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A CONTRIBUTION TO CALVO PRICE STAGGERING

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A CONTRIBUTION TO CALVO PRICE STAGGERING

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In 2007 all ECB publications feature a motif taken from the €20 banknote.

1 Correspondence: peter.mcadam@ecb.int and alpo.willman@ecb.int, Research Dept., European Central Bank, Kaiserstr. 29, D-60311, Frankfurt am Main, Germany. Tel: +.49.69.13.44.6434; Fax: +.49.69.13.44.6575. We thank Jagjit Chadha, Martin Ellison, Keith Kuester, Miguel Leon-Ledesma, Daniel Levy, Joe Pearlman, Frank Smets, one anonymous referee of the ECB Working Paper series as well as seminar audiences at the Sverige Riksbank and at Brunel, Kent, Kiel and Palermo for helpful comments. The views expressed here are not necessarily those of the ECB.

Address
Kaiserstrasse 29
60311 Frankfurt am Main, Germany

Postal address
Postfach 16 03 19
60066 Frankfurt am Main, Germany

Telephone
+49 69 1344 0

Internet
http://www.ecb.europa.eu

Fax
+49 69 1344 6000

Telex
411 144 ecb d

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Abstract

We implement a tractable state-dependent Calvo price-setting signal dependent on inflation and aggregate competitiveness. This allows us to derive a New Keynesian Phillips Curve (NKPC) expressed in terms of the actual levels of variables - rather than in-deviation from "steady state" form - and thus a specification which is not regime-dependent. A consequence of our approach is that ex-ante all firms face the same optimization problem. This state-dependent NKPC nests the conventional hybrid NKPC form as a special case. Finally, we demonstrate the usefulness of our approach by, first, analyzing the persistence and variability of inflation shocks under different inflation regimes and then comparing our state-dependent and time-dependent NKPCs on US data.

JEL Classification: E31, E32.

Keywords: Calvo Price Staggering, New Keynesian Phillips Curves, State-Dependency, Firm-Level Optimization, Regime Dependency.

Non-Technical Summary

The model of price staggering due to Calvo (1983) has become the canonical framework to model nominal rigidities under monopolistic competition. The framework assumes that in each period firms reset prices with a fixed, exogenous probability. This leads to an aggregate-supply relationship commonly referred to as the "New Keynesian Phillips Curve" (NKPC) which relates current inflation to expected inflation plus a measure of activity (typically real marginal costs). The empirical properties of this relationship have been widely examined and its importance underscored by its wide-spread adoption into theoretical and policy models.

Despite its evident popularity a key weakness of Calvo pricing relates to the "time-dependent" nature of its price-resetting signal. That firms change prices only if they receive a random signal with a constant probability is unsatisfactory since it assumes pricing behavior is independent of the state of the economy.

Related to this time-dependent issue is that the Calvo-NKPC framework, when explicitly linked with profit maximization, requires linearizations around a zero inflation steady state. This necessarily implies that the appropriateness of the resulting NKPC deteriorates as we depart from such a regime.

The contribution of our paper is the following. First, we implement a tractable state-dependent price-resetting signal which maps to inflation and aggregate competitiveness. In our framework, the Calvo probability parameter defines not the average price contract length but its upper limit; this may shed light on why empirical findings appear to over-estimate contract length.

Second, using this state-dependent Calvo signal, we derive an NKPC expressed in terms of the log-levels of variables - rather than in-deviation from "steady state" form - and, accordingly, a specification which is not (inflation or monetary) regime-dependent. This State-Dependent NKPC nests the standard NKPC as a special case.

Finally, a corollary of this is that before the signal is received, firms cannot know in which price-setting group they belong ex-post; hence all firms logically face the same ex-ante optimization problem. Unlike the conventional approach, therefore, we do not divide ex-ante firms into profit maximizers and rule-of-thumb price setters. Instead, we assume that the outcome of a time-varying signal defines in which of the three price setting categories (profit maximizers, rule-of-thumb or price-fixer) firms belong ex-post.

1. Introduction

The model of price staggering due to Calvo (1983) has become the canonical framework to model nominal rigidities under monopolistic competition. The framework assumes that in each period firms reset prices with a fixed, exogenous probability. This leads to an aggregate-supply relationship commonly referred to as the "New Keynesian Phillips curve" (NKPC) which relates current inflation to expected inflation plus a measure of activity (typically real marginal costs).

The empirical properties of this relationship have been widely examined (e.g., Roberts, 2005; McAdam and Willman, 2004; Rudd and Whelan, 2005; Welz, 2005) and its importance underscored by its wide-spread adoption into small theoretical models (e.g., Clarida, Galí and Gertler, 1999; Woodford, 2003) as well into larger, open-economy policy models - e.g., the IMF's Global Economy Model (Bayoumi, Laxton and Pesenti, 2004), the ECB's New Area Wide Model (Coenen, McAdam and Straub, 2007).[1]

Despite its evident popularity a key weakness of Calvo pricing relates to the "time-dependent" nature of its price-resetting signal. That firms change prices only if they receive a random signal with a constant probability is unsatisfactory since it assumes pricing behavior (e.g., price stickiness) is independent of the state of the economy. Under "state-dependent" pricing, by contrast, firms review their prices as soon as a shock occurs. Notwithstanding, endogenizing price re-setting has proved a challenging task. Indeed some authors (e.g., Woodford, 2003, p. 142), whilst acknowledging the apparent deficiencies of the traditional Calvo-NKPC assumptions, have questioned the value-added of state-dependent alternatives.

Related to this (time-dependent) issue is that the Calvo-NKPC framework, when explicitly linked with profit maximization (e.g., Galí and Gertler, 1999), requires linearization of the optimal reset and aggregate price around a counter-factual zero-inflation steady state. This suggests that the appropriateness of the resulting NKPC deteriorates as we depart from such a regime.[2]

Consider how both points square with data. Chart 1 shows the historical evolution of US inflation. We observe that the 1970s were periods of high and volatile inflation whereas the 1950s and 1990s were characterized by a more benign outlook. Although on closer inspection, even these periods are distinct since the 1950s were set apart by lower but more volatile inflation than in the 1990s. For certain, however, the zero average (or steady-state) inflation regime is counter factual. Likewise, the implication that price stickiness might be constant over such diffuse historical periods is debatable.

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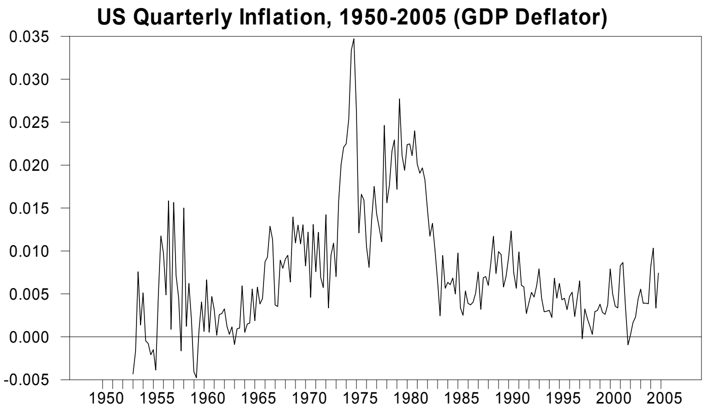
[1] See also Levine, McAdam and Pearlman (2007) for an application of the Calvo staggering framework to interest-rate setting.

[2] This contrasts with Rotemberg's (1987) approach where forward-looking firms set prices to minimize a quadratic loss function that depends on the difference between the reset price over the period it is expected to remain fixed and the optimal price. This leads to an inflation specification derived in terms of the actual levels of variables but leaves the link between the minimization of the loss function and the profit-maximization approach in general at least ambiguous.

Chart 1



Thus, three salient features of the data - non-zero and, occasionally, highly volatile inflation rates as well as the likelihood of time-varying price stickiness does not appear to tally with the Calvo-NKPC framework.

The contributions of our paper are the following:

First, we implement a tractable state-dependent Calvo signal which maps to inflation and market structure. Accordingly, the nature of our state dependency derives not from shocks to "menu costs" and the resulting cost-benefit reset decision of firms (e.g., Dotsey, King and Wolman, 1999; Golosov and Lucas, 2007), but instead directly from endogeneity in the Calvo signal.[3] Thus, we extend the current literature by demonstrating that a half-way house between the time-dependent and state-dependent approaches can be fashioned, grafting enough structure onto the traditional Calvo framework to replicate some features and advantages of state-dependency whilst retaining its overall simplicity and tractability. As a consequence the conventional Calvo probability parameter here defines not the average price contract length but its upper limit; this may shed light on why empirical findings appear to over-estimate contract length.

Second, compared to most studies, our framework does not rely on any linearization around a zero-inflation steady state. Ascari (2004) demonstrated (albeit in a time-dependent model) that analysis based on linearization around a zero inflation steady state may be misleading. Here, by using our state-dependent Calvo signal, we derive a Phillips curve expressed in terms of the log-levels of variables - rather than in deviation-from-"steady-state" form - and, accordingly, a specification which is not (inflation or monetary) regime dependent. This state-dependent form, moreover, nests the NKPC as a special case. This allows us to examine the importance of different monetary and inflation regimes in the inflationary process (e.g., inflation volatility and persistence[4]). In line with the "Lucas critique", we would expect price stickiness to be affected by

[3] For evaluations of the menu-cost sticky price models, e.g., Parkin (1986), McCallum (1994).

[4] "Inflation persistence" is taken to mean the tendency of inflation to converge gradually towards its long-run value following a shock. From a different angle, Senay and Sutherland (2005) also examine the case of the effects of monetary and exchange rate regimes on price setting.

the monetary or inflationary regime, a feature not captured by models with exogenous fixed price resetting costs.

Finally, in order to secure the NKPC into a common profit-maximization framework, there is no requirement in our framework to ex-ante divide firms into profit maximizers, non price changers and rule-of-thumb price setters. Instead, we assume that the outcome of the time-varying signal defines in which of the various price setting categories (profit maximizers, rule-of-thumb or price-fixer) firms belong ex-post. We demonstrate that this follows naturally from the state-dependent Calvo signal.

The paper proceeds as follows. Section 2 outlines our time-varying Calvo-NKPC price setting signal. Section 3 motivates our functional choice of state-dependent Calvo signaling. The subsequent section explains the optimization decision relevant to all firms, which leads to our reformulation of the NKPC in section 5. In Section 6, we use simulation and estimation evidence to compare state-dependent and non-state dependent Phillips curves. Finally, we conclude.

2 A Time-Varying Calvo-Price Setting Signal

In line with many NKPC studies, we start with the most general Calvo-NKPC framework, namely one which incorporates “intrinsic” inflation persistence via, for instance, “rule-of-thumb” price re-setters. This, however, is purely done for generality; our framework follows equally well assuming no such price setters. The novelty of our approach is that the reset signal itself is presumed to be state dependent, a corollary of which is that at the beginning of each period firms receive a time-varying signal regarding price setting in the following three-valued manner:

(1) With a (note: *time-varying*) probability θ_t firm *j* receives the signal indicating that the firm is not allowed to change its price, i.e. $P_t^j = P_{t-1}^j$.

(2) With a probability $(1-\theta_t)\omega$ firm *j* is allowed to change its price following a backward-looking pricing rule, as in Galí and Gertler (1999), $P_t^j = P_t^b = (P_{t-1}/P_{t-2})P_{t-1}^*$, where P_{t-1}^* is the average price level selected by firms able to change price at time *t-1*, and where $\omega \in [0,1]$ represents the fraction of firms able to reset prices but who do so in this rule-of-thumb manner.[5]

(3) With a probability $(1-\theta_t)(1-\omega)$ firm *j* receives the signal that allows it to reset its price on the profit-maximization level, $P_t^j = P_t^f$.

[5] Note, endogenizing ω, the fraction of rule-of-thumb price re-setting firms is beyond the scope of this paper.

As we shall see, the advantage of the three-valued signal is that in the beginning of each period, before the outcome of the signal is known, each firm faces exactly the same optimization problem, (i.e. in an ex-ante sense, all firms are profit-maximizers). By contrast, in the conventional approach there is a fixed, ex-ante classification of firms into profit-maximizers or non profit-maximizers and thus uncertainty concerns only whether the firm is or is not allowed to change its price. If allowed, the firm knows with ex-ante certainty whether it belongs to the rule-of-thumb or profit-maximizing group and hence ex-ante there are two behaviorally different groups of firms: profit maximizers and rule-of-thumb price setters.

It follows that the aggregate price level, p_t, (lower case denoting logs) can be defined as the weighted sum of the reset and lagged price:[6]

$$p_t \equiv (1-\theta_t)p_t^* + \theta_t p_{t-1} \tag{1}$$

where

$$p_t^* = (1-\omega)p_t^f + \omega p_t^b \tag{2}$$

Inserting (2) into (1), and subtracting p_{t-1} from both sides and rearranging, yields,

$$[\theta_t + (1-\theta_t)\omega]\pi_t = (1-\theta_t)\left[(1-\omega)p_t^f + \omega(p_{t-1}^* - p_{t-2})\right] \tag{3}$$

where $\pi_t = \Delta p_t$ denotes inflation. Furthermore, using (1) to solve for p_{t-1}^* and inserting into (3), we derive,

$$\left(\frac{\theta_t}{1-\theta_t} + \omega\right)\pi_t = \frac{\omega}{1-\theta_{t-1}}\pi_{t-1} + (1-\omega)\left(p_t^f - p_t\right) \tag{4}$$

To proceed, we require a derivation of the profit-maximizing price, P_t^f, (see section 4) as well as operationalizing of the time-varying Calvo signal, θ_t, to which we now turn.

3 Regime Dependency of the Signal

Although still exogenous to firms, we assume that the Calvo signal maps to the fundamentals of firms' overall price-setting environment, namely inflation and market structure. Specifically, we assume that in a high-inflation environment price changes are more frequent than otherwise.

[6] Appendix A shows the formal derivation.

Indeed, the ECB's 'Inflation Persistence Network' found that sectors with a higher inflation rate and higher inflation variability, typically exhibit more frequent price changes (e.g., Altissimo, Ehrmann and Smets, 2006, see also Cecchetti, 1986) than otherwise. Moreover, that price stickiness declines with higher inflation rates is a core prediction in menu-cost models, e.g., Kiley (2000). Similarly, and also consistent with the evidence, e.g., Levy et al. (1997), Dutta et al. (2002), price changes in highly-competitive markets are presumed to be more frequent than in less competitive ones (because the sales reactions to a deviation of price from the market average are the larger the more competitive are the markets).

In terms of the fixed-price relationship $\theta_t \sim g(\pi_t, \varepsilon)$, where $-\varepsilon \le -1$ is the price elasticity of demand, this would imply $g_\pi, g_\varepsilon < 0$. The following, simple functional form captures these ideas:

$$\theta_t = \theta \cdot \left(\frac{P_t}{P_{t-1}}\right)^{1-\varepsilon} \equiv \theta \cdot (1+\pi_t)^{1-\varepsilon} \qquad (5)$$

where $\theta \in [0, 1]$ and $\theta_t \in [0, \theta]$ $\forall \pi_t \ge 0$; as we shall demonstrate, functional form (5) is particularly convenient to preclude linearization of the optimal reset price.

Limiting characteristics of state-dependent Calvo signal (5) for the reset probability can be demonstrated for the polar cases of perfect competition and pure monopoly, respectively,

$$\lim_{\varepsilon \to \infty} (1-\theta_t) \to 1; \qquad \lim_{\varepsilon \to 1} (1-\theta_t) \to 1-\theta$$

as well as for hyper and zero inflation:

$$\lim_{\pi \to \infty} (1-\theta_t) \to 1; \qquad \lim_{\pi \to 0} (1-\theta_t) \to 1-\theta$$

Thus the higher is inflation and the more competitive is the economy, the more likely is a probability to reset prices, i.e., $1-\theta_t$ tends to 1.

With positive inflation, parameter θ in (5), note, sets the upper bound to the time-varying price-fixing probability, θ_t, which materializes either under zero inflation or pure monopoly. Indeed, a puzzling feature of estimated NKPCs is their apparent tendency to over-estimate price stickiness, given by $1/(1-\theta)$.[7] One aspect of this puzzle may be that in time-dependent models, firms change prices only on a periodic basis. Accordingly, time-dependent pricing rules might lead

[7] Smets and Wouters (2003), for example, estimate price durations in the euro area at around 2.5 years, which contrasts with comparable micro evidence of around 1 year (e.g., see the IPN summary paper of Altissimo, Ehrmann and Smets, 2006).

to stickier prices than state-dependent ones for a continuum of shocks. In addition, since this duration is typically estimated from non-zero inflation histories (recall Chart 1), some bias might be expected. Our framework, however, might shed light on this puzzle since θ here provides an *upper limit* for average contract duration which, as already stated, precisely materializes under zero inflation (i.e., the maintained hypothesis of the standard NKPC). By contrast, in our framework, price stickiness and duration are time-varying, and can be recursed from (5) for a given ε.

Table 1 elaborates upon the properties of Calvo signal (5). The first four rows demonstrate the dependency of the fixed-price probability and its average duration with respect to the price elasticity of demand. The next four rows express these two relationships corresponding to different θ values, and the last block expresses the corresponding dependency with respect to different quarterly inflation rates. Regarding the last relation, for instance, we see that with $\varepsilon = 10$ (a mark-up of 11%) and $\theta = 0.8$[8] the length of price-fixed duration is nearly halved (from 4.25 to 2.8 quarters) when moving from 2% to 10% annual inflation.

Table 1:
Detailed Illustrative Properties of the State-Dependent Calvo Price Signal

ε	θ	π	θ_t	Duration $= (1-\theta_t)^{-1}$
1.00	0.80	0.005	0.80	5.00
2.00	0.80	0.005	0.80	4.90
10.00	0.80	0.005	0.76	4.25
50.00	0.80	0.005	0.63	2.68
10.00	0.80	0.005	0.76	4.25
10.00	0.75	0.005	0.72	3.53
10.00	0.60	0.005	0.57	2.35
10.00	0.50	0.005	0.48	1.92
10.00	0.80	0.0125	0.72	3.51
10.00	0.80	0.0250	0.64	2.78
10.00	0.80	0.0500	0.52	2.06
10.00	0.80	0.2500	0.11	1.12

4 Factor Demands and the Profit-Maximizing Price Level

Assume that each firm solves its profit-maximization problem in the beginning of the period with full information on all current-period variables - except for the price-setting category to which it belongs ex-post. Regarding the Calvo signal itself, the prior probability distribution is known, i.e. $E_t\theta_t = \theta_t$ and, hence, the j^{th} firm's *expected* price level is,

[8] In the empirical literature, θ is typically estimated at around 0.8 to 0.9, implying a duration of between 5 and 10 quarters.

$$E_t p_t^j = \theta_t p_{t-1}^j + (1-\theta_t)\{(1-\omega)p_t^f + \omega p_{t-1}^*\} \tag{6}$$

Thus, although at the firm level, we find, $E_t p_t^j \neq p_t^j$, $E_t p_t = p_t$ continues to hold at the aggregate level (this can be inferred from taking expectations of (1) and (2)).

Now the profit-maximization problem is identical for each firm independently of the ex-post outcome of the Calvo signal. In general form, therefore, each j^{th} firm maximizes,

$$Max\, E_t \sum_{i=0}^{\infty} R_{t,t+i} \left\{ \frac{P_{t+i}^j}{P_{t+i}} Y_{t+i}^j - \frac{W_{t+i}}{P_{t+i}} N_{t+i}^j - \frac{Q_{t+i}}{P_{t+i}} K_{t+i}^j \right\} \tag{7}$$

subject to its production technology and monopolistic demand curve, respectively:

$$Y_t^j = F\left(K_t^j, N_t^j\right) \tag{7a}$$

$$Y_t^j = Y_t \left(\frac{P_t^j}{P_t} \right)^{-\varepsilon} = P_t \underbrace{Y_t P_t^{\varepsilon-1} \left(P_t^j\right)^{-\varepsilon}}_{Z_t\left(P_t^j\right)} \equiv P_t Z_t\left(P_t^j\right) \tag{7b}$$

where W and Q denote nominal wages and the user-cost-of-capital respectively, Z is a convenient factoring term, F represents some generalized production technology (e.g. Klump, McAdam and Willman, 2007), and $R_{t,t+i} \equiv (1+r_t) E_t \prod_{j=0}^{i} \left(1+r_{t+j}\right)^{-1}$ with r_t being the riskless real interest rate.

After applying expectation rule (6), we can separate $P^j Y^j$ and Y^j conditional on $P_{t+i}^j = P_t^f$, on one hand, and conditional on all other possible prices, i.e. $P_{t+i}^j \neq P_t^f$, on the other:

$$E_t \frac{P_{t+i}^j Y_{t+i}^j}{P_{t+i}} = (1-\theta_t)(1-\omega)\Theta_{t,t+i} P_t^f E_t Z_{t+i}\left(P_{t+i}^j \middle| P_{t+i}^j = P_t^f\right) + E_t \Omega_{t+i}\left[P_{t+i}^j Z_{t+i}\left(P_{t+i}^j \middle| P_{t+i}^j \neq P_t^f\right)\right] \tag{8}$$

$$E_t Y_{t+i}^j = (1-\theta_t)(1-\omega)\Theta_{t,t+i} E_t \left\{P_{t+i} Z_{t+i}\left(P_{t+i}^j \middle| P_{t+i}^j = P_t^f\right)\right\} - E_t \left\{ \frac{P_{t+i}}{P_{t+i}^j} \Omega_{t+i}\left[P_{t+i}^j Z_{t+i}\left(P_{t+i}^j \middle| P_{t+i}^j \neq P_t^f\right)\right]\right\} \tag{9}$$

where $\Theta_{t,t+i} = \frac{1}{\theta_t} \prod_{j=0}^{i} E_t \theta_{t+j} = \theta^i E_t \left(\frac{P_{t+i}}{P_t}\right)^{1-\varepsilon}$ and Ω_{t+i} is the probability-weighted sales corresponding to all possible sales price $P_{t+i}^j \neq P_t^f$ deflated by the aggregate price level.

The profit-maximization problem of (7) can now be presented as,

$$\underset{P_t^f, N_t, \dots N_{t+i}^j}{Max} E_t \sum_{i=0}^{\infty} R_{t,t+i} \left\{ \begin{array}{l} (1-\theta_t)(1-\omega)\Theta_{t,t+i} P_t^f Z_{t+i}\left(P_{t+i}^j \middle| P_{t+i}^j = P_t^f\right) + \Omega_{t+i}\left[P_{t+i}^j Z_{t+i}\left(P_{t+i}^j \middle| P_{t+i}^j \neq P_t^f\right)\right] - \\ \frac{W_{t+i}}{P_{t+i}} N_{t+i}^j - \frac{Q_{t+i}}{P_{t+i}} K_{t+i}^j + \\ \lambda_{t+i} \left\{ \begin{array}{l} F\left(K_t^j, N_t^j\right) - (1-\theta_t)(1-\omega)\Theta_{t,t+i} P_{t+i} Z_{t+i}\left(P_{t+i}^j \middle| P_{t+i}^j = P_t^f\right) - \\ \frac{P_{t+i}}{P_{t+i}^j} \Omega_{t+i}\left[P_{t+i} Z_{t+i}\left(P_{t+i}^j \middle| P_{t+i}^j \neq P_t^f\right)\right] \end{array} \right\} \end{array} \right\} \quad (10)$$

where the terms containing $\Omega_{t+i}(\cdot)$ are independent from maximized variables, i.e., they could equally well be deleted from the maximization problem. The first-order conditions with respect to P_t^f and N_{t+i}^j yield, respectively,

$$(1-\varepsilon)E_t \sum_{i=0}^{\infty} R_{t,t+i}\Theta_{t,t+i} Z_{t+i}\left(P_{t+i}^j \middle| P_{t+i}^j = P_t^f\right) + \varepsilon E_t \sum_{i=0}^{\infty} R_{t,t+i}\Theta_{t,t+i} \frac{\lambda_{t+i}}{P_t^f} Z_{t+i}\left(P_{t+i}^j \middle| P_{t+i}^j = P_t^f\right) = 0 \quad (11)$$

$$R_{t,t+i}\left[-\frac{W_{t+i}}{P_{t+i}} + \lambda_{t+i} F_N\left(N_{t+i}^j\right)\right] = 0 \quad (12)$$

where F_N represents the marginal product of labor.

The demand function defined by (7b) implies that $Z_{t+i}\left(P_{t+i}^j \middle| P_{t+i}^j = P_t^f\right) = \left\{ \frac{Y_{t+i}}{Y_t}\left(\frac{P_{t+i}}{P_t}\right)^{\varepsilon-1} \right\} \cdot Z_t\left(P_t^j \middle| P_t^j = P_t^f\right)$. Using this relation and the definition $\Theta_{t,t+i} = \theta^i E_t\left(\frac{P_{t+i}}{P_t}\right)^{1-\varepsilon}$, these conditions can be transformed into:

$$P_t^f = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{i=0}^{\infty} \theta^i E_t R_{t,t+i} \frac{Y_{t+i}}{Y_t} P_{t+i} \lambda_{t+i}}{\sum_{i=0}^{\infty} \theta^i E_t R_{t,t+i} \frac{Y_{t+i}}{Y_t}} \quad (11')$$

$$\lambda_{t+i} = \frac{W_{t+i}}{P_{t+i} F_N\left(N_{t+i}^j\right)} \equiv MC_{t+i} \quad (12')$$

where MC is the real marginal cost of labor. Thus, from (11') we see that the optimal price has now been reduced to depend on θ instead of the time-varying θ_t.

Furthermore, by assuming that households have access to a complete set of contingent claims, and that identical consumers maximize their intertemporal utility, $\sum_i \beta^i U(C_{t+i})$, we have for the discount rate $R_{t,t+i}$:

$$\frac{1}{R_{t,t+i}}\beta^i E_t\left[\frac{U_c(C_{t+i})}{U_c(C_t)}\right]=1 \tag{13}$$

where C denotes consumption, and β is the discount factor. For the standard case of logarithmic utility and under the assumption that market growth equals consumption growth, i.e. $\frac{Y_{t+i}}{Y_t}=\frac{C_{t+i}}{C_t}$, equations (11')-(13) imply,

$$P_t^f=\frac{\varepsilon}{\varepsilon-1}\frac{\sum_{i=0}^{\infty}(\beta\theta)^i P_{t+i}MC_{t+i}}{\sum_{i=0}^{\infty}(\beta\theta)^i}=(1+\mu)(1-\beta\theta)\sum_{i=0}^{\infty}(\beta\theta)^i E_t MCN_{t+i} \tag{14}$$

where $\mu=\frac{1}{\varepsilon-1}$ and MCN are the mark-up and nominal marginal cost of labor, respectively. The logarithmic approximation of (14) can be written as,[9]

$$p_t^f=(1-\beta\theta)\sum_{i=0}^{\infty}(\beta\theta)^i E_t(mcn_{t+i}+\mu) \tag{14'}$$

Equation (14') resembles a conventional profit-maximizing, price-setting rule, but it is worth noting that it has been derived to hold for the log levels of the left- and right-hand side variables, i.e. not only for the corresponding log differences of the variables from their presumed steady state values. In fact (14') corresponds to the specification implied by Rotemberg's (1987) framework but, unlike that paper, we have explicitly derived it here in the profit-maximization environment. Furthermore, the derivation of (14') crucially requires that the reset probability is state-dependent and of the precise form as defined in (5).

[9] Appendix B shows the relevant transformation.

5 The State-Dependent New Keynesian Phillips Curve

To derive our State-Dependent New Keynesian Phillips Curve (SDPC) we start by inserting equation (14') into (4) to obtain:

$$\left(\frac{\theta_t}{1-\theta_t}+\omega\right)\pi_t=\frac{\omega}{1-\theta_{t-1}}\pi_{t-1}+(1-\omega)\left((1-\beta\theta)\left(\sum_{i=0}^{\infty}(\beta\theta)^i E_t(mcn_{t+i}+\mu)\right)-p_t\right) \tag{15}$$

We next shift (15) forward by one period and take expectations at the beginning of period t.

$$E_t\left[\left(\frac{\theta_{t+1}}{1-\theta_{t+1}}+\omega\right)\pi_{t+1}\right]=\frac{\omega}{1-\theta_t}\pi_t+(1-\omega)\left((1-\beta\theta)\left(\sum_{i=0}^{\infty}(\beta\theta)^i E_t(mcn_{t+1+i}+\mu)\right)-E_t p_{t+1}\right) \tag{16}$$

Now, multiplying both sides of (16) by $\beta\theta$ and subtracting it from (15) we end up, after some manipulations, with the closed-form SDPC:

$$(\theta_t+\omega(1-\theta_t)+\omega\beta\theta)\pi_t=\omega\frac{(1-\theta_t)}{(1-\theta_{t-1})}\pi_{t-1}+(1-\theta_t)\beta\theta E_t\left(\frac{\pi_{t+1}}{1-\theta_{t+1}}\right)+(1-\theta_t)(1-\omega)(1-\beta\theta)(mcn_t+\mu-p_t) \tag{17}$$

If there are no backward-looking price setters, (17) reduces to

$$\theta_t\pi_t=(1-\theta_t)\beta\theta E_t\left(\frac{\pi_{t+1}}{1-\theta_{t+1}}\right)+(1-\theta_t)(1-\beta\theta)(mcn_t+\mu-p_t) \tag{17'}$$

Except for the state dependency of signal probabilities ($\theta_t, \theta_{t-1}, \theta_{t+1}$), equation (17) corresponds to the pure and hybrid NKPC and reduces exactly to them, when $\theta_{t+1}=\theta_t=\theta_{t-1}=\theta$, $\omega=0$ and $\theta_{t+1}=\theta_t=\theta_{t-1}=\theta$ respectively.

The state dependent nature of the signal also implies that our Phillips curve is nonlinear with respect to inflation, unlike the conventional case. A more problematic feature, however, is that (17) contains the complicated expectation of nonlinear function of next period inflation. Therefore, because on the basis of Jensen's inequality $E_t\left(\frac{\pi_{t+1}}{1+\theta(1+\pi_{t+1})^{1-\varepsilon}}\right)\neq\frac{E_t\pi_{t+1}}{1+\theta(1+E_t\pi_{t+1})}$, we operationalize this particular nonlinear expectational term using an expansion around some generic benchmark inflation rate $\tilde{\pi}_t$:

$$E_t\left(\frac{\pi_{t+1}}{1-\theta_{t+1}}\right)\approx\frac{1}{1-\tilde{\theta}_t}\left[E_t\pi_{t+1}+\left(\frac{(1-\varepsilon)\tilde{\theta}_t}{(1+\tilde{\pi}_t)(1-\tilde{\theta}_t)}\right)(E_t\pi_{t+1}-\tilde{\pi}_t)\tilde{\pi}_t\right]$$

where $\widetilde{\theta}_t = \theta(1+\widetilde{\pi}_t)^{1-\varepsilon}$. We see that this approximation allows us to present next-periods non-linear expectations as a *linear* function of next-period inflation. Regarding $\widetilde{\pi}_t$ natural choices might be current inflation, a moving average of past inflation or a (constant) average inflation (i.e., trend inflation). The more price stickiness present in the economy, the more natural it may be to choose current inflation. Conversely, if inflation were strongly mean reverting, then some measure of trend (i.e., sample-average) inflation (or the central bank's inflation target) might be preferred. If current inflation is chosen as the benchmark inflation, then (17) becomes:

$$\pi_t = \gamma_t^b \pi_{t-1} + \gamma_t^f E_t \pi_{t+1} + \lambda_t (mcn_t + \mu - p_t) - \xi_t (E_t \pi_{t+1} - \pi_t) \pi_t \qquad (18)$$

where,

$$\gamma_t^b = \omega \frac{(1-\theta_t)}{(1-\theta_{t-1})} \phi_t^{-1},\ \gamma_t^f = \beta\theta\phi_t^{-1},$$

$$\lambda_t = (1-\omega)(1-\theta_t)(1-\beta\theta)\phi_t^{-1},\ \xi_t = \frac{\theta_t}{(1+\pi_t)(1-\theta_t)\mu} \beta\theta\phi_t^{-1},$$

$$\phi_t = \theta_t + \omega(1-\theta_t) + \omega\beta\theta$$

and $\gamma_t^f, \gamma_t^b, \lambda_t > 0$ and $\xi_t < 0$. A simpler alternative, where all nonlinear terms of (17) are replaced by their linear Taylor approximations around the constant or "trend" inflation rate, is derived in Appendix B.

Although (18) notionally resembles the NKPC, it differs in three key respects:

First, all "coefficients" are time-varying, resulting from the dependency of θ_t on inflation. Furthermore, it can be shown that they are increasing in inflation $\gamma_\pi^b, \gamma_\pi^f, \lambda_\pi > 0$ except for that capturing inflation volatility, $\xi_\pi < 0$. Our formulation therefore turns out therefore to be consistent, for instance, with Ball, Mankiw and Romer's (1988) and Benati's (2007) finding of a positive correlation between the slope of the Phillips curve and inflation (or trend inflation).

Second, our specification differs in being derived directly from the log-levels of the underlying variables, without relying on deviations from a zero (or non-zero) inflation steady state; the standard NKPC is thus valid locally only around a zero-inflation regime, or, at best, a low and stable one. A natural consequence of our framework is that question of monetary and inflation regime dependency can be meaningfully addressed, see Section 6.

Finally, looking at the final right-hand term in (18), the equation contains second-order inflation terms, arising from the treatment of nonlinear expectations. The importance of this is strengthened the higher is inflation and the more volatile it is. However, for steady inflation, i.e., $\pi_{t+1} = \pi_t$, this term disappears independently from the value of that steady rate. Also, to repeat, the time-varying coefficients asymptotically approach those of the NKPC, when inflation converges to zero, i.e. the linearization point of the conventional equation.[10] Likewise the second-order inflation term vanishes when $\pi_t \to 0$. Hence, equation (18) indicates that in an inflation regime at or close to zero, at least, when coupled with relatively small inflation variability, the standard NKPC may be a reasonable approximation to (18), but not otherwise.

6 Quantitative Investigations

In the following, we examine the simulation properties of our state-dependent form benchmarked against a pure and hybrid NKPC (section 6.1) as well as making some historical estimation comparisons (section 6.2).

6.1 The Effects of State Dependency: A Simulation Exercise

Now, we compare our maintained model (equations 19-23) with that of the NKPC (equations 21-24).[11] To simplify our comparison and capture just those dynamics associated with the effect of state-dependence, we assume that on average nominal marginal costs (equation 21) grow in line with equilibrium inflation. For both model types, we assume the cost-push shock, ν_t^{mcn}, follows a first-order autoregressive process with an iid-normal error term, η_t (21, 22):[12]

$$\theta_t = \theta \cdot (1+\pi_t)^{1-\varepsilon} \tag{19}$$

$$\pi_t = \gamma_t^b \pi_{t-1} + \gamma_t^f E_t\pi_{t+1} + \lambda_t(mcn_t + \mu - p_t) - \xi_t(E_t\pi_{t+1} - \pi_t)\pi_t \tag{20}$$

$$\Delta mcn_t = \log[1+\bar{\pi}] + \nu_t^{mcn} \tag{21}$$

$$\nu_t^{mcn} = 0.95 \cdot \nu_{t-1}^{mcn} + \eta_t \tag{22}$$

$$\pi_t = \Delta p_t \tag{23}$$

$$\pi_t = \gamma^b \pi_{t-1} + \gamma^f E_t\pi_{t+1} + \lambda(mcn_t + \mu - p_t) \tag{24}$$

where $\bar{\pi}$ represents the equilibrium inflation rate (i.e., capturing the underlying monetary and inflation regime) and the composite parameters in equation (24) are, as in Galí and Gertler (1999): $\gamma^b = \omega\phi^{-1}$, $\gamma^f = \beta\theta\phi^{-1}$; $\lambda = (1-\omega)(1-\theta)(1-\beta\theta)\phi^{-1}$, $\phi = \theta + \omega[1-\theta(1-\beta)]$.

Charts 2 provide the comparison of the effects of an unanticipated unit impulse in η_t, whose effect is further transmitted by the AR(1) cost-push process. In each case, we show the resulting dynamics from model type (19-23) predicated on different annual inflation regimes,

[10] Although, note that $\xi_t \neq 0 \ \forall \pi_t$.

[11] Both models are simulated using TROLL's sparse-matrix Newton non-linear rational expectations algorithm Newstack, see Juillard, Laxton, McAdam and Pioro (1998). The code is available on request.

[12] Assuming Cobb-Douglas production technology, the mark-up over real marginal costs corresponds to the log of the ratio of the labor income share to its sample average.

$\bar{\pi} \in [0, 0.02, 0.10]$, as well as for the standard NKPC (21-24) with $\bar{\pi} = 0$. The shocks are performed against a 0.0, 0.3 and 0.7 share of ex-post rule-of-thumb price setters.

A clear pattern emerges: the higher is the underlying inflation regime, the higher is the initial jump in inflation and the faster is the reversion to baseline. Interestingly, this is true even in the case where there is no intrinsic persistence ($\omega = 0$). Thus even in the pure NKPC – where inflation persistence is purely caused by that of real marginal costs – SDPCs introduce an additional channel for persistence which derives from the state dependency of the parameters.

Need less to say, the NKPC and the SDPC ($\bar{\pi} = 0$) model behave similarly since for small shocks $\xi_t (E_t \pi_{t+1} - \pi_t) \pi_t \approx 0$ and $\gamma_t^b \approx \gamma^b$ etc. Thus for a zero-inflation regime, the standard NKPC performs satisfactorily (in comparison to our maintained model). In the NKPC, price-fixed duration is a constant, a value on which the SDPC($\bar{\pi} = 0$) naturally converges. However comparing these two cases against those with $\bar{\pi} > 0$ suggests that the conventional NKPC underestimates the volatility in inflation following a shock but over estimates inflation persistence.

The policy implications are striking. If policy makers assume that inflation evolves according to the time-dependent NKPC (with our without intrinsic persistence), they risk seriously mis-understanding inflationary dynamics: misjudging the sensitivity of the initial inflationary reaction to a shock with respect to inflation regime, and the difficulty or ease with which the inflationary may thereafter be reduced. In the case, for instance, where there is a strong inertial component to inflation ($\omega = 0.7$), whilst the reversion of inflation to baseline following a shock will be relatively fast it will also be highly volatile (and complex) requiring a very different policy response to cases where there is no such inertia.

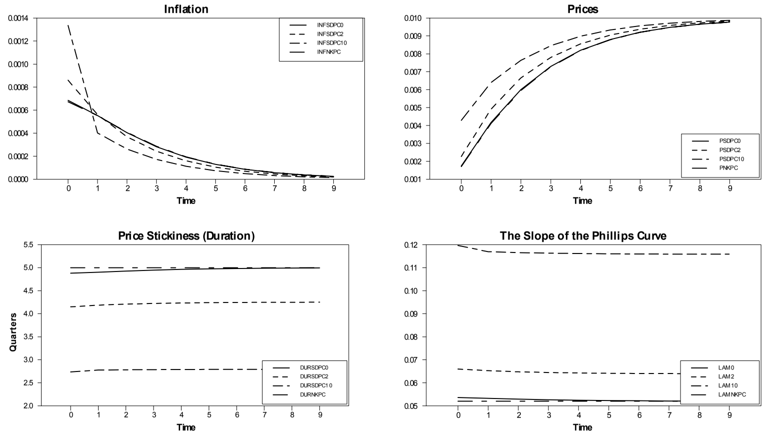
6.2 Estimation of State-Dependent NKPC

Table 2 compares empirical estimates of equation (18) benchmarked against the standard Hybrid NKPC (24). The estimation period covers 1954q1 to 2004q4, with the GDP deflator and labor income share measured in terms of non-housing business sector. The instrument set for the regressions were one and 3-period lags of the log of output from its quadratic trend, 1-period lags of the labor income share, the growth rates of crude oil price and the price of the energy component of CPI, 2-period lags of inflation, the interest rate spread (defined as the difference of 5-year and 3-month Treasury bond yields) and hourly compensation growth. Results are shown for constrained and unconstrained discounting as well as plausible aggregate elasticity ranges (a markup from 20% to 5%).

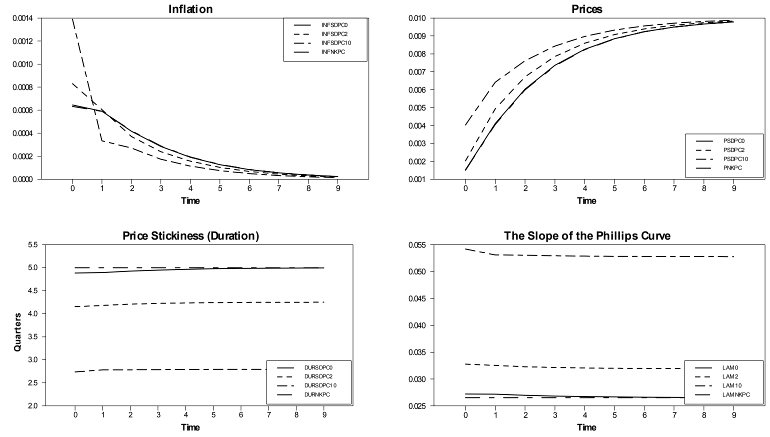
Results confirm that our SDPC can be taken successfully to the data. All parameters are significant at the 1%, and we find reasonable discount-factor estimates when freely estimated and (at just over a year in the time-dependent case) plausible price-stickiness values. We also, at least for the freely-estimated discount factor case, find that as the elasticity increases (markup decreases), price stickiness falls. This concurs with our motivation for our SD Calvo signal (Table 1): the more competitive is the economy, the more rapidly are prices changed. Another striking feature is that our ω estimates tend to be high (mostly above 0.5) relative to the standard NKPC (e.g., the first two columns). This therefore puts our works closer to other studies that suggest inflation is strongly backward looking, e.g., Fuhrer (1997), Rudebusch (2002), Rudd and Whelan (2005). Finally, **Chart 3** shows the implied price duration of the SDPC (for the standard $\varepsilon = 11$ case) graphed against actual inflation. According to the Chart, since the mid 1950s until early 1970s price duration varied in the range of around 5-6 quarters. Thereafter, reflecting accelerated inflation the duration shortened to around 3.5-4.5 quarters and, since early 1980s, it lengthened back to around 5-6 quarter range.

Charts 2: Model Responses to Cost-Push Shock

The Effects of State Dependency (Omega=0)



The Effects of State Dependency (Omega=0.3)



The Effects of State Dependency (Omega=0.7)

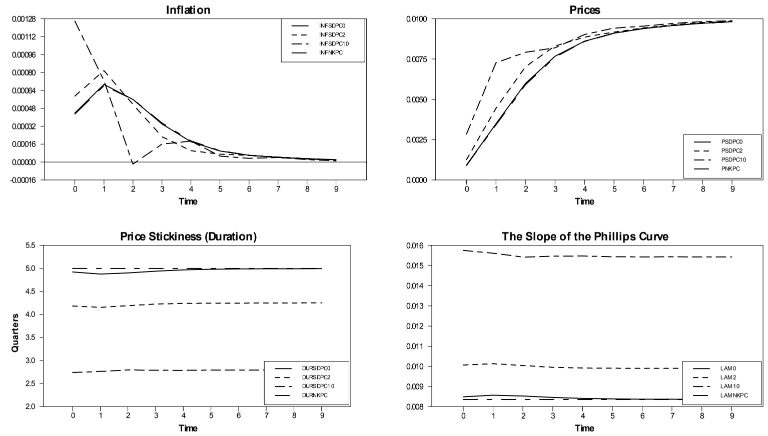


Table 2:

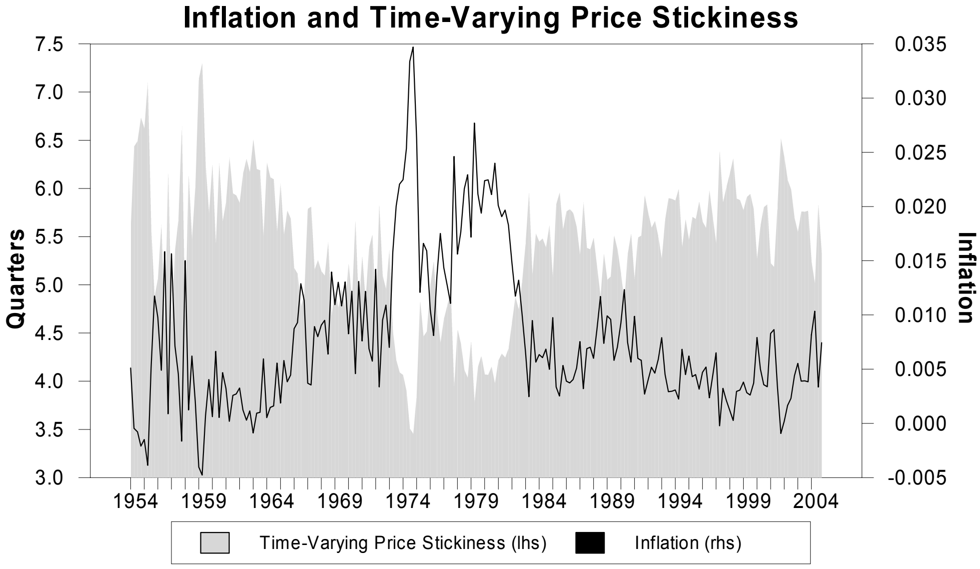
Estimates of Time-Dependent and State-Dependent New Keynesian Phillips Curves

Parameter	Time-Dependent Case [1]		State-Dependent Case [2]							
θ	0.843 (0.042)	0.833 (0.041)	0.843[3] (0.052)	0.888 (0.054)	0.893 (0.008)	0.863 (0.010)	0.926 (0.021)	0.905 (0.026)	0.881 (0.024)	0.852 (0.023)
β	0.99 (–)	1.000 (0.026)	0.99 (–)				0.906 (0.010)	0.877 (0.033)	0.880 (0.051)	0.898 (0.065)
ω	0.386 (0.135)	0.369 (0.137)	0.715 (0.058)	0.849 (0.048)	0.930 (0.022)	0.940 (0.019)	0.407 (0.080)	0.751 (0.070)	0.852 (0.049)	0.894 (0.033)
J-test	[0.930]	[0.902]	[0.843]	[0.805]	[0.594]	[0.551]	[0.997]	[0.989]	[0.973]	[0.955]
Duration (quarters)	6.371	5.970	–	–	–	–	–	–	–	–
ε	–	–	6.00 (–)	11.00 (–)	16.00 (–)	21.00 (–)	6.00 (–)	11.00 (–)	16.00 (–)	21.00 (–)

Notes:
[1] Corresponds to equation (24), the standard hybrid NKPC
[2] Corresponds to equation (18).
[3] Case used in Chart 3.

Standard errors in parenthesis, p-values in squared brackets.

Chart 3



Note: The Figure corresponds to third column results in Table 2

7 Conclusions

In this paper, we extended the Calvo price staggering framework to allow for state-dependency whereby the reset probability maps to inflation and market structure. In our framework, the normal Calvo probability defines not the average price contract length but its upper limit; this may shed light on the typical empirical over-estimation of contract length. Our approach augments the NKPC by transforming its otherwise constant coefficients into time-varying, state-dependent ones and adds a second-order term in inflation (capturing inflation volatility), when non-linear expectations are treated. The resulting model is quite general, with the standard NKPC arising as a limiting case.

Moreover our time-varying signaling mechanism implies that before the signal is received, firms cannot know in which price-setting group they belong ex-post; hence all firms logically face the same ex-ante optimization problem. Thus, we provide a more traditional firm-based treatment of the Calvo framework since our optimization framework applies uniformly to all firms, independently of their ex-post price setting.

Our resulting state-dependent form thus departs from the NKPC in a number of attractive ways. Our form holds for the log levels of variables, i.e. not only for the corresponding log differences of the variables from their presumed “steady state”. Accordingly, issues of regime dependence can be addressed. Our simulation exercises suggested that the economy’s response to a shock to inflation fundamentals is markedly regime dependent. In particular, the standard NKPC tends to under-estimate inflation volatility but over-estimate inflation persistence; only around a zero inflation regime, is the NKPC a sufficient approximation to inflation dynamics in the context of our maintained model.

A number of future directions are suggested by this work. One extension might be to examine the (numerical) characteristics of optimal monetary policy under our form of state-dependency. Similarly, an important consideration would be to examine the output-inflation trade off under different inflation regimes. Finally, since dividing agents into informationally-differentiated groups is a common theoretical construct (e.g., Ricardian, non-Ricardian consumers), our work could stimulate research into how their shares could be endogenized and how, in turn, this changes agents' ex-ante decision strategies. We leave these open for future work.

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Appendix A: Derivation of the Aggregate Price Level

Using the definition of the aggregate price and the fact that all re-setting firms choose the same optimal price, P_t^f :

$$P_t = \left[\int_0^1 P_t^{j\,1-\varepsilon} dj\right]^{\frac{1}{1-\varepsilon}} = \left[\int^{\theta_t} P_{t-1}^{1-\varepsilon} dj + \int^{(1-\theta_t)(1-\omega)} P_t^{f\,1-\varepsilon} dj + \int^{(1-\theta_t)\omega} P_t^{b\,1-\varepsilon} dj\right]^{\frac{1}{1-\varepsilon}} = \left[\theta_t P_{t-1}^{1-\varepsilon} + (1-\theta_t)(1-\omega)P_t^{f\,1-\varepsilon} + (1-\theta_t)\omega P_t^{b\,1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

Divide through by P_t and re-arrange:

$$1 = \left[\theta_t\left(\frac{P_{t-1}}{P_t}\right)^{1-\varepsilon} + (1-\theta_t)(1-\omega)\left(\frac{P_t^f}{P_t}\right)^{1-\varepsilon} + (1-\theta_t)\omega\left(\frac{P_t^b}{P_t}\right)^{1-\varepsilon}\right].$$

Denote these relative prices as $X_t^1 = \frac{P_{t-1}}{P_t}, X_t^2 = \frac{P_t^f}{P_t}, X_t^3 = \frac{P_t^b}{P_t}$. In the steady state all relative price ratios are necessarily constant and can be normalized to unity. Accordingly, and taking a log expansion:

$$0 = \left[(1-\varepsilon)\theta_t\left(X_t^1 - 1\right) + (1-\varepsilon)(1-\theta_t)(1-\omega)\left(X_t^2 - 1\right) + (1-\varepsilon)(1-\theta_t)\omega\left(X_t^3 - 1\right)\right]$$

$$\Rightarrow 0 = \left[\theta_t\left(\frac{P_{t-1} - P_t}{P_t}\right) + (1-\theta_t)(1-\omega)\left(\frac{P_t^f - P_t}{P_t}\right) + (1-\theta_t)\omega\left(\frac{P_t^b - P_t}{P_t}\right)\right]$$

$$\Rightarrow \theta_t \log\left(\frac{P_{t-1}}{P_t}\right) + (1-\theta_t)(1-\omega)\log\left(\frac{P_t^f}{P_t}\right) + (1-\theta_t)\omega\left(\frac{P_t^b}{P_t}\right) \Leftrightarrow \log P_t = \theta_t \log(P_{t-1}) + (1-\theta_t)(1-\omega)\log\left(P_t^f\right) + (1-\theta_t)\omega\left(P_t^b\right) \blacksquare$$

Appendix B: Derivation of (14')

Multiply both sides of equation (14) by $\beta\theta$ and lead

$$\beta\theta P_t^f = \beta\theta(1+\mu)(1-\beta\theta)\sum_{i=0}^{\infty}(\beta\theta)^i E_t MCN_{t+1+i}.$$

Subtract this expression from (14):

$$P_t^f - \beta\theta P_{t1}^f = (1+\mu)(1-\beta\theta)MCN_t \Rightarrow P_t^f - (1+\mu)MCN_t = \beta\theta\left(E_t P_{t+1}^f - (1+\mu)MCN_t\right)$$

$$\Rightarrow \frac{P_t^f - (1+\mu)MCN_t}{(1+\mu)MCN_t} = \frac{\beta\theta\left(E_t P_{t+1}^f - (1+\mu)MCN_t\right)}{(1+\mu)MCN_t}.$$

Noting that, $\frac{P_t^f - (1+\mu)MCN_t}{(1+\mu)MCN_t} \approx \log\left[\frac{P_t^f}{(1+\mu)MCN_t}\right]$ and $\frac{P_{t+1}^f - (1+\mu)MCN_t}{(1+\mu)MCN_t} \approx \log\left[\frac{P_{t+1}^f}{(1+\mu)MCN_t}\right]$. This implies,

$$\log P_t^f = \beta\theta \log P_{t+1}^f + (1-\beta\theta)\log MCN_t + (1-\beta\theta)\log(1+\mu) = (1-\beta\theta)\sum_i(\beta\theta)^i(\log MCN_{t+i} + \log(1+\mu)) \blacksquare$$

Appendix C: Specification of the State-Dependent NKPC around a Given Inflation Rate

Let us expand the time-varying coefficients of (15) around a given inflation rate, $\bar{\pi}$ (where $\bar{\pi}$ may denote trend inflation (i.e., the sample average or the central bank's inflation target):

$$\left(\frac{\theta_t}{1-\theta_t}\right) \approx \frac{\bar{\theta}}{1-\bar{\theta}} + \left(\frac{\bar{\theta}(1-\varepsilon)}{(1+\bar{\pi})(1-\bar{\theta})^2}\right)\{\pi_t - \bar{\pi}\}$$

$$\left(\frac{\omega}{1-\theta_{t-1}}\right) \approx \frac{\omega}{1-\bar{\theta}} + \frac{\omega\bar{\theta}(1-\varepsilon)}{(1+\bar{\pi})(1-\bar{\theta})^2}\{\pi_{t-1} - \bar{\pi}\}$$

where $\bar{\theta} = \theta\cdot(1+\bar{\pi})^{1-\varepsilon}$. After inserting these terms, equation (16) can be re-written as,

$$\left(\frac{\bar{\theta}}{1-\bar{\theta}}+\omega\right)\pi_t-\frac{\bar{\theta}}{(1+\bar{\pi})(1-\bar{\theta})^2\mu}(\pi_t-\bar{\pi})\pi_t=$$

$$\frac{\omega}{1-\bar{\theta}}\pi_{t-1}-\frac{\omega\bar{\theta}}{(1+\bar{\pi})(1-\bar{\theta})^2\mu}(\pi_{t-1}-\bar{\pi})\pi_{t-1}+(1-\omega)\left\{(1+\beta\theta)\left[\sum_{i=0}^{\infty}(\beta\theta)^i E_t(mcn_{t+i}+\mu)\right]-p_t\right\}$$

After substituting forward we derive,

$$\pi_t=\gamma^b\pi_{t-1}+\gamma^f E_t\pi_{t+1}+\lambda(mcn_t+\mu-p_t)+\psi\left[(1-\beta\theta\omega)\{(\pi_t-\bar{\pi})\pi_t\}-\omega\{(\pi_{t-1}-\bar{\pi})\pi_{t-1}\}-\beta\theta\{(E_t\pi_{t+1}-\bar{\pi})\bar{\pi}\}\right]$$

where

$$\gamma^b=\omega\phi^{-1},\ \gamma^f=\beta\theta\phi^{-1};\lambda=(1-\omega)(1-\bar{\theta})(1-\beta\theta)\phi^{-1},$$
$$\psi=\frac{\bar{\theta}}{(1+\bar{\pi})(1-\bar{\theta})\mu}\phi^{-1},\phi=\bar{\theta}+\omega\left[1-\bar{\theta}(1-\beta)\right]$$

In the special case of a zero expansion point, i.e. $\bar{\pi}=0$ and $\bar{\theta}=\theta$, coefficients multiplying lagged and expected inflation as well as the mark-up over real marginal costs correspond to those of NKPC as in Galí and Gertler (1999).

Notwithstanding, however, note that the last non-linear terms in the squared brackets *does not* disappear when $\bar{\pi}=0$, since the term inside the square brackets would reduce to $(1-\beta\theta\omega)\pi_t^2-\omega\pi_{t-1}^2$. We see, though, that the practical importance of this term declines rapidly the smaller is the variation of actual inflation around the regime average.

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