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CONTENTS

Abstract

In certain market environments, a large investor may benefit from building up a futures position first and trading subsequently in the spot market (Kumar and Seppi, 1992). The present paper identifies a variation of this type of manipulation that might occur in money markets with an interest rate corridor. We show that manipulation involving the use of central bank facilities would be observable only sporadically. The probability of manipulation decreases when the central bank uses an active liquidity management. Manipulation can also be reduced by widening the interest rate corridor.

JEL classification: D84, E52

Keywords: Money market, corridor system, manipulation

Non-technical summary

Money markets with interest rate corridors differ institutionally from other markets such as stock markets or markets for fixed income instruments. This paper identifies a kind of manipulation that might occur in such a money market. The strategy is to build up a significant position in short-term interest rate instruments (swaps or futures), and to affect the short-term market rate subsequently using the standing facilities provided by the central bank. In contrast to alternative forms of manipulation, the impact on short-term prices is achieved not mainly by trading activities, but by a strategic use of the standing facilities of the central bank. The paper offers a model that allows to derive the manipulator's optimal strategy in a market that is aware of the possibility of manipulation.

The main prediction of the model is that manipulation in money markets would occur only from time to time. Two comparative statics results are obtained. Firstly, the likelihood of manipulation decreases with the size of the reaction that the market rate exhibits in response to a strategic recourse to the central bank facilities. Thus, with an active liquidity management that neutralizes manipulative actions in due time, the central bank has the means to ensure that attempts to control the market rate will in general not be successful. The analysis also shows that manipulation becomes less likely in systems with a wider interest rate corridor. This suggest a new theoretical rationale for having the facility rates not "too close" to the target or policy rate.

1. Introduction

Major central banks around the world increasingly focus on steering some short-term money market interest rate in their implementation of the monetary policy stance. This is, for example, the case of the Federal Reserve in the US, the European Central Bank (ECB) in the euro area, and the Bank of England in the UK. More broadly, central banks around the world seem to increasingly attach greater value to stable, day-to-day and even intra-day money market conditions. With this aim, so-called corridor systems have been adopted, for example, in Australia, Canada, the euro area, New Zealand and the US. More recently the Bank of England announced that it is also considering adopting such a system (see Bank of England [4]).

In a corridor system, the central bank stands ready to provide overnight liquidity in unlimited amounts, generally against collateral, at a rate some basis points above market rates (lending facility); and stands ready to absorb liquidity overnight in unlimited amounts at a rate some basis points below market rates (deposit facility). By setting a corridor around the central bank target or policy rate, the range of variation of overnight interest rates will be bounded, on a day-to-day and intra-day basis, by the rates on the standing facilities, allowing short-term market interest rates to be steered with limited volatility around the desired level. This reduces the noise that short-term liquidity conditions may cause to the signalling of the monetary policy stance (see Woodford [12]).

Further stabilization of money market interest rates can be achieved by introducing a reserve requirement system with an averaging provision. Under this framework, banks have to maintain a daily positive reserve balance at the central bank, which has to reach a certain value on average over a maintenance or statement period. The stabilization of overnight interest rates works through an arbitrage mechanism. As a tool for liquidity management by commerical banks that is complementary to transacting in the inter-bank market, the reserve account at the central bank can be run down or increased on a daily basis. Thus, intuitively speaking, temporary or small liquidity imbalances can be absorbed by variations in quantities rather than prices. In fact, during most of the maintenance period, an individual bank's demand schedule will be highly elastic around the interest rate level expected to prevail at the end of the maintenance period. However, when the end of the period approaches, the demand schedule becomes increasingly inelastic as quantitative targets for reserves must be met.

The combination of a corridor system with an averaging mechanism provides a powerful framework to stabilize the overnight interest rate, as shown in Quirós and Mendizábal [10]. However, it appears that the issue of the appropriate design of standing facilities is still largely unexplored in the literature. Recently, Furfine [6, 7] shows how an improper design of the marginal lending facility may lead to its use being greater than what would be expected from the characteristics of the interbank market; and that a stigma from using the standing facilities may lead to its use being lower than what would be desirable to reduce interest rate volatility.

This paper wishes to contribute to the ongoing discussion on the appropriate design of corridor systems by showing that, from a theoretical perspective, manipulation is a potential issue in such money markets. Specifically, a commercial bank, obliged to hold minimum reserves on average over a statement period, might have build up a significant red position over the period for some reason, so that it would look as an attractive possibility if market rates were temporarily lower to ease refinancing. To create lower rates, the bank may take up a credit from the central bank, lending the money subsequently out into the market. Under certain conditions, this would generate a drop in the market rate, adding value to the red position. In fact, this strategy could be even more successful when the manipulator builds up a swap or futures position beforehand. We will discuss under which conditions this is a profitable strategy, and which incentive effects are created by this possibility. We will also discuss some of the means at the disposal of the central bank to eliminate this kind of behavior.

Market manipulation is a topic that has attracted significant attention during the last two decades. According to a useful classification by Allen and Gale ([1]), different forms of manipulation can be sorted into the three categories action-based (e.g., the analysis of manipulation around takeovers contained in Bagnoli and Lipman [3] and Vila [11]), information-based (e.g., by gurus as suggested by Benabou and Laroque [5]), and trade-based, which can either be informed (e.g., the study of manipulation around seasoned equity offerings by Gerard and Nanda [8]) or uninformed (e.g., the study of stock price manipulation due to Allen and Gorton [2]).[4]

[4] Other more recent forms of manipulation that are currently discussed in the media are so-called market timing and late trading in the context of the funds industry (esp. in the US).

Our model is a variant of a model used by Kumar and Seppi [9] to study the profitability of manipulation in futures markets with cash settlement. Their model falls into the category of uninformed trade-based manipulation, while ours could be interpreted as one of action-based manipulation. The difference between our model and Kumar and Seppi's is the cost structure underlying manipulative strategies. In our model, the manipulator can affect the market only by having recourse to standing facilities, which means that a non-marginal spread must be paid for a manipulation of the market rate. As we will see, this affects the optimal strategy of the manipulator, when compared to Kumar and Seppi's prediction, in a potentially relevant way.

The rest of the paper is structured as follows. In Section 2, we analyze the problem of the manipulator under the assumption of an exogenous price effect in the swap market. We discuss the consequences of endogenizing the price effect on the manipulator's strategy in Section 3. Section 4 concludes. The appendix contains technical derivations.

2. Exogenous price effect in the swap market

The time structure of our model is as follows (cf. Figure 1). At time $t = 0$, there is a swap market where banks and larger companies trade interest rate swaps that cover the last day of the statement period. At time $t = 1$, that is, on the evening before the last day of the statement period, there is the option available to the manipulator to have recourse to a credit or deposit facility. The net recourse to central bank facilities is published in the morning of the last day of the period. In $t = 2$, that is, during the last day of the statement period, spot trading occurs.

The manipulator approaches the swap market in $t = 0$, having an initial endowment $X_0 \sim N(0, \sigma_0^2)$ in the swap. This initial position may be the result of trading with non-bank customers, and is assumed to be private information to the manipulator. We will use the convention that a long position (e.g., $X_0 > 0$) means that the manipulator receives the variable leg, and that a short position means that the manipulator pays the variable leg. This convention has the consequence that a long position in the swap makes increasing market rates desirable, and declining rates undesirable for a manipulator with a long position.

At time $t = 0$, the manipulator submits a market order of X_1. In addition to the market maker, there are noise traders in the swap market, submitting an additional order volume of $Y \sim N(0, \sigma_Y^2)$. Total order volume is then given by

$$Z = X_1 + Y.$$

We will assume initially that a market maker is willing to clear the swap market at a rate

$$r^*(Z) = r^0 + \lambda Z,$$

for some *exogenous* $\lambda > 0$. It will become clear that the market maker, when assumed to follow the usual no-profit condition, would use a non-linear pricing rule. An extension of the model incorporating this possibility will be discussed in Section 3.

We assume that the market at time $t = 2$ does not suffer from informational frictions, so that the market rate depends only on the liquidity in the market. Ignoring autonomous factor shocks (they would cancel out in expectation), we may assume that the market rate depends only on the net recourse to standing facilities S, so that the spot rate amounts to

$$r(S) = r^0 - \rho S,$$

where $\rho > 0$ is the liquidity effect on the last day. Standing facility rates are given by r^L for the marginal lending facility and $r^D < r^L$ for the deposit facility. We assume that the corridor is symmetric around the central bank's target rate r_0, i.e.,

$$r^0 = \frac{r^D + r^L}{2}.$$

Controlling the market rate. Expected profit for the manipulator is given by the sum of the net returns on the individual positions in the swap and spot markets, i.e.,

$$\pi(S) = X_0(r(S) - r^0) + X_1(r(S) - r^*) + S(r(S) - r^{L/D}(S)),$$

where we write

$$r^{L/D}(S) = \begin{cases} r^L & S > 0 \\ r^0 & S = 0 \\ r^D & S < 0 \end{cases}$$

for the interest rate that the manipulator either pays for having recourse to the marginal lending facility or that she receives for depositing money with the central bank.

The model is solved backwards. The first order condition in $t = 1$ equalizes the marginal benefit from manipulation with the marginal cost of manipulation

$$\rho(X_0 + X_1 + S) = r(S) - r^{L/D}(S).$$

Rearranging and taking care of the discontinuity at zero yields our first intermediary result. See Figure 2 for illustration.

Proposition 1. *For given swap position $X = X_0 + X_1$, the optimal strategic use of standing facilities at date $t = 1$ is*

$$S^*(X) = \begin{cases} -\dfrac{X}{2} - \dfrac{r^L - r^0}{2\rho} & \text{if } X \leq -\dfrac{r^L - r^0}{\rho} \\ 0 & \text{if } -\dfrac{r^L - r^0}{\rho} < X < \dfrac{r^0 - r^D}{\rho} \\ -\dfrac{X}{2} + \dfrac{r^0 - r^D}{2\rho} & \text{if } X \geq \dfrac{r^0 - r^D}{\rho}. \end{cases}$$

Proof. See the appendix.¶

Thus, a manipulator who has build up a sufficiently large red position in the swap market ($X \ll 0$) will have recourse to the marginal lending facility ($S > 0$) to cause prices to fall, while a manipulator with a sufficiently large black position ($X \gg 0$) will have recourse to the deposit facility ($S < 0$), and profit from the tightening of the market.[5] In contrast to Kumar and Seppi's [9] model, we find that manipulation is not always profitable. Specifically, there is an intermediate range for the swap position X where manipulation of the spot rate does not pay off. This is due to the non-marginal cost of having recourse to standing facilities, which is, as mentioned earlier, the main difference between our model and the one used by Kumar and Seppi.

Building up a position. Proceding backwards, we now pose the question of how the manipulator chooses the swap order. Plugging the optimal usage

[5] In practice, this position taking could also be accomplished by satisfying reserve requirements unevenly over time, but we ignore this possibility for reasons of simplicity.

of the standing facilities into the objective function of the manipulator and carefully considering the resulting problem for the manipulator at date $t = 0$, we obtain our next main result:

Proposition 2. *Assume that the liquidity effect in the spot market is not too large when compared to the price impact in the swap market, i.e., $\rho < 4\lambda$. Then, for a given initial swap position X_0, the optimal swap order size is finite and given by*

$$X_1^* = \begin{cases} -\dfrac{\rho}{4\lambda - \rho}\left(X_0 - \dfrac{r^0 - r^D}{\rho}\right) & \text{if } X_0 \geq \dfrac{r^0 - r^D}{\rho} \\ 0 & \text{if } -\dfrac{r^L - r^0}{\rho} < X_0 < \dfrac{r^0 - r^D}{\rho} \\ \dfrac{\rho}{4\lambda - \rho}\left(X_0 + \dfrac{r^L - r^0}{\rho}\right) & \text{if } X_0 \leq -\dfrac{r^L - r^0}{\rho} \end{cases}.$$

Proof. See the appendix.¶

The above result stresses the effect that a market participant may, in anticipation of profitable opportunities to control the market rate, have an incentive to leverage her position in the swap market. When the initial position is sufficiently red ($X_0 \ll 0$), the manipulator will increase her exposure by going further short in the swap market ($X_1 < 0$). Subsequently, she will inflate reserves in the market by having recourse to the marginal lending facility ($S > 0$). Conversely, when the initial position is sufficiently black ($X_0 \gg 0$), then the long position will be further enlarged ($X_1 > 0$), and the manipulator will subsequently draw reserves from the market.

Proposition 2 illustrates another difference to Kumar and Seppi's model. In their model, there is (endogenously) no price effect in the futures market, so that the manipulator would always want to build up an infinite position unless hindered by a margin requirement. In the present model, the optimal position in the swap market may be either finite or infinite, depending on the relative size of price effect in the swap market and liquidity effect.[6] In the case considered in the Proposition, we find an optimal *finite* position. The difference to Kumar and Seppi's model that drives this effect seems to be that in their model, a large order in the futures market implies that the price in the spot market will increase not only due to manipulative trading, but also due to the changing expectations of the market specialist in the spot market. Note that this link is not present in our model. As a consequence, the futures (swap) rate in our model exhibits a reaction to the order flow, which is not the case in Kumar and Seppi's analysis.

The probability that the manipulator does not leverage her position in equilibrium is given by

$$\begin{aligned} \text{pr}\{X_1 = 0\} &= \text{pr}\{-\frac{r^L - r^0}{\rho} < X_0 < \frac{r^0 - r^D}{\rho}\} \\ &= \Phi(\frac{r^0 - r^D}{\rho\sigma_0}) - \Phi(-\frac{r^L - r^0}{\rho\sigma_0}). \end{aligned} \tag{1}$$

It is not difficult to see that the expression on the right-hand side decreases in ρ. Thus, the larger the liquidity effect ρ, the more likely will it be that the manipulator will leverage the initial position. Similarly, if the interest rate corridor is tightened by either decreasing the lending rate r^L or by increasing the deposit rate r^D or both, then the probability of manipulation increases

[6] In fact, as we will see, there are parameter values for which the *endogenously* determined price effect in the swap market generates finite positions. So the finiteness of the position is not an artifact of the exogenity of the price effect.

as well. This latter comparative statics result suggests that tightening the interest rate corridor "too much" may be detrimental to the objectives of monetary policy implementation.

Several manipulators The main prediction of the present paper is that in a corridor system of monetary policy implementation, and under suitable conditions, an individual bank may find it worthwhile to make strategic use of central bank standing facilities. This provokes the question as to why there should be only one potential manipulator. If sufficiently profitable, we would expect any commercial bank to stand ready for such activities. However, the theoretical analysis suggests that a strategic recourse to central bank facilities would be profitable only under the very restrictive condition that the potential manipulator possesses a secretly acquired and sufficiently large position in short-term interest rate instruments. In fact, if the initial position were public information, then it should be expected that the market maker would request a mark-up on futures prices, making manipulation unprofitable. Thus, manipulation would be very unlikely, and the coincidence of two or more banks making simultaneously strategic usage of central bank facilities may be neglected without much loss.

In practice, the decision to manipulate will depend not only on the initial endowment but also on (i) the overall trading and collateral capacities of the bank, (ii) its general readiness to take strategic measures in the search of profit opportunities, including the involved daringness vis-à-vis the central bank and potentially other regulatory institutions. For these reasons, we would expect that even in a large currency area, only few banks may be prepared for manipulative actions such as those described in this paper. Depending on the central bank's stance on this issue, it may also be difficult for an individual bank to repeat an unwanted manipulative strategy. In sum, it appears that in practice other reasons may amplify the sporadic nature of manipulation, so that the restriction to just one manipulator does not seem as a serious qualification for the validity of the predictions.

3. Endogenous price effect

We have noted before that the linear price rule of the market maker in the swap market may not correspond to a zero-profit condition. Indeed, the non-linear strategy used by the manipulator suggests that the pricing rule should take this behavior into account. In this section, we will discuss the question of endogenizing the pricing rule in this set-up. We will allow for a general pricing rule

$$r^*(Z) = r^0 + \lambda(Z),$$

for some twice differentiable *function* $\lambda(.)$. Note that we are back in the previously studied case when $\lambda(Z) \equiv \lambda Z$.

The plan is now to calculate, from $\lambda(.)$, first the optimal order size for the manipulator, then the resulting distribution of orders X_1, and finally, the market maker's posterior beliefs about X_1 given Z. When these posterior beliefs correspond to $\lambda(.)$, we have identified an equilibrium.

Note that the more general form of the pricing rule in the swap market does not affect the manipulator's problem in the spot market. Thus, Proposition 1 remains valid without change. Our starting point is therefore the manipulator's problem of choosing X_1 in the swap market so as to maximize expected profit.

Proposition 3. *For an initial swap position*

$$X_0 \in (-\frac{r^L - r^0}{\rho}; \frac{r^0 - r^D}{\rho}),$$

the manipulator will not leverage the initial position, i.e., $X_1 = 0$*. Outside of this interval, i.e., for either*

$$X_0 \leq -\frac{r^L - r^0}{\rho} \text{ or } X_0 \geq \frac{r^0 - r^D}{\rho},$$

the optimal swap order is given implicity by $H(X_1) = X_0$*, where*

$$H(X_1) = \frac{2}{\rho}\{\widehat{\lambda}(X_1) + X_1\widehat{\lambda}'(X_1)\} - X_1 + \frac{r^0 - r^{L/D}(X_1)}{\rho}. \tag{2}$$

Proof. See the appendix.¶

For illustration, the reader may refer to Figure 3, which shows the size of the swap order X_1 as a function of the initial position X_0 in a simulated example. The illustration suggests two features of the equilibrium. On the one hand, as before, the manipulator refrains from leveraging small initial positions. On the other hand, and in contrast to the model with exogenous pricing, the extent of leverage declines for larger initial positions. The reason is that in the model with endogenous pricing, the market maker responds to sizable orders by adjusting the price, which makes leveraging more expensive for the manipulator.

The strategy of the manipulator will generate a distribution of market orders X_1 with an atom at $X_0 = 0$. Denote by $f(.)$ the density of X_1, satisfying

$$\text{pr}\{X_1 = 0\} + \int_{X_1' \neq 0} f(X_1)dX_1 = 1.$$

In equation (1), we have derived a formula for $\text{pr}\{X_1 = 0\}$, i.e., for the probability mass of the atom. With the help of the previous proposition, the density of the distribution for values $X_1 \neq 0$ can be calculated as a transformation of the distribution of the initial position.

Proposition 4. *For $X_1 \neq 0$, the density function is given by*

$$f(X_1) = |\frac{H'(X_1)}{\sigma_0}|\phi(\frac{H(X_1)}{\sigma_0}), \tag{3}$$

where

$$\phi(t) := \frac{1}{\sqrt{2\pi}} \exp(-\frac{t^2}{2})$$

is the density of the standard normal distribution.

Proof. See the appendix.¶

We proceed by calculating the price effect from the optimal strategy. The market maker in the swap market will form expectations about the extent of manipulation in the swap market, i.e., about S. The equilibrium condition is that

$$\begin{aligned} r^*(Z) &= r^0 + \lambda(Z) \\ &= r^0 - \rho E[S|X_1 + Y = Z]. \end{aligned}$$

By Proposition 1,

$$\lambda(Z) = -\int \{\widehat{\lambda}(X_1) + X_1\lambda'(X_1)\} d\nu(X_1|Z), \tag{4}$$

where $d\nu(X_1|Z)$ is the conditional distribution of X_1 given Z. By Bayes' rule,

$$d\nu(X_1|Z) = \frac{f(X_1)\phi(\frac{Z - X_1}{\sigma_Y})}{\text{pr}\{X_1 = 0\}\phi(\frac{Z}{\sigma_Y}) + \int_{X_1' \neq 0} f(X_1')\phi(\frac{Z - X_1'}{\sigma_Y})dX_1'} \tag{5}$$

for $X_1 \neq 0$. The functional equation (4), complemented by equations (2), (3) and (5), specifies the endogenous price effect in the swap market.[7]

Numerical computation. The functional equation that determines the price effect in the swap market is highly non-linear so that we doubt that an explicit solution is feasible. We have therefore used numerical methods to discuss the properties of the equilibrium. We think that the intuitions gained are interesting, so that we will briefly describe the computations and the results.

To find the non-linear equilibrium, we have used the standard method of approximating fixed points of a functional operator by iterating the operator. The functional operator has been programmed as follows. Starting from an approximation $\lambda_0(Z)$ of the pricing rule, given by its values on a number of sampling points in a pre-specified interval. We have then used methods of numerical integration to calculate sampling points of the smoothed function $\widehat{\lambda}(X_1)$ on a somewhat smaller interval. The derivatives $\widehat{\lambda}'(X_1)$ and $\widehat{\lambda}''(X_1)$

[7]There exists an analogous formula for the case $X_1 = 0$, but this formula need not be spelled out because the integrand in (4) vanishes at zero.

have been calculated similarly, based on the explicit formulas

$$\begin{aligned}\widehat{\lambda}'(X_1) &= \frac{1}{\sqrt{2\pi}\sigma_Y}\int \lambda'(X_1+Y)\exp(-\frac{Y^2}{2\sigma_Y^2})dY \\ &= \frac{1}{\sqrt{2\pi}\sigma_Y^3}\int \lambda(X_1+Y)Y\exp(-\frac{Y^2}{2\sigma_Y^2})dY,\end{aligned}$$

and

$$\begin{aligned}\widehat{\lambda}''(X_1) &= \frac{1}{\sqrt{2\pi}\sigma_Y^3}\int \lambda'(X_1+Y)Y\exp(-\frac{Y^2}{2\sigma_Y^2})dY \\ &= -\frac{1}{\sqrt{2\pi}\sigma_Y^3}\int \lambda(X_1+Y)(1-\frac{Y^2}{\sigma_Y^2})\exp(-\frac{Y^2}{2\sigma_Y^2})dY,\end{aligned}$$

which can be found using integration by parts. From the thereby derived approximations, the density function $f(X_1)$ could be calculated on sampling points using the expression given in Proposition 4. This in turn allowed to determine the conditional density $d\nu(X_1|Z)$ for any given value of Z, again on a number of sampling points. This makes it feasible to numerically approximate $\lambda(Z)$ as given by (4), for a given Z. Varying now Z over the same set of sampling points, we obtain a new approximation $\lambda_1(Z)$ for the price effect in the swap market. The obtained approximation was then extended linearly to the larger set of sampling points (corresponding to the larger interval). This procedure was iterated, so that a sequence of approximations $(\lambda_n(.))_{n\geq 0}$ was generated, where each $\lambda_n(.)$ was defined on the same set of sampling points. Unfortulately, we have no formal argument that the apparent point-wise limit of the established sequence is an equilibrium of our model with endogenous price effect. However, the results are intuitive.

The resulting pricing effect in the swap market is predicted to be very low for small values of Z, and somewhat larger when Z becomes big (cf. Figure 4). This property of the pricing rule is due to the fact that for small absolute values of Z, the market maker's posterior assigns a relatively large probability to the atom $X_1 = 0$, so that the likelihood of a leverage strategy is perceived to be small in the swap market. However, for larger values of Z, the posterior probability assigned by the market maker to a manipulative strategy increases overproportionally with Z. This is because the posterior probability of the atom $X_1 = 0$ decreases exponentially when Z grows linearly. The range of values for X_1 in which the manipulator leverages her position remains the same as before. However, as Figure 5 illustrates, the non-linear price effect in the swap market induces the manipulator to leverage her position somewhat stronger for smaller (in absolute terms) initial positions when compared to the linear set-up studied before. This behavior yields a very particular shape of the ex-post distribution of the market orders X_1. Specifically, the density $f(X_1)$ is bimodal (cf. Figure 6). The density again determines the specific form of the posterior, which suggests that the numerical computations have in fact approximated an equilibrium, and the results of Section 2 appear to be robust with respect to endogenizing the price effect in the swap market.

4. Conclusion

In this paper, we have pointed out that in money markets that are embedded in a corridor system, composed of central bank lending and deposit facilities, there is the potential for manipulative action that abuses these facilities in a strategic way. Specifically, we mentioned the example of a bank that builds up a large short position in the market for short-term interest-rate instruments, and that subsequently offers interbank credit at very low rates, financed from the central bank's lending facility. We have shown that this type of manipulation can be profitable for a bank with suitable ex-ante characteristics. In addition, we have seen that manipulation remains a feature of the equilibrium even if the market responds to the possibility of manipulation with a rational-expectations adaptation of the pricing rule.

The comparative statics analysis showed that the likelihood of manipulation increases with the liquidity effect. This supports the common understanding that with suitably chosen fine-tuning operations, the central bank has the means to ensure that attempts to control the market rate will in general not be successful. In fact, one could argue that the availability of fine-tuning is a necessary condition for avoiding such kind of manipulative recourses. The second insight from the comparative statics analysis is that the likelihood of manipulation decreases with the width of the interest rate corridor. This suggest a new theoretical rationale for having the rates not "too close" to the target or policy rate. In fact, when these costs of a tighter corridor are balanced with the benefit of cutting off the spikes of non-strategic interest rate volatility, the analysis provides a intuitive foundation for an optimal size of the interest rate corridor.

Appendix

Proof of Proposition 1. Assume first that $X \geq 0$. In this case, it cannot be optimal to choose $S > 0$, i.e., to manipulate the price downwards, because $S = 0$ avoids the costs for having recourse to the marginal lending facility, and does not lower the value of the overall position. Thus, $S^* \leq 0$ for $X \geq 0$.

However, the profit function is continuous and twice differentiable for $S \leq 0$. First and second order derivatives in this domain are given by

$$\frac{\partial \pi}{\partial S} = -X\rho + r^0 - 2\rho S - r^D$$
$$\frac{\partial^2 \pi}{\partial S^2} = -2\rho < 0.$$

From the concavity of the objective function it follows that the first-order condition

$$S^+ = \frac{1}{2\rho}\{-X\rho + r^0 - r^D\}$$

characterizes the solution whenever $S^+ \leq 0$, i.e., when

$$X \geq \frac{r^0 - r^D}{\rho},$$

while the boundary solution $S^* = 0$ is optimal whenever $S^+ > 0$, i.e., when

$$0 \leq X < \frac{r^0 - r^D}{\rho}.$$

A completely analogous argument can be made for the case $X < 0$, which yields the assertion.¶

Proof of Proposition 2. The proof consists of three steps.

Step A. Let

$$\underline{X}_1 := -\frac{r^L - r^0}{\rho} - X_0$$
$$\overline{X}_1 := \frac{r^0 - r^D}{\rho} - X_0.$$

Consider first the problem where the manipulator is restrained to make an order $X_1 \leq \underline{X}_1$. Then, by Proposition 1,

$$S^*(X_1) = \frac{1}{2}(\underline{X}_1 - X_1) \geq 0, \tag{6}$$

and the profit function is given by

$$\pi(X_1) = X_0(r(S) - r^0) + X_1(r(S) - E[r^*(Z)]) \tag{7}$$

$$+ Sr(S) - Sr^L$$

$$= -X_0\rho S - X_1(\rho S + \lambda X_1) + S(r^0 - \rho S) - Sr^L. \tag{8}$$

Plugging (6) into (8) and differentiating twice with respect to X_1 gives

$$\frac{\partial \pi}{\partial X_1} = \frac{1}{2}X_0\rho - (\rho S + \lambda X_1) + (\frac{\rho}{2} - \lambda)X_1$$

$$+ S\frac{\rho}{2} - \frac{1}{2}(r^0 - \rho S) + \frac{1}{2}r^L$$

$$\frac{\partial^2 \pi}{\partial X_1^2} = \frac{\rho}{2} - \lambda + \frac{\rho}{2} - \lambda - \frac{\rho}{4} - \frac{\rho}{4}$$

$$= \frac{\rho}{2} - 2\lambda.$$

The constrained problem has a solution if and only if the second-order condition $\rho < 4\lambda$ is satisfied. When the solution is interior, the first-order condition yields

$$X_1^- = \frac{1}{4\lambda - \rho}\{\rho X_0 + r^L - r^0\}$$

$$= -\frac{\rho}{4\lambda - \rho}\underline{X}_1. \tag{9}$$

However, since $X_1 \leq \underline{X}_1$ by assumption, the interior solution cannot be optimal for $\underline{X}_1 < 0$. Thus, for $\underline{X}_1 < 0$, i.e., for a sufficiently large initial long position, the optimal solution of the unconstrained problem satisfies $X_1 \geq \underline{X}_1$, so that the manipulator does not use the marginal lending facility.

Step B. Consider now the manipulator's problem under the constraint $X_1 \in [\underline{X}_1; \overline{X}_1]$. From Proposition 1, we know that in this case, $S = 0$. This implies

$$\pi(X_1) = X_1(r^0 - E[r^*(Z)])$$

$$= -\lambda(X_1)^2.$$

Thus, under the restriction $X_1 \in [\underline{X}_1; \overline{X}_1]$, the optimal choice for X_1 is

$$X_1^* = \begin{cases} 0 & \text{if } \overline{X}_1 \geq 0 \\ \overline{X}_1 & \text{if } \overline{X}_1 < 0 \end{cases} . \tag{10}$$

We have seen in Step A that for $\underline{X}_1 < 0$, the optimal solution of the unrestrained problem satisfies $X_1 \geq \underline{X}_1$. By completely analogous arguments, one can show that for $\overline{X}_1 > 0$, the optimal solution of the unconstrained problem satisfies $X_1 \leq \overline{X}_1$. Thus, if $\underline{X}_1 < 0$ and $\overline{X}_1 > 0$ are satisfied simultaneously, then the optimal solution lies in the interval $X_1 \in [\underline{X}_1; \overline{X}_1]$, and from (7), we obtain $X_1^* = 0$.

Step C. Assume now that $\underline{X}_1 < \overline{X}_1 \leq 0$. From Step B, we know that then the optimum must lie in the interval $[\overline{X}_1; \infty)$. But in this interval, the problem is concave, and the first-order condition (derived as in Step A) is satisfied for

$$X_1^+ = -\frac{\rho}{4\lambda - \rho} \underline{X}_1 \geq \overline{X}_1.$$

Hence, for $\underline{X}_1 < \overline{X}_1 \leq 0$, we get $X_1^* = X_1^+ \geq 0$. Now assume that $\underline{X}_1 \geq 0$. Then clearly $\overline{X}_1 > 0$. By considerations analogous to those performed in the previous two steps, we obtain that in this case $X_1^* := X_1^- < 0$. Summarizing, we have that

$$X_1^* = \begin{cases} X_1^+ & \text{if } \underline{X}_1 < \overline{X}_1 \leq 0 \\ 0 & \text{if } \underline{X}_1 < 0 < \overline{X}_1 \\ X_1^- & \text{if } 0 \leq \underline{X}_1 < \overline{X}_1 \end{cases} .$$

This completes the proof of Proposition 2.¶

Proof of Proposition 3. The manipulator chooses X_1 in the swap market so as to maximize expected profits

$$\widehat{\pi}(X_0, X_1, Z, S) = X_0(r(S) - r^0) + X_1(r(S) - r^*(Z)) + S(r(S) - r^{L/D}(S)).$$

Taking expectations, we obtain

$$E[\widehat{\pi}(X_1,S)] = -X_0\rho S - X_1(\rho S + \widehat{\lambda}(X_1)) + S(r^0 - \rho S) - Sr^{L/D}(S),$$

where

$$\begin{aligned}\widehat{\lambda}(X_1) &= E[\lambda(X_1+Y)] \\ &= \frac{1}{\sqrt{2\pi}\sigma_Y}\int \lambda(X_1+Y)\exp(-\frac{Y^2}{2\sigma_Y^2})dY.\end{aligned}$$

Assuming first that either

$$X_0 \leq -\frac{r^L - r^0}{\rho},$$

we get the first-order condition

$$\widehat{\lambda}(X_1) + X_1\widehat{\lambda}'(X_1) - \frac{\rho}{2}X_1 = \frac{\rho}{2}(X_0 + \frac{r^{L/D} - r^0}{\rho}),$$

or, more concisely $X_0 = H^-(X_1)$, where

$$H^-(X_1) = -\frac{r^L - r^0}{\rho} + \frac{2}{\rho}\{\widehat{\lambda}(X_1) + X_1\widehat{\lambda}'(X_1)\} - X_1. \tag{11}$$

We start from the hypothesis that this equation gives an implicit expression for the optimal order X_1^- in the swap market for a non-linear pricing rule in the considered. The corresponding equation for the case is given by $X_0 = H^+(X_1)$, where

$$H^+(X_1) = \frac{r^0 - r^D}{\rho} + \frac{2}{\rho}\{\widehat{\lambda}(X_1) + X_1\widehat{\lambda}'(X_1)\} - X_1.$$

whose implicit solution will be denoted by X_1^+.¶

Proof of Proposition 4. To determine $f(.)$, denote by $F(.)$ the cumulative distribution function of X_1. For $X_1 < 0$, assuming strict monotonicity of X_1^-

with respect to X_0, we have

$$F(X_1) = \text{pr}\{X_1^- \leq X_1\}$$
$$= \text{pr}\{X_0 \leq H^-(X_1)\}$$
$$= \Phi(\frac{H^-(X_1)}{\sigma_0}).$$

A similar argument can be made for $X_1 > 0$. This proves the assertion.¶

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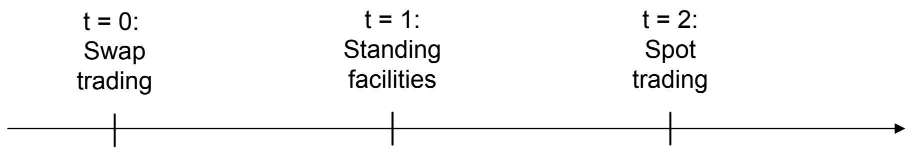

Figure 1. Time structure of the model.

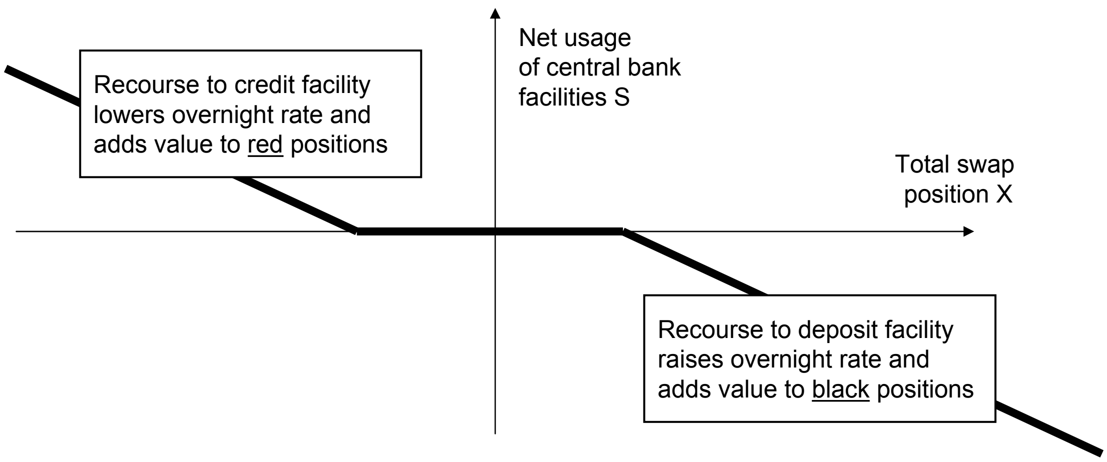

Figure 2. Net usage of standing facilities as a function of the manipulator's total swap position.

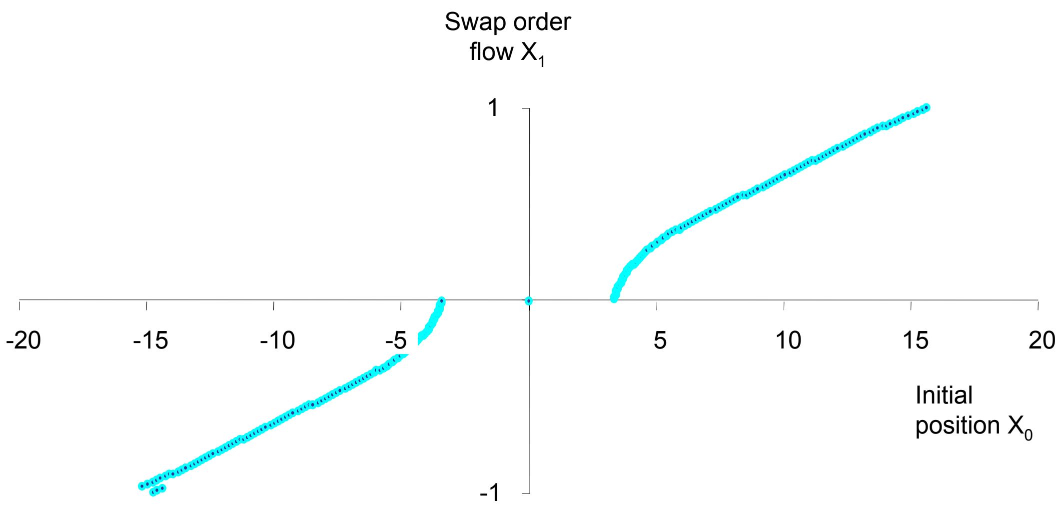

Figure 3. Equilibrium swap trading as a function of the manipulator's initial position. The curvature of the graph shows that small initial positions are leveraged at lower marginal costs.

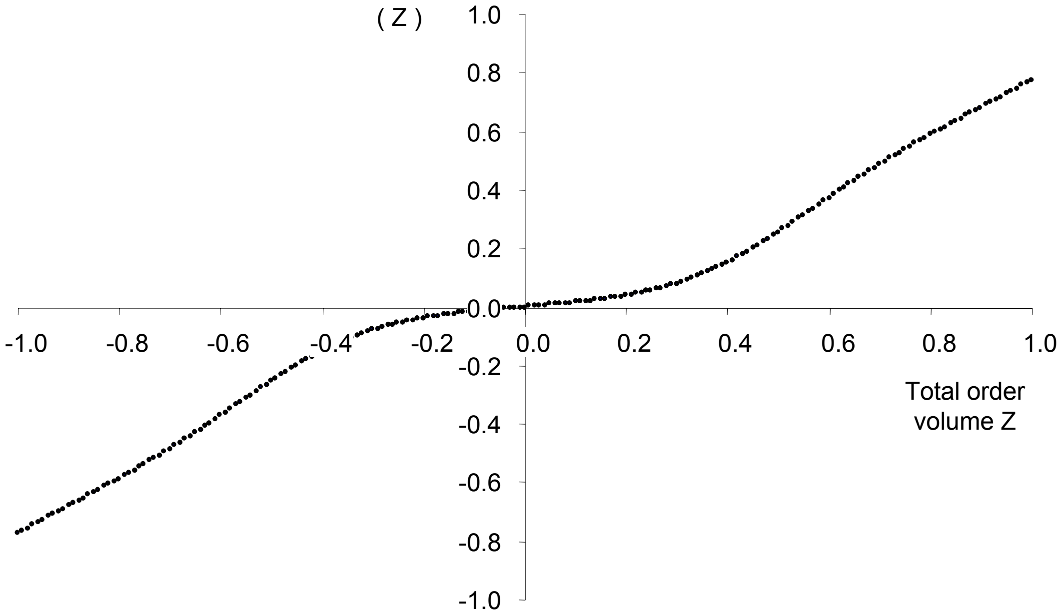

Figure 4. Equilibrium swap spread as a function of the total order volume. The price effect of demand in the swap market can be seen to be comparably low unless manipulation is apparent.

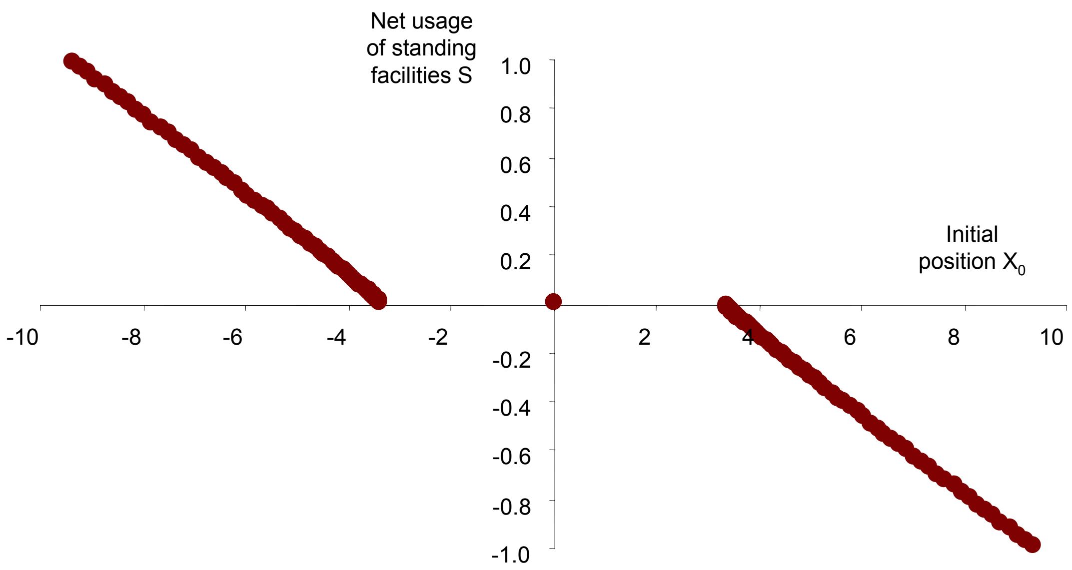

Figure 5. Net usage of standing facilities as a function of the initial position. Manipulation occurs with probability smaller than one.

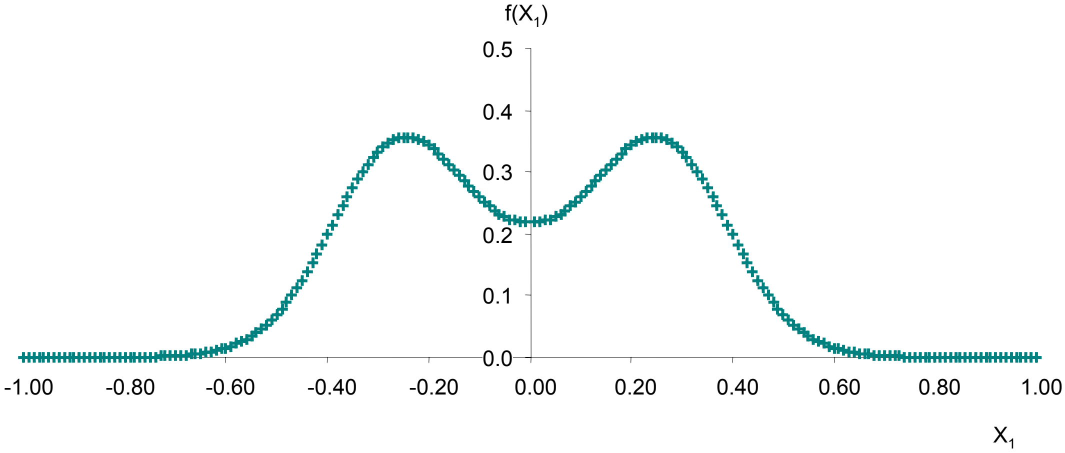

Figure 6. Non-normal ex-post density of the manipulator's swap order flow.
Note: the distribution possesses an atom at $X_1 = 0$.

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