}$$

instead of (8), with $c_0=\lambda_0$, $c_1=\lambda_1 B$, $C=\Lambda B$. Put $\gamma^j = (\Lambda + A^j)B$. It is easily checked that the no-arbitrage pricing conditions (9) of Proposition 2 imply the following equations for the observed yields:

$$\begin{aligned} y_{t+1}^j &= -c_0 - \frac{\left|\gamma^j\right|^2}{2} - \left(c_1 + \gamma^j \Phi\right)u_t + \left(\gamma^j - C\right)u_{t+1} \\ &= -c_0 - \frac{\left|C + q^j\right|^2}{2} - \mu u_t + q^j \varepsilon_{t+1} = -m_{t+1} - \frac{\left|\gamma^j\right|^2}{2} + \gamma^j \varepsilon_{t+1},\ j=1,\dots,N, \end{aligned} \tag{12}$$

where $\mu=c_1+C\Phi$ and $q^j=\gamma^j-C$.

With the chosen normal state variable errors, implementation of conditions (4) has lead to relatively simple return formulae (12) with linear-quadratic coefficient constraints. Handling non-linearity in (5) is less straightforward. We will need to make a natural assumption that the autarchic FX return z has the same structure of risk factor dependence as other autarky returns:

$$z_{t+1} = \varsigma_1 u_t + \varsigma u_{t+1} = \xi u_t + \varsigma \varepsilon_{t+1}, \tag{13}$$

with $\xi=\zeta_1+\zeta\Phi$. Observe the missing constant term in (13): we do not assume any equilibrium trends in the autarchic exchange rate.

Introduce the certainty-equivalence shorthand notation:

$$\eta_t = c_0 + (c_1 + C\Phi)u_t + \frac{|C|^2}{2},\ \eta_t^* = c_0 + \left(c_1 + \varsigma_1 + (C+\varsigma)\Phi\right)u_t + \frac{|C+\varsigma|^2}{2}.$$

Then (5) can be rewritten as

$$h(\varphi_t^i) = \frac{e^{\rho_{t+1}^0 + \eta_t} - 1}{e^{\rho_{t+1}^i + \eta_t^*} - 1}. \tag{14}$$

Since one needs a linear structure in the unobserved states to apply the conventional Kalman filter technique, (14) must be further transformed. Qualitatively, this equation connects the aggregate FX order flow φ^i with the risk-adjusted interest rate difference ρ^0-ρ^i+η-η^*, plus higher order terms. Therefore, the easiest linearized version of (14) to use would be

$$f_t = \frac{|C+\varsigma|^2}{2} - \frac{|C|^2}{2} + (\varsigma_1 + \varsigma\Phi)u_t + \alpha\varphi_t^i = \varsigma\left(\frac{\varsigma}{2} + C\right) + \alpha\varphi_t^i + \xi u_t, \tag{15}$$

where f=ρ^0-ρ^i is the forward exchange premium/discount and α a positive constant. This would correspond to the marginal transaction function in the forex to be of the time-dependent form

$$h(\varphi_t^i) = H(t, \varphi_t^i) = \frac{1 - e^{-\rho_{t+1}^0 - \eta_t}}{e^{-\alpha\varphi_t^i} - e^{-\rho_{t+1}^i - \eta_t^*}}. \tag{16}$$

One can easily check that the marginal transaction function defined in this way has the right properties, i.e. is increasing, strictly positive for admissible realizations of the model variables, and takes the value of unity at the origin.

Equation (15) relates the aggregate FX order flow (the modeled investor-initiated net FX purchases if positive, sales if negative) and the forward premium in a seemingly counterintuitive way. Namely, high forward premia (i.e. a relatively high domestic short interest rate, risk-adjustment taken into account) correspond to a flow out of the domestic currency, and vice versa. On the other hand, what may appear unnatural from a naive portfolio adjustment point of view, is in line with the observed reality. The first look at the data (Fig. 1) renders a clearly positive relationship between the strengthening foreign currency, the growing home-foreign interest rate differential and the order flow into the foreign cash.

In the model, the observed sign of the OF-dependence is due to the fact that high forward premia correspond to high shadow/autarchic foreign currency values. (Recall that it is for the shadow prices that UIP holds in the model.) Shadow currency values above the actual ones induce an order flow into it, reverting the actual exchange rate back to its autarky level. Simultaneously with the accomplishment of this reversion, the forward premium falls to its neutral level and the positive order flow dies out. Thus, the model offers a qualitative explanation of the positive relation between the order flow and the forward premium observed in the data. Quantitatively, the FX risk premium coming out of (15) is non-stationary. Beside the time dependence contained in the order flow, the residuals of the form ξu_t are multivariate-autoregressive, due to (10), and conditionally correlated with the residuals in the asset return equations, due to (12).

The definitive formulation of our model will have the vector state process u satisfying the first order multivariate autoregressive equation (10). The observation equations are (12) and (15). The model written in state-space form (10), (12), (15) contains restrictions on the fixed coefficients. After having estimated it by means of the Kalman filter method and obtained the (hyper)coefficients Φ, c_0, c_1, C, γ, we can reconstruct the observation equation coefficients a_0, a_1 by means of (9) and the coefficient matrix A by using the definition of γ:

$$A^j = \gamma^j B^{-1} - \Lambda = \left(\gamma^j - C\right)B^{-1},\ j=1,\ldots,N. \tag{17}$$

This will complete the estimation procedure for the pricing kernel.

One period interest rate as a basic security in the benchmark model

In a standard pricing kernel paradigm for asset pricing (i.e. in an ICCAPM without the order flow variables or any other source of discrepancy between autarchic and actual asset prices), the one period domestic interest rate would be the single risk-free security return. Different specifications of the risk-free rate dependence on latent factors would give rise to different yield curve models. The present model is not constructed by directly fitting the term structure of interest rates. However, if one works with monthly data, it is easy to derive the pricing kernel estimate in a benchmark frictionless asset pricing model without independent order flow risks.

In the benchmark model, the one-month risk-free interest rate is one of the basic securities, to which we assign superscript 1. Then y^1_{t+1} is the continuously compounded one period rate between t and t+1. We incorporate it into the model by imposing the requirement A^1=0 in (6) for j=1. Equivalently, one must have γ^1=ΛB=C, the first observation equation degenerates to

$$y^1_{t+1} = -c_0 - \frac{|C|^2}{2} - \left(c_1 + C\Phi\right)u_t\,, \tag{18}$$

and the coefficient recovery formulae (17) are applicable for j=2,…,N.

All other points in the yield curve (with longer maturities) generate uncertain one period yields, so that there is no need for further specialization in case one would want to include any of them in the list of basic securities.

In the benchmark model, the *excess returns* on risky securities over the risk-free rate have a simple form

$$er^j_{t+1} = y^j_{t+1} - y^1_{t+1} = \frac{|C|^2}{2} - \frac{|\gamma^j|^2}{2} + (\gamma^j - C)\varepsilon_{t+1} = -Q^j\left(\frac{Q^j}{2} + C\right) + Q^j\varepsilon_{t+1}\,, j=2,\ldots,N, \tag{19}$$

with Q^j=γ^j-C. That is, basic asset excess returns are correlated white noise fluctuations around time-independent means. This is a standard CCAPM outcome for the risk structure defined in the present model.

Identification of the autarky exchange rate
If one limits attention to the vantage point of the domestically based investor, then the autarchic FX return, z, described by (13), remains unidentified. One way how to make identification possible, is to change the perspective to the one of a foreign investor with the same information and investment opportunity set as the domestic one defined earlier. In the FX order flow model, the foreign investor may be assumed to face a similar supply uncertainty. Since both variants of the model have been defined as symmetric with respect to the investor residency, the first order optimality conditions of Subsection 2.3 can be obtained for the foreigner by a simple adjustment of notations. Then, the autarky exchange rate identification can be achieved by making the following

Assumption 1 *The foreign investors are identical and face the same underlying risks, characterized by* (7), *as the domestic investor. There is no transfer of basic asset holdings from the foreigners to either the domestic investor or any third party that would cause a difference between the autarchic and observed basic asset prices.*

Assumption 1 says that autarky prices X^d, X^f can be used instead of the actual ones in the foreign investor first order conditions and all subsequent analysis. It can be shown that one case in which this is true is when the foreign investor weight in the basic asset markets is negligible compared to the home resident. If their sizes are comparable, Assumption 1 is still justified provided that the FX supply uncertainty is identical for investors of both residencies and is generated by third parties (e.g. the central banks or inventory traders that we do not model explicitly).

As follows from the mentioned symmetry of the model, the foreign investor faces the basic asset one period returns

$$y_{t+1}^{d*} = \log \frac{X_t(\Gamma_t^d + X_{t+1}^d)}{X_{t+1} P_t^d}, \quad y_{t+1}^{f*} = \log \frac{\Gamma_t^f + X_{t+1}^f}{X_t^f}.$$

Accordingly, the autarchic FX return z satisfies the equalities $y^{d*}=y^d+z$, $y^f=y^{f*}+z$ for all d and f. Given the no-arbitrage pricing conditions (12), the assumed underlying risk structure and Assumption 1, the above equalities all follow from the single condition on the pricing kernel coefficients for the two investors:

$$\zeta = C^* - C, \quad \varsigma_1 = c_1^* - c_1. \tag{20}$$

Here, asterisks denote the pricing kernel coefficients of the foreigner. Therefore, the autarchic exchange rate coefficients are available if the pricing kernel coefficient estimates of both the domestic and the foreign investor can be obtained. As an example, Fig. 2 features the actual and the *expected autarchic* (i.e. variable z less the current period errors, or the term $\xi \cdot u(-1)$ in (13)) monthly EUR/CZK returns according to both models.

4. Estimation

The estimation procedure for the model defined in the previous two sections has to be adapted to the short length of available time series covering the Czech capital market, as well as the substantial structural changes that this market underwent at the beginning of 1999. Particularly, the features of the Czech koruna FX spot segment have changed dramatically after the withdrawal of most short interest rate speculators, discouraged by the decrease of the Czech-EMU interest rate differential. So, there would make little sense to analyze the data from earlier periods.

We work with monthly data. This choice has two main reasons. First, the study should be usable on a recurrent basis with the possibility to update the sample continuously. However, collection of daily FX transaction data from the banks under the supervision of the Czech National Bank terminated in 2001. Therefore, one is forced to use as a proxy, the data currently collected in monthly periodicity on Czech koruna demand deposit accounts of non-residents (reported by banks to the balance of payments statistics). We discuss the reason in 4.1 below. Second, a higher than monthly frequency would make the one-period risk-free interest rate an unobservable variable. Indeed, the actual overnight and weekly money-market interest rates are heavily distorted by liquidity-induced noise. Other excess volatility for which the models of the considered class are unable to account concerns the high frequency asset return data (both the exchange rate and others).

Given these limitations, the model cannot be analyzed by a standard one-shot Kalman filter calculation conducted on the system (10), (12), (15). Instead, we focus on the central result of the model concerning the FX order flow and the forward premium decomposition (equation (15)). This equation is a key to establishing the magnitude of the exchange rate return deviation from its unobservable autarky level. We provide a state-space estimate of (15) together with the macro-risk observation equations that generate the first n-1 risk factors. (Recall that the last, nth factor is the unobservable risk needed to explain the FX order flow within the model.) In this way, the matrix of multivariate risk factor autoregression coefficients in (10) is estimated. Subsequently, the observed asset returns can be used to (calibrate and) estimate the coefficients in the pricing kernel equation (11).

To gauge the extent of change that the introduction of explicit FX order flow risk causes in a standard asset price model, we shall look at the coefficients of the pricing kernel in two setups. In Subsection 4.2, we consider the pricing kernel projected on the space of basic risk factors (i. e. the benchmark model is calibrated to the selected asset return statistics). After that, in Subsection 4.3, the basic risk factors are augmented by the FX order flow one, so that the full FX order flow risk model can be estimated.

4.1 Data

Our sample is of monthly data between 9:1999 and 6:2003. The observed seven macro risks are: German industrial production index, German CPI index, 1M EURIBOR, Czech-German 5Y government bond yield differential, Czech industrial production index, Czech CPI index, 1M PRIBOR (Prague interbank offer rate). The seven basic monthly asset returns are on (in the order appearing in the estimated equations and used in the Cholesky decomposition): Altana common stock, Volkswagen common stock, DAX, 1Y EURIBOR, 1Y PRIBOR, Ceska pojistovna common stock, PX50 (the main index of Prague Stock Exchange). Additionally, 10Y German and Czech bond returns will be used for calibrating the pricing kernel in the full FX OF risk model (Subsection 4.3).

The above choice was motivated by the pursuit of the following objectives:

- To include assets that are not perfectly correlated with the market indices (that is why Altana is included on the EU side and the major insurance company Ceska pojistovna – on the Czech side)
- To include an asset in whose price both small country and large country macro risks are likely to be represented (VW in our case)
- To select relevant risk factors in a thin capital market of the small accession country (that is why the 10 bonds and 1Y interest rates are included).

Depending on the reference currency of the modeled investor, ex post returns on the other country assets are exchange rate-adjusted. The chosen order of assets for Cholesky-decomposition purposes is: Altana common stock, VW common stock, DAX, 1Y EURIBOR, 1Y PRIBOR, Ceska pojistovna common stock, PX50 index.

Given the short sample, inclusion of more than seven basic assets is not justified.

As the aggregate order flow of the type appropriate for the present model is usually unobservable, we are using a proxy in the form of non-resident Czech koruna-denominated demand deposits in the Czech banking system. Formally, the received FX cross-border order flow is mostly accounted for in the bank statistics as koruna transactions with non-residents, whereas active (outgoing) FX order flow of the same banks falls into the category of foreign currency transactions. This is why the banking statistics are able to give a fair approximation of the net FX cross-border order flow into and out of koruna, for each reported month.

4.2 Projected pricing kernel in the benchmark model

We have calculated the pricing kernel projected on the seven "Czech-German" assets defined above, considered from the perspective of a standard frictionless Walrasian asset pricing model with no specific role for either exchange rate or the FX order flow. That is, we assume that there is a representative investor who can place his wealth equally easily on both sides of the border.

Determination of coefficients Q and C in (19) is a matter of calibration rather than estimation. The unconditional covariance matrix of excess returns renders row vectors Q^j by Cholesky decomposition, as soon as one fixes the order of assets. Subsequently, the row vector C can be recovered from the excess return unconditional means. It follows that the pricing kernel determination in this case hinges on the one period lead short rate multivariate representation (18). Clearly, such a representation is not unique. A feasible course of action – the one we take in the present study – is to select N macroeconomic and financial risk factors of interest in fixed order, one of which is the next period domestic short rate, and orthogonalize them in a structural multivariate AR(1)-model with state space defined by (10). The selected seven macro risks represent the signals. The imposed structure we choose is lower-triangularity of Φ. (The macroeconomic risk factors were ordered for the estimation purpose as: German industrial production index, German CPI index, 1M EURIBOR with one period lead, Czech-German 5Y government bond yield differential, Czech industrial production index, Czech CPI index, 1M PRIBOR, i.e. the Czech inter-bank offer rate, with one period lead.) The corresponding row of Φ can be then used to recover the coefficient row ($c_1+C\Phi$) in (18). Given that we already know C and have just estimated Φ, the estimate for c_1 follows.

The estimated state variable autoregressive structure is reported in Table 1. The calibrated log-pricing kernel coefficients for the benchmark model are given in Table 3, upper panel.

4.3 Order flow as an independent risk factor

As was explained in 4.1, the average change in the CZK demand deposits, w, is the variable that we are using in (15) instead of φ^i. Formally, this means that linearization of the no-arbitrage condition (3b) is performed with the marginal transaction function H defined in (16), with argument w replacing φ^i. Since both the actual order flow and the marginal transaction function are unobservable, this formal substitution does not impose additional restrictions on the model.

In the present model with the inter-currency order flow there is no risk-free interest rate in the proper sense. The pricing kernel does not integrate to the one period risk-free bond price. As the liquidity constraint condition (3b) indicates, the pricing kernel expectation times the one period risk-free return is less than one (cf. He and Modest, 1995, or Campbell et al, 1997, Chapter 8.3, p. 315; our condition (3b) specializes their inequality (8.3.1)). Accordingly, one cannot work with excess asset returns as in the benchmark model. Moreover, the short sample at our disposal does not allow for a full-fledged equation system estimation in the Czech-EU case. Instead, one has to apply ad hoc techniques, which combine Kalman filter estimation of equation subsystems with calibration of certain parameters. In particular, the orthogonalized macroeconomic risk factors $u_1,\ldots,u_7$ that were obtained for the benchmark model estimation purposes, have been supplemented by the eighth latent factor u_8 responsible for the FX order flow uncertainties. The state space system this time contained the seven previously used macroeconomic indicators (German IPI, German HICP, 1M EURIBOR, 5Y government bond yield differential, Czech IPI, Czech CPI, 1M PRIBOR) and the 1M forward premium as signal variables (equation (15) was used as the eighth observation equation). This system was estimated to obtain the last row of the state variable autoregression matrix Φ, the coefficient by w (the order flow proxy) and the risk factor loadings ξ in (15).

The outcome is given in Table 2.

For the sake of comparison with the benchmark model, we have obtained the pricing kernel coefficients by a calibration method similar to the one used in 4.1. Since this time, the risk-free short rate is unavailable (equation (18) can no longer be obtained from the first order condition (3b) which, instead, give rise to (15)), the usual excess returns, as in (19), cannot serve the purpose. To proceed by analogy with 4.1, we have chosen to consider the excess returns over the 1Y EURIBOR. For 8 components of vector C, corresponding to 8 latent risk factors, to be calculated, one needs 8 excess returns. We have only got 6 at our disposal so far (recall that 1Y EURIBOR was one of the risky returns in 4.1). One needs two more assets of which one could reasonably assume they are spanned by the same 8 risk factors as the others. That is, these two assets would have to be derivatives of the 7 risky assets defined earlier. In this paper, we shall make the following assumption about the existence of the two sought derivatives:

Assumption 2 *The EUR-denominated returns on 10Y German and Czech government bonds are spanned by the eight risk factors defined in 4.1. In addition, the German 10Y-bond yield is independent on the 3 "Czech-specific" risk factors.*

This assumption is in line with the majority of literature on term structure: the already introduced indicators more than exhaust the list of those macroeconomic and financial variables that are usually being employed to explain the long end behavior of the default-free yield curve.

The monthly return on the 1Y EURIBOR denoted by y^e, we form the eight excess returns $v^j=y^j-y^e$, j=1,…,8, and utilize the equations which are direct analogues of (19):

$$v^j_{t+1} = y^j_{t+1} - y^e_{t+1} = \frac{\left|\gamma^e\right|^2}{2} - \frac{\left|\gamma^j\right|^2}{2} + (\gamma^j - \gamma^e)\varepsilon_{t+1}$$

$$- (C + Q^e) \cdot p^j - \frac{\left|p^j\right|^2}{2} + p^j \varepsilon_{t+1}, j=1,\ldots,8, \qquad (20)$$

where we have used the notation $p^j=Q^j-Q^e$. Note that in the benchmark model, Q^e was zero (the risk loading vector of the risk-free one period interest rate), so that p^j was equal to Q^j for all seven assets considered there.

Equation (20) allows one to recover the scalar products $(C+Q^e)\cdot p^j$ from the unconditional means and variances of excess returns v^j. By Cholesky-decomposing the unconditional covariance matrix for $[v^1,\ldots,v^8]$, one also obtains the matrix formed by rows p^j. (In view of Assumption 2, the 10Y German bond return takes the 4th position after the three other German assets, and the 10Y Czech bond return takes the last, 8th positon.) Further, the error term $Q^e\cdot\varepsilon$ in (12) for $j=e$ is the residual of the corresponding signal equation. Its covariances with the residuals of state equations (10) must be equal to the components of Q^e. Therefore, (20) can be used to calibrate the vector of contemporaneous risk loadings C in the pricing kernel. Similarly, the constant in the signal equation for 1Y EURIBOR can be used to recover the pricing kernel constant c_0. In this way we reconstruct the full set of coefficients in the pricing kernel formula (11), as well as the autarchic exchange rate coefficients. The results are summarized in Table 3, lower panel.

5. Conclusion

The constructed international asset pricing model with an explicit FX order flow risk provides a method for estimating the divergence of the actual and the "fundamental", i.e. inducing no excess currency demand, exchange rate. The two variants of the model – the benchmark, which follows the usual representative investor ICCAPM paradigm, and the full FX order flow risk setting, if analyzed in parallel, allow one to assess the significance of the forex-specific risk factor in the investor decision making. Accordingly, one becomes able to form and regularly update an opinion on the presence of free space for an active monetary authority policy in the national currency market.

Specifically, the analysis presented in the paper could be a useful supporting tool at the phase of the ERMII-entry of a fresh EU-member and the selection of the euro central parity for the national currency. Supposing one establishes significance of the full FX OF risk model and its superior performance compared to the benchmark one, this could suggest the need to augment a purely macroeconomic view of the exchange rate behavior with the analysis of an additional latent component. Since the latter cannot be readily linked to the conventional set of macro fundamentals, one is confronted with a poorly classifiable source of nominal exchange rate volatility. Some central banks might choose at times to counteract it by FX interventions. The present paper does not express itself to the chance of such interventions to succeed. Instead, it proposes a methodology helping the central bank to assess the extent of latent pressure it would have to face in the forex.

Under other circumstances, a deterioration of performance of both models, the benchmark and the full FX OF ones, at once (such as a considerable shift in the estimated coefficients) would indicate a potential market pressure on the central parity, should it be fixed at the currently observed exchange rate level. Namely, such a development would imply that the international investors' management of the member country financial risks undergoes a revision that is not limited to the forex.

The prevailing informal evidence suggests that the most important determinants of the Czech koruna exchange rate expectation revision (and subsequent potential threats to the ERM II-compliance) are in the area of capital flows. The presently obtained results on a sample of nine selected Czech and euro area asset returns and a CZK/EUR order flow proxy, support this view. Shifts in relative productivity, terms of trade and other real exchange rate determinants do not constitute a significant source of *unexpected* exchange rate movements. Most of them are being learned by the market gradually with no need of a sudden revision of expectations. Indeed, these fundamental shocks are being relatively sluggishly transmitted into the representative Czech-EU asset return differentials, the effects being dampened, lagged and selective. On the contrary, the cross-border FX order flow, reflected in the reported Czech bank spot FX transactions with non-residents (this order flow is eventually reflected in the financial account of the balance of payments), explains a large portion of the koruna/euro rate deviations from the uncovered asset return parity. This happens irrespective of whether specific episodes of pronounced one-sided flow have or do not have clearly identifiable fundamental reasons. Our analysis indicates that there is a whole range of risk factors influencing the observed FX order flow – from standard macro fundamentals to idiosyncratic liquidity management-related – that the central bank should keep track of. The relative importance of these factors has to be gauged before deciding the exact form and extent of the central bank presence in the market.

Appendix: Proofs

Proposition 1

Denote by $G(x)$ the value of the intertemporal utility to be maximized, after the decision variables c and φ have been expressed through asset holdings x by means of the budget constraints (2). It can be checked that G is strictly concave in each scalar argument, so that the first order conditions w.r.t. x describe the optimal decisions. Fix $t \geq 1$ and calculate the partial derivative of G w.r.t. x_t^i. We obtain

$$\frac{\partial G}{\partial x_t^i} = \delta^t \left\{ \frac{S_t u_l(l_t, c_t)}{P_t} - \frac{S_t h(\varphi_t^i) u_c(l_t, c_t)}{P_t} + \delta E_t \left[\frac{S_{t+1} h(\varphi_{t+1}^i) u_c(l_{t+1}, c_{t+1})}{P_{t+1}} (1 + r_{t+1}^i) \right] \right\}.$$

Equating this expression to zero, dividing by $S_t h(\varphi_t^i) u_c(l_t, c_t)$, applying the definitions of M_t^{t+1} and L_t and recalling that $S_t = X_t / h(\varphi_t^i)$, we obtain the second equation in (3b). The remaining equations are proved analogously •

Proposition 2

The no-arbitrage pricing relation for asset j can be written as $E_t\left[e^{m_{t+1}+y_{t+1}^j}\right]=1$. Since both m and y^j are normally distributed with known conditional expectations and variances for each date t, the last equation can be rewritten as

$$E_t\left[m_{t+1}+y_{t+1}^j\right]+\frac{1}{2}Var_t\left[m_{t+1}+y_{t+1}^j\right]=0.$$

In accordance with (1)-(3), this is equivalent to

$$\lambda_0+\lambda_1 x_t+\Lambda b x_t+a_0^j+a_1^j x_t+A^j b x_t+\frac{1}{2}\left|(\Lambda+A^j)B\right|^2=0 \qquad \text{(A1)}$$

for all t for each j. Equations (A1) can be considered as identities involving a non-trivial vector autoregressive state process x. They are satisfied if and only if all coefficients on the left hand side of (A1) are identically zero. This means

$$a_0^j+\lambda_0+\frac{\left|(\Lambda+A^j)B\right|^2}{2}=0,\ a_1^j+\lambda_1+(\Lambda+A^j)b=0$$

for all j, which is equivalent to (4) •

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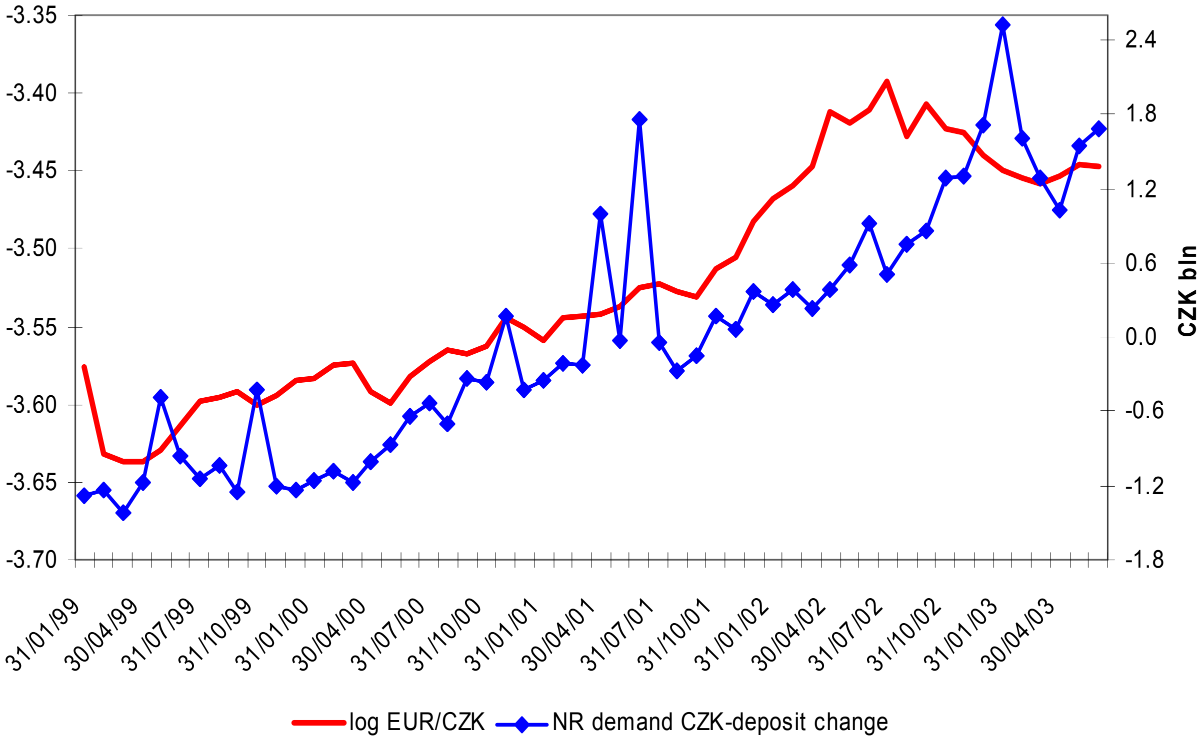
Fig. 1 Non-resident demand deposits in Czech koruna and the CZK/EUR market

(a) deposit change and the spot EUR/CZK rate



(b) deposit change and 1M EUR/CZK forward premium

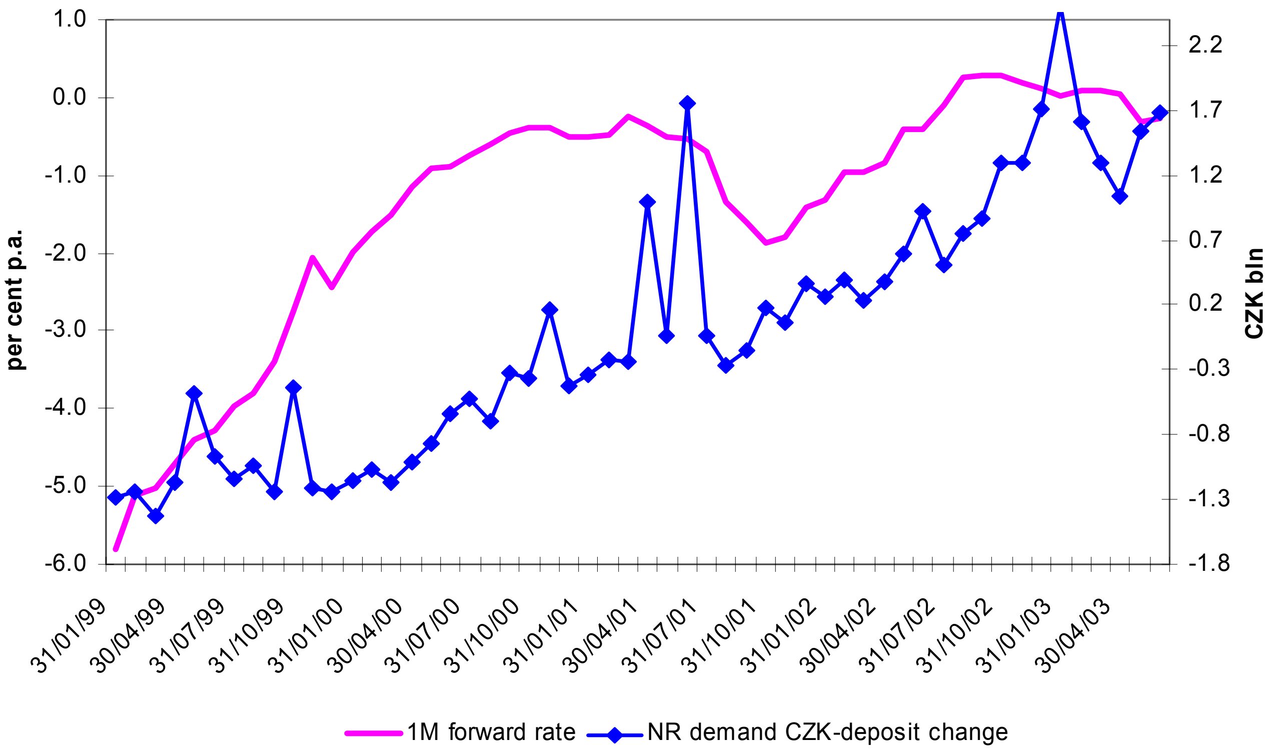


Fig. 2 Actual and expected autarchic monthly EUR/CZK returns

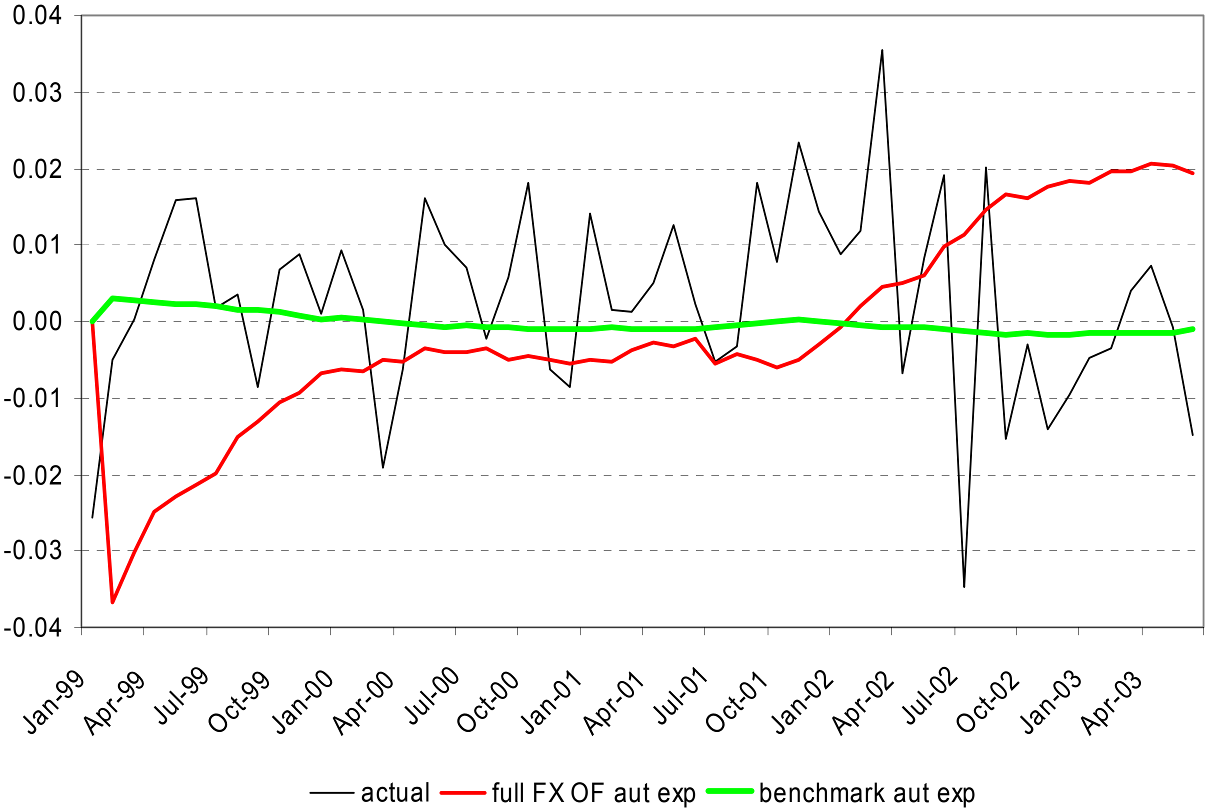


Table 1 State variable decomposition of the macro risks

	Normalized risk variables, lagged		$u_1(-1)$	$u_2(-1)$	$u_3(-1)$	$u_4(-1)$	$u_5(-1)$	$u_6(-1)$	$u_7(-1)$
Macro risk factors	*a*	*b*	Φ^1	Φ^2	Φ^3	Φ^4	Φ^5	Φ^6	Φ^7
Gerprod	0.00089	0.010848 (4.408947)	-0.468 (-1.664)						
Gercpi	0.01285	0.003293 (3.327244)	-0.137 (-0.470)	0.985 (9.291)					
Eu1m	0.00295	0.000143 (4.795073)	0.249 (0.446)	-0.020 (-0.074)	0.983 (12.257)				
Mr10yd	-0.00119	0.000192 (3.599884)	-0.076 (-0.210)	0.220 (0.952)	-0.122 (-1.499)	0.921 (10.461)			
Czprod	0.00376	0.03393 (3.590533)	0.247 (0.778)	-0.037 (-0.208)	-0.015 (-0.198)	-0.020 (-0.329)	-0.581 (-2.444)		
Czcpi	0.02561	0.003707 (3.062780)	0.156 (0.347)	0.286 (0.565)	0.137 (0.959)	-0.093 (-0.701)	0.085 (0.244)	0.789 (4.306)	
Cz1m	0.00410	0.000106 (3.709979)	-0.039 (-0.076)	-0.122 (-0.306)	0.006 (0.029)	-0.324 (-1.046)	0.032 (0.097)	0.216 (0.832)	0.769 (5.781)

Each macro risk factor, rf_j, is a linear function of a single normalized risk variable, u_j: $rf_j = a_j + b_j u_j$. Coefficients *a* were set equal to unconditional means of the risk factors. The normalized risk variables are multivariate autoregressive: $u_j = \sum_{i=1}^{7} \Phi^i_j u_i(-1) + \varepsilon_j$, with ε being standard normal independent disturbances. Φ^is are columns of the estimated matrix Φ. Individual elements of Φ, as well as coefficients *a* and *b*, are given in the table together with the z-statistics in parentheses.

The variable names stand for: Gerprod – German IPI, Gercpi – German CPI, Eu1m – 1M EURIBOR, Mr10yd – German-Czech monthly return differential on 10Y government bonds, Czprod – Czech IPI, Czcpi – Czech CPI, Cz1m – 1M PRIBOR.

Table 2 FX order flow risk model estimation output: Kalman filter for the eight risk factor model

Risk variables:	$u_1(-1)$	$u_2(-1)$	$u_3(-1)$	$u_4(-1)$	$u_5(-1)$	$u_6(-1)$	$u_7(-1)$	$u_8(-1)$
Parameter								
Φ_8	0.065	-0.217	-0.019	-0.392	-0.578	0.303	-0.306	0.998
z-statistic	0.181	-1.237	-0.223	-2.959	-1.894	2.275	-3.767	1256.490
ξ	0.00013	0.00019	-0.00024	-0.00003	0.00032	-0.00080	0.00027	-0.00103
z-statistic	0.590	1.005	-1.283	-0.136	1.937	-3.517	1.437	-7.314
α	0.00086							
z-statistic	2.525							
β	-0.044							
z-statistic	-59.048							

The estimated state space model consists of the observation equation

$$f = \beta + \alpha w + \sum_{i=1}^{8} \xi_i u_i(-1)$$

and the seven previously used macro risk factor observation equations (coefficients fixed at the levels obtained by the previous estimation). Here, f is the forward premium (the difference 1M EURIBOR-1M PRIBOR), and w is the change in the non-resident CZK demand deposits in the Czech banking system, our proxy for the order flow into/out of the Czech koruna. The state equations are the seven ones reported in Table 1 (coefficients fixed at the previously estimated levels) plus the one for the latent risk variable:

$$u_8 = \sum_{i=1}^{8} \Phi_8^i u_i(-1) + \varepsilon_8 .$$

The error term ε_8 is a standard normal noise uncorrelated with $\varepsilon_1, \ldots, \varepsilon_7$. Φ_8 is the last (8^{th}) row of Φ.

Table 3 Pricing kernel and autarchic FX return coefficients: comparison of the benchmark and the full FOREX order flow risk models of asset pricing

Benchmark model				
	EU-investor	*CZ-investor*	*Autarchic exchange rate*	
Parameter	Estimated/ calibrated value	Estimated/ calibrated value	Parameter	Estimated/ calibrated value
$C(1)$	-0.33	0.16	$\zeta(1)$	0.49
$C(2)$	0.08	0.54	$\zeta(2)$	0.45
$C(3)$	0.09	0.69	$\zeta(3)$	0.60
$C(4)$	-0.61	2.23	$\zeta(4)$	2.84
$C(5)$	-0.33	2.32	$\zeta(5)$	2.65
$C(6)$	-0.49	-0.74	$\zeta(6)$	-0.25
$C(7)$	0.12	-0.26	$\zeta(7)$	-0.38
$c_1(1)$	-0.05	-0.32	$\zeta_1(1)$	-0.27
$c_1(2)$	0.20	-0.74	$\zeta_1(2)$	-0.94
$c_1(3)$	-0.10	-0.27	$\zeta_1(3)$	-0.17
$c_1(4)$	0.55	-2.16	$\zeta_1(4)$	-2.71
$c_1(5)$	-0.15	1.42	$\zeta_1(5)$	1.57
$c_1(6)$	0.36	0.64	$\zeta_1(6)$	0.28
$c_1(7)$	-0.09	0.20	$\zeta_1(7)$	0.29
c_0	-0.43	-5.87		
FX order flow risk model				
	EU-investor	*CZ-investor*	*Autarchic exchange rate*	
$C(1)$	-0.33	-0.28	$\zeta(1)$	0.04
$C(2)$	0.08	0.13	$\zeta(2)$	0.05
$C(3)$	0.09	0.16	$\zeta(3)$	0.07
$C(4)$	-2.10	0.34	$\zeta(4)$	2.43
$C(5)$	-0.57	2.15	$\zeta(5)$	2.72
$C(6)$	-0.08	-0.08	$\zeta(6)$	0.00
$C(7)$	0.79	0.80	$\zeta(7)$	0.01
$C(8)$	-3.27	-3.26	$\zeta(8)$	0.01
$c_1(1)$	0.07	-0.41	$\zeta_1(1)$	-0.48
$c_1(2)$	-0.23	-0.71	$\zeta_1(2)$	-0.48
$c_1(3)$	-0.41	-0.14	$\zeta_1(3)$	0.27
$c_1(4)$	0.88	-1.29	$\zeta_1(4)$	-2.18
$c_1(5)$	-2.24	-0.65	$\zeta_1(5)$	1.59
$c_1(6)$	0.89	0.88	$\zeta_1(6)$	-0.01
$c_1(7)$	-1.61	-1.61	$\zeta_1(7)$	-0.01
$c_1(8)$	3.27	3.26	$\zeta_1(8)$	-0.01
c_0	-8.10	-8.09		

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