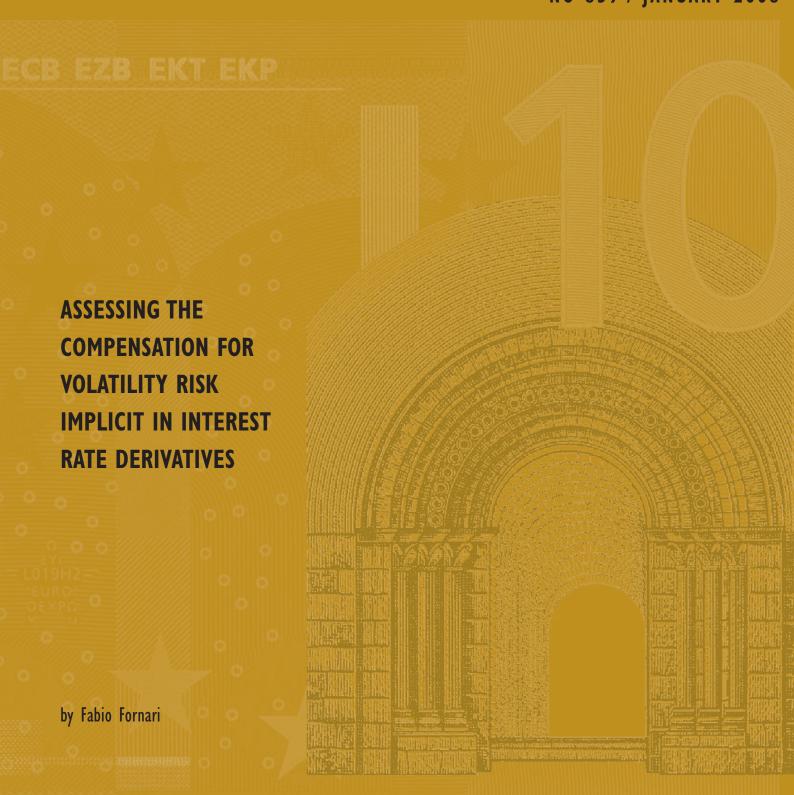


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# ASSESSING THE COMPENSATION FOR VOLATILITY RISK IMPLICIT IN INTEREST RATE DERIVATIVES 1

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#### Abstract

Volatilities implied from interest rate swaptions are used to assess the size and the sign of the compensation for volatility risk, for dollar, euro and pound rates at a daily frequency, between October 1998 and August 2006. The measurement of the volatility risk premium rests on a simple model according to which variance forecasts are generated under the objective probability measure. Results show that especially between September 2001 and mid-2003 dollar implieds were embodying a large - negative compensation for volatility risk, a component which was smaller in absolute terms but not relative to the level of the respective implied volatilities - for the other two currencies. While the negative compensation for volatility risk is in line with previous studies focusing on other asset classes, we also document that it exhibits a term structure, more evident for dollar and euro rates than for pound rates. The volatility risk premium is strongly changing through time but much less than implied volatilities. Estimates of risk aversion based on the physical skewness and kurtosis of interest rate changes suggest that (minus) the volatility risk premium can almost directly be read as risk aversion, as its proportionality with such risk aversion measure is about 0.8. Also, compensation for volatility risk is positively related to expected volatility, although the relation is not completely linear. Daily compensation for volatility risk is influenced, as expected, by the level of the short term rate and its volatility as well as by a small but robust number of macroeconomic surprises. The latter induce more sizeable changes on compensation for volatility risk of dollar rates than of euro or pound rates.

JEL classification: G120, G130, G140.

**Keywords:** Volatility risk premium, risk aversion, economic surprises.

Non-technical summary

The paper investigates the existence of a gap between implied volatilities extracted

from options written on swap rates and expectations of such swap rate volatilities. The

latter are obtained through simulations of a model which filters historical volatilities

from lagged swap rate changes. The existence of a systematic gap between implied

volatilities and expectations of realised volatilities would indicate that market

participants attach a significant price to volatility risk, while allowing at the same time

their degree of risk aversion to be estimated. Risk aversion can be recovered by

exploiting its proportionality to compensation for volatility risk.

The methodology is applied to daily dollar, euro and pound sterling swap rate

volatilities observed between October 1998 and August 2006. Coherently with

previous literature, volatility risk is found to be negative and volatility seems to have

been significantly priced for all three currencies, albeit with a strong time variation,

with a relative peak between September 2001 and June 2003. An alternative estimator

of the risk aversion coefficient based on the joint use of the conditional skewness and

kurtosis of swap rates suggests that volatility risk premium represents a good proxy

for risk aversion

Looking at the determinants of the volatility risk premium (i.e. risk aversion), the

short-term interest rate, its conditional volatility as well as macroeconomic releases,

with a small but robust number of US variables playing the key effect on all three

currencies, seems to be prominent.

СВ

# 1 Introduction

Over the last 25 years a huge literature has evidenced that financial volatility is stochastic. Asset pricing models taking into account the presence of stochastic volatility have yielded good results in applications with Government bonds, equities, foreign currencies and corporate bonds (see Andersen et al., 2002). Allowing for stochastic volatility in the physical dynamics of the state variables requires that for pricing assets - a task in which the risk neutral dynamics of the state variables is needed - the functional form of the volatility risk premium be specified. However, both the functional form of this premium and its sign are topics which remain largely unexplored, and represent the focus of this paper.

One can suppose that compensation for volatility risk has a non-zero mean because if, as said, volatility is stochastic, then rational agents must ask a compensation for its fluctuations.<sup>1</sup> Alternatively, one can see implied volatilities (and implied density functions) of asset prices as close or unbiased indicators of future expected volatilities (and future physical densities) of that asset price, thus implying that volatility risk is unpriced. However, the few results presented so far in the literature suggest that it would be wrong to assume compensation for volatility risk to be zero. Then, if agents ask a rising compensation to hold volatility-dependent instruments (i.e. options) in presence of rising uncertainty about future volatility, it is valuable to increase our knowledge about the volatility risk premium (henceforth vrp) so to correctly assess the practical implications of implied volatilities for our 'physical' world.

The issue can be illustrated by looking at the surge in dollar swaptions volatilities between September 2001 and June 2004, a development which was much less evident for euro rates (Figure 1). As before September 2001 dollar and euro implieds were nearly equal, the higher swaptions' implieds on dollar rates must have had a US-specific origin or must have been induced by increased riskiness in factors to which dollar rates have higher exposure. Further, dollar volatilities did not increase uniformly along the swaptions maturity spectrum (the 6-month maturity is reported in the Figure). These developments led all dollar swap rates to exhibit a strongly negatively sloped term structure of implied volatilities. Since mid-2003, and more significantly since mid-2004, the higher average

<sup>&</sup>lt;sup>1</sup>However, despite the many econometric analyses dealing with modelling stochastic volatility and the empirical success of asset pricing models with stochastic volatility, only a few papers (discussed in detail later in this section) tackle the issue of how the volatility risk premium behaves and what sign and size it has

volatility of dollar swap rates relative to euro and pound rates and the relative volatility peak at short horizons have receded. Since end-March 2005 the volatility term structures have been almost flat with implied volatilities of dollar interest rates close or, at times, below those observed for euro rates.

# [insert Figure 1 about here]

Such developments lead us to investigate what has been observed in the nearly threeyear period after September 2001. That is, were higher dollar implieds reflecting i) expectations of higher interest rate volatility and stability in the required compensation for volatility risk, ii) stable expected volatilities but higher compensation for volatility risk or iii) a combination of the two previous outcomes?

In this paper we use a simulation-based approach which rests on a simple Garch model to assess if, in nearly eight years of daily data, implied volatilities have been coherent with model-based expectations of realised volatilities (hence there has been negligible or small compensation for volatility risk) or if, by contrast, implied deviated significantly from such predictions (therefore implying that volatility risk was significantly priced). As explained in detail in the next section, the difference between implied volatilities and expected realised volatilities represents, in the discrete time setting of this paper, the counterpart of the compensation for volatility risk examined in the approach of Bollerslev et al. (2007). In doing so we take the gap between implied volatilities and model-based expected realised volatilities as indication that volatility risk has commanded a significant compensation, although part of the gap may also reflect potential misspecification of the model employed to generate the volatility forecasts. The analysis reported in this paper refers to at-the-money implied volatilities of dollar, euro and pound swap rates. Among a large set of swaptions, we examine those regarding the 1, 2, 5 and 10-year swap rates with option maturity spanning up to 2 years. To anticipate, the main result is that, coherently with previous evidence focusing mainly on equity indices and currencies, compensation for volatility risk has been negative and sizeable - especially for dollar rates in absolute terms, i.e. relative to the level of the corresponding implied volatility - in the period of high implieds between September 2001 and June 2003. Compensation was lower in absolute terms for euro and pound swap rates relative to dollar rates, although higher than before September 2001 also for these two currencies. In relative terms, i.e. scaling vrp by implied

volatility, compensation for volatility risk has been higher for euro and pound rates than for dollar rates. Evidence favouring the presence of a priced volatility risk is also obtained by looking at the relation between the sign of delta-hedged gains from portfolios of swaptions and past realised volatility. Looking at time series behavior, vrp changes significantly through time but on average moves much less than implied volatility. Hence, changes in volatility are not fully translated in changes in the compensation for volatility risk. The interest rate level and its volatility are the main drivers of the compensation for volatility risk and a small but robust number of macroeconomic surprises - mainly the US nonfarm payrolls, Chicago PMI and industrial production - lead agents to systematically revise vrp, beyond influencing implied volatilities. There is also evidence that news releases in one country affect the compensation for volatility risk of other currencies, although the impact of US macroeconomic surprises on euro and pound rates tends to be more sizeable than the effect played by European surprises on dollar rates.

Looking at previous literature, a formal analysis on volatility risk compensation was initiated by Bakshi and Kapadia (2003). They recovered hints on the presence of a volatility risk premium looking at delta-hedged gains (henceforth dhg) of options' portfolios, defined as  $\Pi_{t,t+\tau} = C_{t+\tau} - C_t - \int_t^{t+\tau} \Delta_u dS_u - \int_t^{t+\tau} r(C_u - \Delta_u S_u) du$ , where  $C_t$  is the option's price as of time t and  $\Delta_t$  ( $\partial C_t/\partial S_t$ ) denotes the delta of the option. In a general stochastic volatility specification of an asset price changes  $dS_t$ , the expectation of the delta-hedged gain is

$$E\left(\Pi_{t,t+\tau}\right) = \int_{t}^{t+\tau} E_{t}\left(\frac{\partial C_{u}}{\partial \sigma_{u}}\lambda_{u}\right) du \tag{1}$$

where  $\partial C_u/\partial \sigma_u$  is the vega of the option and the expectation is taken under the physical probability measure. It is obvious that if volatility risk is not priced ( $\lambda_t = 0$  in (1)) then the delta-hedged gain is expected to be nil on average.

By factoring the delta-hedged gain in a part related to the price of the underlying asset and in a second component which depends on the volatility, the authors showed that the time series of scaled dhg<sub>s</sub>  $E_t(\Pi_{t,t+\tau})/S_t$  should vary with volatility, while its cross sections, obtained by fixing  $\sigma$  at its day t level, should covary with the option's moneyness (y) and time to expiry  $(\tau)$ . For Heston's (1993) model they obtained  $E_t(\Pi_{t,t+\tau}) = \lambda \cdot \sigma_t \cdot S_t \cdot \psi_t(\tau)$ . Hence, a relation between  $E_t(\Pi_{t,t+\tau})/S_t$  and any functional form of the volatility would reveal the existence of a non-zero volatility risk premium.<sup>2</sup> It is important to point out

<sup>&</sup>lt;sup>2</sup> As for the second test, given that the vega of an option is highest when the option is at the money and

that this analysis helps identify the presence and the sign only, but not the size, of the volatility risk premium.

Empirically, delta-hedged gains are calculated as:

$$\pi_{t,t+\tau} = \max \left( S_{t+\tau} - K, 0 \right) - C_t - \sum_{n=0}^{N-1} \Delta t_n (S_{t_{n+1}} - S_{t_n}) - \sum_{n=0}^{N-1} r_n (C_t - \Delta t_n S_{t_n}) \tau / N$$

where  $t_0 = t$  and  $t_N = t + \tau$ , the delta  $(\Delta t_n)$  is the Black-Scholes delta and the volatility employed in the calculations of the delta comes from a garch estimate of the conditional volatility of the underlying asset. Bakshi and Kapadia show that for all at-the-money calls, as well as for each options' maturity, the delta-hedging strategy loses money, 0.10% of the index level. The buyer of the call is thus paying a premium to the seller, nearly 43 cents per call, 28 when the option is in the money. Also, gains grow with the maturity of the options, and at the 31-60-day horizon they are nearly twice as sizeable as for the 14-30-day maturity.

Bakshi and Madan (2006) prove that for the class of pricing kernels m(R) satisfying the Taylor series expansion around zero,<sup>3</sup> the theoretical percentage spread between riskneutral ( $\sigma_{rn}^2$ ) and physical ( $\sigma_p^2$ ) volatilities, under a power utility function, is

$$\frac{\sigma_{rn}^2(t,\tau) - \sigma_p^2(t,\tau)}{\sigma_p^2(t,\tau)} = -\gamma \cdot (\sigma_p^2(t,\tau))^{0.5} \cdot \theta_p(t,\tau) + \frac{\gamma^2}{2} (\sigma_p^2(t,\tau)) \cdot (k_p(t,\tau) - 3)$$
 (2)

where  $\gamma$  is risk aversion and  $\theta_p$  and  $k_p$  are the physical skewness and kurtosis. Therefore, the volatility spread is largest when p(R), the physical marginal density function of asset returns, is negatively skewed and fat tailed. The conditional skewness and kurtosis contribute to the spread with weights  $-\gamma \cdot \sigma_p(t,\tau)$  and  $\frac{\gamma^2}{2}(\sigma_p^2(t,\tau))$  respectively. The authors report that between January 1984 and December 1999 the risk-neutral volatility of S&P100 index options was 3 percentage points higher than the realised volatility, calculated as in Andersen et al. (2003), with an average volatility spread of 23.45% and implieds exceeding realised volatilities in 74% of the months. Subsample analysis evidenced that positive volatility spread are a post-crash phenomenon, with average spread after October

then tends to decrease, the hypothesis of a non-zero volatility risk premium can be rejected if this relation is not found between  $E_t\left(\Pi_{t,t+\tau}\right)/S_t$  and  $y_{i,t}$ , with i denoting the moneyness of the option.

 $<sup>^{3}</sup>m(R) = 1 - A_{1} \cdot R + 0.5 \cdot A_{2} \cdot R^{2} + O(R^{3})$  where m(0)=1,  $A_{1} = \frac{-\partial m}{\partial R}$ ,  $A_{2} = \frac{\partial^{2} m}{\partial R^{2}}$ .

1987 being 30.49%. They also estimate the value of risk aversion via gmm through the orthogonality between  $\varepsilon_{t+1}$ 

$$\varepsilon_{t+1} = \frac{\sigma_{rn}^2(t+1) - \sigma_p^2(t+1)}{\sigma_p^2(t+1)} + \gamma \cdot (\sigma_p^2(t+1))^{0.5} \cdot \theta_p(t+1) - \frac{\gamma^2}{2}(\sigma_p^2(t+1)) \cdot (k_p(t+1) - 3)$$

and some time-t information-based variables  $Z_t$ , minimising  $J_T = \arg\min_{\gamma} g_T' \cdot W_T \cdot g_T$  where  $g_T(\gamma) = (1/T) \sum_{t=1}^T \varepsilon_{t+1} \otimes Z_t$  and  $W_T$  a suitable covariance matrix. Empirically, skewness and kurtosis were calculated on a rolling basis via the S&P100 returns and updated each 90 days, while the set of instruments  $Z_t$  employed were the risk-neutral volatilities dated t, t-1 and t-2. The estimated average risk aversion was reported to be approximately 17.

Looking at more empirical papers, Carr and Wu (2007) use options on 5 stock indices and on 35 individual stocks. Interestingly, they get evidence of a significant volatility risk premium in options written on the S&P500 index, the S&P100 and the Dow Jones Industrial Average. For individual stocks, however, the presence of vrp is weak and the magnitudes of risk compensation in such cases are overall smaller than for the three broad indices. The authors investigate if the negative correlation between stock index and stock index variance (the leverage effect) can account for the negative variance risk premia by regressing the latter on the broad stock index return and a constant. Although the slope of the regression is negative, the constant has nearly the same magnitude as the average volatility premium, evidencing that there is no information content in the stock return. Additional factors, among which those identified by Fama and French, do not improve the negative intercept. Hence, they conclude that either there is inefficiency in the market for index variance or that the market is pricing an independent, unknown, risk factor.

Guo and Neely (2004) transpose the Bakshi and Kapadia (2003) analysis to options on currency futures. They analyse options on the US dollar against pound sterling, Deutsche Mark, Japanese yen and Swiss franc between 1975 and 1999. Their delta hedged strategies are related to realised volatility but negatively related the options' maturity, unlike the prediction of Bakshi and Kapadia. Beyond these findings, they claim that the fundamental issue is whether the trading profit arising from delta hedged portfolios remains significant after adjusting for known risk factors (based on the Capm and on the Fama and French

(1993) model). Pooling the delta hedged gains across currencies they are unable to reject that delta hedged gains are zero. Also Bakshi et al. (2000) and Buraschi and Jackwerth (2001) show that equity index options are non-redundant securities, suggesting that other risks, among which variance risks, may be priced. Coval and Shumway (2001) use daily straddles and argue that the variance risk premium is negative. A rather wide literature also employs structural models calibrated to options data to recover the volatility risk premium, among which Guo (1998), Bates (2000), Chernov and Ghysels (2000), Benzoni (2002), Fornari and Mele (2001) and Pan (2002).

A work that is closely related to the present paper is Bollerslev et al. (2007), where estimates of the volatility risk compensation are presented and put in relation with macroeconomic and financial determinants. The authors derive the compensation from comparing model-free implied (IV\*) and realised (V) volatilities.<sup>4</sup> Using monthly implied and realised model-free volatilities for the S&P500 index from January 1990 through May 2004 they concluded that both measures increased during the latter half of the sample, that realised volatilities are systematically lower than implieds and that their unconditional distribution tends to display larger deviations from a normal. On average the risk premium is reported to have been about -2 percentage points. Looking at determinants of the volatility risk premium, realised volatility has had the biggest impact, followed by the AAA corporate bond spread over Treasuries and the P/E ratio.<sup>5</sup>

# 2 Forming expectations of swap volatilities

Implied volatilities represent market expectations of future realised volatilities over the life of the option. However, when volatility is stochastic and volatility risk is priced, they

$$\begin{array}{ll} physical & risk \ neutral \\ dp_t = \mu_t(\cdot)dt + \sqrt{V_t}dB_t & dp_t = r_t dt + \sqrt{V_t}dB_t^* \\ dV_t = k \cdot (\theta - V_t)dt + \sigma_t(\cdot)dW_t & dV_t = k^* \cdot (\theta^* - V_t)dt + \sigma_t(\cdot)dW_t^* \\ corr(dB_t, dW_t) = \rho & corr(dB_t^*, dW_t^*) = \rho \end{array}$$

To retrieve IV\* and V they follow Andersen et al. (2003) in estimating  $V_{t,t+\Delta}$  as sums of squared high-frequency returns over the interval  $\Delta$ . The model-free measure of risk-neutral integrated volatility is instead derived from options prices (see Britten-Jones and Neuberger, 2000; Carr and Wu, 2007; Jiang and Tian, 2005) as  $IV_{t,t+\Delta}^* = 2 \int_0^\infty \frac{C(t+\Delta,K)-C(t,K)}{K^2} dK$ . For expected volatilities it is then easy to show that  $E(V_{t+\Delta,t+2\Delta}) = \alpha_\Delta E(V_{t,t+\Delta}) + \beta_\Delta$  and given that  $IV_{t,t+\Delta}^* = E^*(V_{t,t+\Delta})$  then  $E(V_{t,t+\Delta}) = A_\Delta IV_{t,t+\Delta}^* + B_\Delta$ , where  $A_\Delta = \frac{(1-e^{-k\Delta})/k}{(1-e^{-k^*\Delta})/k^*}$  and  $B_\Delta = \theta[\Delta - (1-e^{-k\Delta})/k] - A_\Delta \theta^*[\Delta - (1-e^{-k^*\Delta})/k^*]$  and the parameters reported in the above formulas come from the physical and risk neutral dynamics of the price of the asset, i.e.

<sup>&</sup>lt;sup>5</sup>Similarly to the analysis in Bollerslev et al. (2007), Chernov (2003), Eraker (2004), Jones (2003) and Pan (2002), analyse volatility risk premia and return risk premia by fitting option pricing models.

will also include a time varying compensation to stand its fluctuations through time. As said at the end of the previous section, we use a simple methodology, based on modeling realised variances as Garch processes, and then forecasting from such models, to measure the portion of the implied volatility that reflects expected volatility. Namely, we specify a model for the behaviour of the historical volatility, from which we can draw volatility forecasts over various horizons, conditionally on being at a given point in time. The assumption made here is that the historical interest rate volatility is captured by a parsimonious asymmetric garch(1,1):

$$r_t = \mu + \phi r_{t-1} + \sigma_t z_t$$

$$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2 + \gamma \max(0, -\varepsilon_{t-1})^2$$

$$\varepsilon_t = \sigma_t z_t | I_{t-1} N(0, \sigma_t^2); z_t NID(0, 1)$$

where  $(\mu, \phi, \omega, \alpha, \beta)$  are real parameters,  $r_t$  denotes the logarithmic daily rates of change of a given swap rate and  $\sigma_t$  is its conditional volatility. Denoting by  $I_{t-1}$  the information set, i.e. the past history of the interest rate series, forecast errors  $\varepsilon_t = \sigma_t z_t$  are conditionally normal. Alternative distributional assumptions for forecast errors or different parametrizations of the above equations based, for example, on the huge number of members of the Arch family, are not explored, mainly because of the way in which  $z_t$  is sampled in the simulation exercise. Rather than from a normal distribution, as the "strong garch" assumption postulates (Drost and Werker, 1995),  $z_t$  is bootstrapped from the historical series of forecast errors (i.e. from realised  $z_t$ ) so that any model misspecification ending up in non-normal  $z_t$  will be eventually re-sampled through observed forecast errors.<sup>6</sup>

To reproduce the time-t expectations of economic agents about future interest rate volatility, the estimation of the models is performed on expanding samples, the shortest of which starts on 23 January 1997 and ends on 15 October 1998, hence containing 450 daily observations. In this way, volatility forecasts rely only on information available

<sup>&</sup>lt;sup>6</sup>Of course different specifications would lead to different paths for simulated volatilities as the drift of the volatility process would differ from what assumed in the asymmetric garch(1,1) employed in the paper. This would change the relative weight of the 'signal term'  $\beta \cdot \sigma_{t-1}^2$  with respect to the noise term  $\alpha \cdot \varepsilon_{t-1}^2$ . As an example one could employ the asymmetric power Arch of Ding et al. (1993) studied in more detail in Fornari and Mele (1997) especially as concerns distributional assumptions that lead to a better fit of the tail-thickness of financial data or the component Arch model of Engle and Lee (1999), basically a reparametrization of a garch(2,2) where the conditional volatility is made up of a long-run (slow changing) component and a short-run (fast changing) component, which may allow for a more precise identification of the expected volatility drift.

when forecasts were made. For each day we retain the parameters of the models, based on an expanding sample starting on 23 January 1997 and ending at time t, the time series of forecast errors  $(\varepsilon_t)$  and the realised filtered volatilities  $(\sigma_t)$ . In a second step we use such ingredients to produce, for each calendar day after 15 October 1998, forecasts of the realised volatility over various horizons. Each day we generate 5000 future paths of the interest rate and its volatility, for each of the four chosen interest rates (1-, 2-, 5and 10-year) and for five forecast horizons, 1, 3, 6, 12 and 24 months. For each of these horizons we compute the expected volatility by averaging the simulated daily volatilities first across the option's maturity and then across the 5000 replications. This average expected volatility is then compared, for each day in the sample, to the implied volatility taken from options with the same swap rate as underlying and with the same life to maturity. To make a practical example, the expected six-month physical volatility is the average across the 5000 replications of the average volatility simulated in the six months (126 working days) after a particular day. Estimating expected volatility conditioning on information available only at time t is important because this information may affect the volatility of volatility, and changes in this parameter potentially lead economic agents to adjust the swaptions price relative to what suggested by the historical volatilities.

The structure of the simulation scheme has no major differences relative to the asymmetric Garch(1,1) described above. The only difference, as already mentioned, comes from the distributional assumption placed on the standardized forecast errors ( $\varepsilon_t/\sigma_t = z_t$ ). Since the implicit Garch assumption that  $z_t$  are independently and identically normally distributed is rejected, especially in high-frequency data, due to the presence of asymmetry in excess of zero and kurtosis in excess of three, these non-normal features are plugged back into the simulation to account for the in-sample mispecification displayed by the model. For each day in the sample, we randomly select an element from the time series of  $z_t$  and then loop over the following two equations up to a two-year horizon (506 days) to generate a path of  $\sigma_{t+i|t}^2$  and  $r_{t+i|t}$  with j=1,2,..., 506:

<sup>&</sup>lt;sup>7</sup>The alternative and computationally much simpler procedure of comparing the current implied volatility to the current historical volatility (and not to the average expected volatility between t+1 and  $t+\tau$ ,  $\tau$  being the maturity of the swaption) forces the compensation for volatility risk to depend on the assumption that future volatility is a random walk process, which contrasts with the findings of the Garch literature. methodology.

$$\sigma_{t+j|t}^2 = \omega + (\sigma_{t+j-1} \cdot z_{t+j-1})^2 + \beta \cdot \sigma_{t+j-1}^2 + \gamma \cdot \max(0, -(\sigma_{t+j-1} \cdot z_{t+j-1}))^2$$
$$r_{t+j|t} = \mu + \phi \cdot r_{t+j-1} + \sigma_{t+j} \cdot z_{t+j}$$

We also consider block boostrap-based simulations with a fixed block size of 10 working days, to reproduce more closely the clustering of errors through time, typical of high frequency financial data. Furthermore, as in each day we have 5000 values for the expected volatility of each interest rate over the five forecast horizons, we can recover the simulated marginal density function of expected volatilities. From this we take the 2.5 and the 97.5 percentiles and use them to build a non-parametric 95% confidence interval for the expected volatility. The confidence interval helps to tell days or periods of normal compensation for volatility risk from days or periods of excessive compensation for volatility risk, where normal and excessive have to be intended as functions of the confidence interval generated by the process according to which expected volatility is formed.

#### 3 **Evidence**

#### 3.1 Data and delta-hedged gains

All results in this paper are based on implied volatilities extracted from at-the-money swaptions<sup>8</sup> referred to dollar, euro and pound swap rates. Between January 1997 and December 1998 Deutsche Mark swaptions were used as a substitute for euro swaptions. As said, underlying rates are the 1-, 2-, 5- and the 10-year swap rates while the maturity of the swaptions was selected as 1, 3, 6, 12 and 24 months. All data are taken from Bloomberg. For all currencies the analysed sample goes from 23 January 1997 to 28 August 2006, but vrp time series are available only after 11 October 1998 (ie 450 days after the beginning of the sample) as the initial 450 days are needed to generate the first forecast volatilities. Figure 2 reports some additional information about the time series

<sup>&</sup>lt;sup>8</sup>Swaptions are options on forward swap rates. As a swap rate is made up of a given number of predetermined fixed payments and stochastic floating payments related to future libor rates, they amount to options on a portfolio made up of such future libor rates. The number of these libor rates is determined by the life of the swaption plus the life of the swap rate itself. Swaptions are quoted in terms of implied volatility, i.e. the volatility value which plugged into the Black (1976) formula, gives thee desired swaption price. At-the-money swaptions are those for which, at the inception of the contract, the exercise (strike) price is set equal to the forward interest rate.

behavior of implieds, namely those of the 2-, 5- and 10-year swap rates over swaptions maturities of 6 months and 2 years, which were not evidenced in Figure 1. The two left panels of the Figure show again that the term structure of dollar implied volatilities was virtually flat across all interest rates and options' expirations before September 2001. After this date, both term structures became negatively sloped, with shorter rates perceived as more volatile than longer rates, for a given expectation horizon, and shorter expectation horizons more volatile than longer expectation horizons, for a given interest rate. Since 2005 the two economic areas have had again broadly comparable volatility levels and flat volatility term structures.

The area between the vertical bars identified by the symbol '2' in Figure 2 evidences the prolonged phase of US monetary policy easing that took place between 2001 and mid-2004 while the area between the vertical bars identified by the symbol '1' highlights the US recession between March and November 2001.<sup>9</sup> It is worth noticing that dollar implieds started to rise a couple of months before the official NBER-based start of the recession and that they kept rising throughout the contractionary phase, peaking at the trough of the US cycle.

After a few months of pause around these levels, implieds returned to rise, reaching new historical maxima in late 2002 and in the first half of 2003. Afterwards they tended to fall. Euro implieds moved broadly in accordance with dollar implieds, rising across the US recession, thereafter stabilising and then surging again since mid-2002. However they reached lower levels than those seen for dollar rates. Looking at spring 2003, the 2-year dollar rate implied volatility peaked at about 60% annualised while the corresponding euro implied hardly touched 35% annualised. In addition, euro implieds have been recording a significant fall before 2001, while dollar implieds were substantially stable over the same period.

# [insert Figure 2 about here]

Before quantifying the compensation for volatility risk using the garch-based simulation introduced in the previous section, a preliminary indirect evidence of its presence and

<sup>&</sup>lt;sup>9</sup>Although the paper does not aim at measuring how much of the increase in compensation for volatility risk comes from monetary policy uncertainty, it is straightforward to see that implied volatilities reflected or have been correlated with market concerns about the modality of US monetary policy reaction to the 'deflation scare' in the United States in 2002-2003 and then to evidence of robust growth in the first half of 2004, before the Federal Reserve started to raise official interest rates.

its sign can be obtained through the model-free method of Bakshi and Kapadia. This analysis will first tell us the gains/losses achieved by a delta-hedged strategy. Then it will be the characteristics of such delta-hedged returns, and their relation with lagged physical interest rate volatility, to reflect the presence of a priced volatility risk. The discrete time delta-hedged gains  $(\pi_{t,t+\tau}^k)$  are calculated as

$$\pi_{t,t+\tau}^k = \max(F_{t+\tau}^k - F_t^k, 0) - C_t^k(\sigma_t^{imp,k}) - \sum_{n=0}^{N-1} \Delta t_n(\sigma_t^{rea,k}) (F_{t_{n+1}}^k - F_{t_n}^k) - \sum_{n=0}^{N-1} r_n (C_t - \Delta t_n(\sigma_t^{rea,k}) F_{t_n}^k) \tau_{reb}$$

where the first two terms on the right hand side capture the return to holding the option, k denotes the maturity of the swap rate  $F^k$  (1, 2 and 5 years),  $C_t^k$  the price of the swaption, which before maturity depends on the quoted implied volatility through the Black (1976) formula,  $\Delta t_n(\sigma_t^{rea,k})$  is the delta  $(\partial C/\partial F^k)$  of the swaption with maturity k, which we assume to be dependent on a measure of time-varying realised volatility of that rate, proxied by a garch(1,1) as in Bakshi and Kapadia. Last,  $r_n$  is the observed annualised 3-month rate, N expresses calendar days (ie N=126 for a 6-month horizon) while  $\tau_{reb}$  is equal to one day, i.e. the portfolio is rebalanced each day.

The delta-hedged gains are reported in Figure 3 for the swaptions on the dollar 2-year rate at the 3- and 12-month horizons, but as evidenced from Table 1 this pattern is typical of other rates. On average the gains have been negative, with means ranging between  $-2.66(\cdot 10^{-5})$  and  $-1.36(\cdot 10^{-3})$ , with the only exception of the swaption with 6-month life written on the 1-year rate, whose return has been approximately zero. As a ratio of the spot rates such mean losses on the delta-hedged strategy are not negligible, ranging, on average for the full sample (minus the last 21, 63, 126 and 253 days for the 1-, 3-, 6and the 12-month horizons, respectively) between -1% and -8%, while as a ratio of the swaption price they ranged from -10% to -40%. Overall, they are not distant from what obtained by Bakshi and Kapadia for a different underlying asset (the S&P500 index) and across a different sample (January 1988 - December 1995). For options with moneyness ranging between 0 and -5%, they estimated the losses to range between 7% and 34% of the value of the underlying asset. Looking also at the number of negative occurrences in the time series of the delta-hedged gains, Bakshi and Kapadia get on average a figure of 68% for options with moneyness between -2.5% and 2.5%. In the case of our interest rate swaptions, delta-hedged gains for at-the-money instruments are negative in about 65% of times for the 1-year maturity and in about 80% of times for the 6-month maturity. Results show that the negative return of a delta-hedged strategy has been less the case for euro rates. In this case means are negative but much smaller than for dollar rates. Losses range between -1% and -3% of the swap rate level and between -1% and -75% of the swaption price.

# [insert Table 1 about here]

To highlight the presence of a compensation for volatility risk, we now test for a relation between the delta-hedged gains, scaled by the relevant swap rate, and the lagged interest rate physical volatility. Given the strong autocorrelation present in the daily time series of the delta-hedged gains (which overlap for a substantial amount of time) we have resampled them at a monthly frequency and subsequently such re-sampled monthly gains, scaled by the value of the corresponding underlying asset (i.e. the interest rate on which the swaptions are written), have been regressed on their first lag and on the first lag of the physical (garch) log-volatility. Predictability (R-squared are in Table 1) is quite large, about 70% for dollar rates and 60% for euro rates. Although significant autocorrelation remains also at the monthly frequency, the first-order autocorrelation coefficient being on average 0.8 excluding the volatile gains at the 1-month horizon, the sign of the lagged logarithm of the conditional volatility is negative and significant in most cases. Figure 4 reports the same time series of delta-hedged gains for the euro denominated swaptions. For euro rates, although delta-hedged gains are on average negative through the sample, as mentioned above, regression do not fully support the presence of a negative volatility risk premium, as lagged garch log-volatility of the euro swap rates does not happen to be significantly negative (only 2 cases out of 12). This may provide first of all indirect evidence for the hypothesis that in the market for euro interest rate swaptions volatility risk may have been perceived less strongly than for dollar rates. On the other hand, as the euro swap market only developed after 1999, agents may have been affected by the fact that no historical volatility of euro rates was available, which in turn may have reduced the weight initially attached to volatility risk. The findings in Carr and Wu (2007) may also imply that US rate volatility could have been perceived a priced source of risk for international portfolios while euro rates volatility did not represent a key risk factor for their portfolios.

# 3.2 Garch-based compensation for volatility risk

Figures 5-7 report (minus) the volatility risk compensation  $(-\lambda_t)$  for dollar, euro and pound rates, at various horizons, together with the corresponding 1-year swap rates. As described in Section 2, such series are obtained as difference between implied volatilities and Garch-based expectations of realised volatilities. As in Figure 2, the area between the vertical bars identified by the '2' symbols highlights the long phase of monetary easing in the US, while the area between '1' evidences the 2001 US recession. The 1-year swap rate has been included to highlight the fact that, despite what one may think, there has not been a strong direct relation between the interest rate level and the interest rate volatility, i.e. higher volatilities have not been trivially related to the falling interest rate level, even in the period of monetary policy easing delimited by the '2' symbol.

As said before vrp moves proportionally to risk aversion. In fact, starting from the fact that  $\lambda_t = -cov(\frac{dm_t}{m_t}, d\sigma_t^2)$ , where  $m_t$  is the pricing kernel, and assuming that investors have a power utility function ( $m_t = S_t^{1-\gamma}$ , with  $S_t$  the asset price) and that a two factor model governs the dynamics of the interest rate and its stochastic volatility<sup>10</sup> then  $\lambda_t = \frac{\gamma_t \cdot \rho}{\eta}$  where  $\gamma_t$  is the coefficient of relative risk aversion as of time t,  $\rho$  the correlation between the two brownian motions characterising the bivariate process governing the dynamics of the interest rate and its volatility and  $\eta$  is the volatility of volatility parameter. Bollerslev et al. (2007) obtain  $\frac{\rho}{\eta} \simeq 1$  so that in principle one could expect that at least for equity index options vrp  $\simeq$  risk aversion. However, as the simulation-based approach employed in this paper cannot guarantee that minus the vrp stays positive, as one would expect the coefficient of relative risk aversion to be, vrp will be mainly treated as 'volatility risk compensation', although it will help to recall its proportionality with relative risk aversion when interpreting its movements across time and its reaction to economic surprises in Section 4.

All in all, the occurrence of negative values in (minus) vrp is not particularly high and is indeed much smaller than the number of times the delta-hedged gains are positive, both for the swaptions analysed in this paper and for the equity index options studied

<sup>&</sup>lt;sup>10</sup>Which is the case of the Garch(1,1) model, converging to a continuous time model resembling Heston (1993). See Bollerslev et al., (2007).

in Bakshi and Kapadia. Table 2 shows the percentage of days, out of the full sample, in which (minus) vrp turned out to be negative. For dollar rates such a percentage was about 70% for the 1-year swap rate and above 85% for 2-, 5- and 10-year rates. For euro rates the percentages are even higher and especially so for the 1- and 2-year rates. Assuming for the time being the proportionality factor between vrp and risk aversion to be approximately equal to one (more on this in the next subsection), Figure 5 suggests that at the end of August 2006 risk aversion was very low by historical standards for all the interest rates analysed. Between 1998 and August 2006 it has been on average 4 across the three currencies, just one fourth of the gmm-based value of 17 reported in Bollerslev et al. (2007) based on the S&P500 index over the sample 1990-2004.

# [insert Figures 5-7 about here]

Visual inspection of Figures 5-7 suggests that (the negative of the) compensation for volatility risk is very correlated across rates and maturities. The overall comovement of vrp across rates and horizons, however, does not imply that vrp is stable across these dimensions. Figure 8 shows, for dollar and euro rates only, the unconditional term structures of vrp, together with the corresponding unconditional term structure of implied volatilities. The curves for the dollar and the euro have a similar shape in most cases - downward sloping or inverted U-shaped - although overall vrp levels and the slope of the vrp curves (scaled by the average value of vrp or by implied volatility) tend to be higher for euro than for dollar rates. Therefore, although - in absolute terms - vrp levels are higher for dollar rates than for euro or pound rates, in relative terms - i.e. scaling vrp by the relevant implied volatilities - compensation for volatility risk was on average higher for the latter rates than for dollar rates.

For all currencies, however, vrp is much lower than the implied volatility of corresponding rates and maturities, on average between one quarter and one third of the latter, with the difference being most pronounced between the 1- and the 6-month horizons. As the vrp term structure is rather flat across maturities compared to implieds' term structure, the observed rise in implieds after September 2001 was likely reflecting genuine expectations of higher volatilities over short horizons (1- and 3-month) but was instead driven by

higher compensation required for volatility risk at the 6- and the 12-month horizons. 11 12

# [insert Table 2 about here]

Despite the high correlation across rates and maturities which one perceives from Figures 5-7, compensation for volatility risk has evolved quite differently between dollar and euro rates (beyond the already mentioned differences in the levels). Between Summer 1998 and May 2001 (minus) the vrp has been falling for euro rates, while it has been broadly stable for dollar rates. The decline in relative risk aversion in the euro area in this period could be seen as the result of the decline in both interest rate level and volatility in the run up to the single currency and after its inception. It is also interesting to note that movements in dollar vrp between short and long maturities - for a given rate - have become more correlated after the September 2001 events, possibly reflecting the fact that in turbulent periods common factors tend to dominate, therefore curbing the weigth of idiosyncratic risk factors. However, this was not the case for euro rates, whose term structure of volatility risk premiums became almost uncorrelated at short and long horizons, despite the fact that before this date correlations were sizeable and similar to what observed for dollar rates. After 2004, when interest rates started to rise in main areas, the movements in vrp across areas have become again less correlated: a decline in dollar vrp at all horizons has been accompanied by a rise or a stability for euro rates.

# [insert Table 3 about here]

Unconditional correlations reveal that comovement was indeed higher within countries than across countries, possibly reflecting business cycle correlation (vrp proxies for risk aversion). For dollar swap rates, and across maturities, correlation was on average 0.8,

<sup>&</sup>lt;sup>11</sup>The term structure of pound volatilities tends instead to differ from what observed for the other two currencies. First of all, it is slightly upward sloping for the 1-, 2- and 5-year rates compared to the other currencies and its steepness is extremely small. In addition the term structure of volatility premiums, with the exception of the 2-year rate (the horizon which is generally reported to be most influenced by news releases) is basically flat across maturities. In other words compensation for volatility risk in pound swaptions does not appear to be influenced by the maturity of the contract. This may be related to the fact that the yield curve of Government bonds yields has also been flatter in the UK than in other major industrial areas.

<sup>&</sup>lt;sup>12</sup>For sake of brevity, the results based on the block bootstrap are not reported in the paper. Overall, vrp estimates based on block bootstrap did not exhibit noticeable differences with respect to what reported in Figures 5 to 7. Of course some differences tended to emerge in periods of extremely low or extremely high volatility, when persistence tended to change more than on average, but they were not sizeable enough to modify either vrp developments or average vrp estimates.

more or less the same value as for euro rates, across the whole sample. Across countries, the dollar-euro correlation exceeded 0.6 for the 1-, 2- and 5-year swap rates at the 3-month and the 2-year horizons (Table 4) only in one eighth of the cases. While at the 3-month horizon, and especially for the 1- and the 2-year rates, correlations are broadly similar and high across countries, at the 2-year horizon they weaken somewhat in the dollar-euro comparison and fall noticeably in the dollar-pound comparison (Table 5). Comparing horizons, i.e. looking for example at the 3-month horizon in a country and to the 24-month horizon in another, correlations drop further. Overall it seems that the long-end of the volatility risk premium curve for the euro and the pound is not strongly correlated with both the short and the long end of the term structure of dollar vrp. All in all, the existence of a term-structure of vrp correlations in the three analysed currencies is a phenomenon first highlighted in this paper. While higher correlations across vrp at short horizons may reflect more correlated business cycle developments at short frequencies (and/or high spillovers across business cycles), the lower correlation at long horizons may indicate that country-specific components tend to be more influential for business cycle developments, and therefore for risk aversion, at longer horizons. To summarise, compensation for volatility risk is rather correlated, within a given country, across rates and forecast horizons. By contrast, correlations across countries are strong but tend to weaken when the forecast horizon in one country differs from the forecast horizon in another country.

## [insert Tables 4-5 about here]

The estimated volatility risk premium can also be used to shed light on the functional form of its relation with the implied volatility. To this aim, vrp has been regressed on a polynomial function of the implied volatility, including the linear, quadratic and cubic terms plus a logarithm. Regressions have been run for vrp referred to the 1- and 5-year rates at the 1-, 3-, 6- and 12-month horizons and are reported in Figure 9, where shaded areas represent the observed range of implied volatilities. For euro rates the relation between vrp and implied volatility is well approximated by a linear function, with vrp ranging, for the 1-year rate, from about 2% to about 15% when corresponding levels of the implied volatility range from 12% to 32%. This amounts to saying that, on average, a 30% implied volatility originated from a 18% expected volatility and a 12% compensation

for volatility risk. For dollar rates there is a much stronger evidence of nonlinearity, with vrp, always for the 1-year rate, ranging from approximately 0% to about 20% when corresponding implied volatility levels ranged from 10% to 55%. This result could provide a useful insight for continuous time asset pricing, where the specification of the volatility risk premium is generally assumed to be a linear function of the volatility, an hypothesis that seems to be rather weak for dollar rates while, at least in our estimates, finding support for euro rates.

As expected volatilities are derived as averages across 5000 simulations, confidence interval for the average expected realised volatility can be easily built, allowing the identification of extreme movements in the compensation for volatility risk. In this context 'extreme movements' represent those values of the implied volatilities which lie near or outside the confidence interval for the realised volatility referred to the same rate and to the same horizon. Figure 10 shows the 95% confidence interval for the expected volatilities on dollar and euro rates only, together with the observed implied volatilities taken from swaptions, for the 2-year rate at selected maturities (6- and 24-month). For the majority of the dates included in the sample, the implied volatility for both dollar (first row) and euro rates (last row) ranged within the confidence intervals. Put simply, this suggests that investors were not discounting the occurrence of sizeable jumps in the volatility dynamics at the selected horizons. The areas between the vertical bars identified by the '1' symbols in the Figures highlight the period from September 2001 to Spring 2003, when implied volatilities for both dollar and euro rates, at both maturities, stayed at times outside the confidence interval for the forecast realised volatilities or were persistently very close to the upper end of the band.

> [insert Figure 8 about here] [insert Figure 9 about here]

# 3.3 Reality check

As said, the negative of the volatility risk premium moves proportionally to risk aversion and Bollerslev et al. (2007) found the constant of proportionality to be close to one, at least with reference to their stock index options dataset. To establish a link between the negative of the volatility risk premium, as calculated in the present paper, and the coefficient of absolute risk aversion, we can follow Bakshi and Madan (2006) result that

there exists a link between the volatility spread and the skewness  $(\theta_p)$  and kurtosis  $(k_p)$  of the physical data (as evidenced in eq. 2), i.e.:

$$\frac{\sigma_{rn}^{2}(t,\tau) - \sigma_{p}^{2}(t,\tau)}{\sigma_{p}^{2}(t,\tau)} = -\gamma \cdot (\sigma_{p}^{2}(t,\tau))^{0.5} \cdot \theta_{p}(t,\tau) + \frac{\gamma^{2}}{2} (\sigma_{p}^{2}(t,\tau)) \cdot (k_{p}(t,\tau) - 3)$$

To exploit this link we have calculated a series of 20-day logarithmic changes in 1-, 2-, 5- and 10-year swap rates. From such series, daily time series of volatility, skewness and kurtosis have been calculated on moving windows of 20 days, between October 1998 and end-August 2006. For each currency, putting together the volatility spreads (the lhs of the above equation) for the 4 swap rates and 3 swaptions maturities (6-, 12- and 24-month) and using as instruments the first three lags of both the implied volatility and the forecast expected volatility, in addition to a constant (19 instruments as a whole) results in 228 moment conditions from which the risk aversion coefficient is recovered via the generalised method of moments. Using only dollar rates,  $\gamma$  equalled 3.32, with a t-stat of 74.4. The Jtest was  $J_{227} = 216.9$ , with a extremely high p-value. The relative contribution of skewness and kurtosis to compensation for volatility risk can be calculated by simply putting to zero and three, in turn, the time series of the skewness or the kurtosis in equation (2), and then running again the gmm. When  $\theta_p(t,\tau)=0$ , the gmm delivers a risk aversion value of  $\gamma=3.60$  with a t-stat of over 100 while when  $k_p(t,\tau)=3$  we get  $\gamma=28.1$  with a t-stat of 31. In both cases the J-test is rather significant, with p-values of over 50%. As restrictions are valid in both cases, both the conditional third and fourth moments matter for the compensation for volatility risk, in line with what found by Bakshi and Kapadia for the equity market. However, a thing they also found, risk aversion spikes up when the kurtosis is set to three, implying that such a restriction is too stringent or, equivalently, that conditional skewness alone is not enough to characterise volatility risk premiums, while the conditional physical fourth moment could be used almost alone to get a reliable estimate of the risk aversion coefficient. When euro and dollar rates are used together in the gmm, the risk aversion estimate does not show significant changes, reaching a value of  $\gamma = 3.73$  (using both skewness and kurtosis). Adding pound sterling rates produces instead a rise to about  $\gamma = 4.9$ . In both cases  $\gamma$  is significant and restrictions are accepted. Running again the gmm for dollar rates but limiting the sample to end-June 2004, a date after which implied volatilities scaled significantly back compared to their peaks in 2002 and 2003, the coefficient of risk aversion rises to  $\gamma=8.08$ , with a t-stat of 153.8, evidencing that gmm estimation also produces a pattern of time variation in risk aversion similar to what suggested by the vrp series of Figure 5. Last, we report the average value of the negative volatility risk premiums across the same dollar rates and the same swaptions maturities employed in the gmm, which happens to be 4.07. As this value stands slightly over 20 percent higher than the risk aversion obtained through gmm, i.e. 3.32, we are led to estimate the constant of proportionality between risk aversion and compensation for volatility risk to be approximately 0.8 (i.e. a vrp=-5 would correspond to  $\gamma=4$ ), all in all not extremely distant from the unit value reported in Bollerslev et al. (2007). Looking at the sample ending on 30 June 2004, the average value of (minus) the vrp is about 8, which is almost equal to the risk aversion coefficient calculated via gmm (8.08), further supporting the conjecture that the proportionality factor is about one.

The risk aversion figures presented so far are dependent on the statistical model chosen to generate volatility forecasts as well as on the ability of swaptions' volatilities to be an accurate representation of markets' risk neutral expectations. As an additional check that the estimated time series of risk aversion behave in accordance with the typical notion of risk aversion that one has in mind, one can look at their developments in coincidence with the occurrence of events known to have generated distress in financial markets. Figure 11 shows the implied volatilities and the vrp for the three currencies, extracted from 6-month swaptions on the 2-year rate. The vertical bars identify the specific events considered, namely the Russian debt crisis, the beginning of the burst of the technology bubble, the US 2001 recession, the events of 9/11/2001, the period of the US jobless recovery and the deflation scare, the Madrid train bombing, the start of the Fed tightening, the downgrading of Ford and GM debt, the global equity markets drop between May and July 2006.

The dotted lines - corresponding to the three vrp series - evidence that in accordance with one's expectations, risk aversion rose in the aftermath of the Russian crisis and in anticipation of the strong contraction in technology equity prices. After reverting in 2000 it rose again a few months before the beginning of the March 2001 recession, peaking at the end of the slowdown. The sharpest rise, especially for dollar rates, occurred through the deflation scare period. Risk aversion was also historically high around the beginning of the Fed tightening phase between March and June 2004 and is therefore difficult to disentangle the movements related to news about future monetary policy from those orig-

inating, instead, from higher geopolitical risk in the aftermath of the Madrid events of 11 March 2004. However, in contrast with previous events, around the last two episodes and especially in relation to the turbulences in major equity markets between May and July 2006, risk aversion did not record significant changes, although the tensions in the corporate debt market in May 2005 produced minor spikes, more pronounced in implied volatilities than in risk aversion. This may provide indication that either the model chosen to forecast realised volatility does not fully capture movements in expectations or that, as the last two events were more specific to the equity and corporate debt markets, interest rate derivatives do not always provide a full picture of developments in global risk aversion. To shed more light on this issue, exchange rate and stock index options could be employed to broaden the set of derivatives upon which risk aversion estimates rest.

Looking at each currency in turn, the implied volatility and the associated vrp reported in Figure 11 exhibit significant comovement. However, despite what suggested by a simple visual inspection of the graph, correlations among the two variables have not been always high. The rolling correlations reported in Figure 12, based on moving windows of 1 year, evidence periods of significant disconnect between the quantity of risk and the price of risk. On average, correlation tends to be extremely high, almost unity, when implied volatilities are low (see the period before September 2001 and after mid-2006), but in significant portions of the sample, when volatility oscillates around medium-to-high values, periods of high correlation (approximately in the 0.8-0.9 range) are suddenly followed by periods in which correlation remains extremely low, between 0.1 and 0.5. The period between September 2001 and June 2002 is an example of disconnect between the two quantities, with implied volatilities rising and risk aversion remaining broadly stable. From an asset pricing perspective, the existence of such large changes in correlation between the two quantities would suggest that it should not be assumed that additional uncertainty mechanically leads to higher risk premia, as risk aversion, the parameter to which risk premia are related, may remain relatively stable in presence of rapidly moving expectations about future risks.

[insert Figures 10 - 12 about here]

# 4 Determinants of the compensation for volatility risk

As seen, between 1998 and 2006 volatility risk compensation has been sizeable across all three currencies but on average - in absolute terms - higher for dollar rates than for euro and pound rates. Overall, it was been especially in the volatile September 2001 - June 2004 subsample that vrp (always in absolute terms) was much higher for dollar rates and exhibited a remarkable time variation. Given these features, an obvious question is what explains its time variation and its size. According to standard finance theory vrp should be related to the state variables which determine the payoff of the swaption, in the present application the interest rate level and its volatility. From a more empirical standpoint, one may nonetheless suppose that other variables, which could be ultimately seen as conditioning information for these two main factors, are able to affect market perception of future volatility risk. Among these, we consider the slope of the term structure, the slope of the volatility term structure, the corporate spread, macroeconomic variables and macroeconomic surprises.

#### 4.1 Financial variables

To shed light on the determinants of the compensation for volatility risk at a daily level we would ideally regress the premium, measured as the difference between the implied variance and the forecast realised variance for the four interest rates and the five maturities examined so far (the series reported in the Figures 5-7) on the level of the three-month rate, on a measure of interest rate physical volatility, on the slope of the yield curve and of the volatility term structure and on some measures of corporate spread. However, as compensation for volatility risk is borderline stationary, as also the interest rate level and its volatility, we adopt the following specification in first difference, allowing for mean reversion in vrp:

$$\Delta vrp_{t}^{(i,k)} = c + \alpha_{1} \cdot r_{t-1} + \left[\alpha_{2} \cdot \sigma^{RN(i,k)}_{t-1} - \alpha_{3} \cdot \sigma^{P(i,k)}_{t-1}\right] + \alpha_{4} \cdot S_{t-1} + \alpha_{5} \cdot S_{t-1}^{\sigma} + \alpha_{6} \cdot CR_{t-1} + \alpha_{7} \cdot CR_{t-1}^{1} + \sum_{h=1}^{L} \sum_{j=1}^{N} \gamma_{j}^{(h)} \cdot \Delta V_{t-h}^{(j)} + \varepsilon_{t}$$

where  $\sigma_t^{RN(i,k)}$  is the implied swaption volatility for interest rate i and maturity k,  $\sigma_t^{P(i,k)}$  the expected physical volatility (so that  $\sigma_t^{RN(i,k)} - \sigma_t^{P(i,k)}$  denotes the volatility risk pre-

mium for interest rate i and maturity k as of time t),  $r_t$  is the 3-month interest rate,  $S_t$  and  $S_t^{\sigma}$  are the slope of the yield curve and of the volatility term structure,  $CR_t$  is the 10-year swap rate minus the 10-year T-bond rate while  $CR_t^1$  is the baa rate (or equivalent) minus the 10-year T-bond rate,  $\Delta$  is the first difference operator and  $V_t^{(j)}$  denotes a vector which collects the financial variables employed in levels in the regression. For brevity, results are only reported for the premium required on US rates (Table 6) but are representative also of the other two currencies. The vrp is mainly dependent on the expected determinants, i.e. the level of the three-month rate and the level of short-term implied volatility. Given the estimated coefficients of the two variables in the regressions and their means in the sample, the compensation derives almost equally from interest rate and interest rate volatility.

# [insert Table 6 about here]

Looking at the coefficients for the variables in levels, reported in the first seven columns of Table 6, the short rate level has overall a positive effect on the compensation for the 1, 2 and 5-year rates but not for the 10-year rate, although the coefficients are significant especially for the 1- and the 5-year rate. The risk neutral implied has a negative impact on compensation and is decreasing across maturities, i.e. its effect is stronger at short horizons. On the contrary, physical expected volatility has a positive impact and its effect is also more sizeable at short maturities. The coefficient of the physical volatility is on average twice as big as that of the implied volatility, partly compensating the fact that the latter volatility measure is generally higher than the former. Overall the net effect is positive, i.e. high volatility tends to lead to higher compensation for volatility risk. The slope behaves pretty much as the interest rate level (a positive slope typically indicates rising interest rates and therefore increases the compensation for volatility risk). Out of the two measures of corporate spreads, the swap-T-bond rate differential is overall negative and barely significant; the spread between the baa yield and the 10-year T-bond happens to be positive and highly significant for most rates and maturities. Looking at first differences most variables have a negative sign, i.e. yesterday's increases in these variables produce a fall in today's compensation, with the exception of the interest rate level on the 1-year swap rate and of the physical volatility on the 5-year swap rate. Corporate risk measures are instead positively related to future changes in volatility risk premiums. Results for the remaining two currencies are in line with what seen for dollar compensation.

# 4.2 Macroeconomic surprises

Results in the previous subsection show that the compensation for volatility risk in interest rate derivatives is statistically related to the two variables which affect the convenience of buying or selling a swaption, the interest rate level and its volatility. This subsection focuses instead in detail on the reaction of volatility risk premiums and implied volatilities to economic surprises.<sup>13</sup> Looking at the impact of surprises on vrp is important insofar as surprises can be seen as unexpected shocks or innovations derived out of a reduced form model as a traditional Var. Therefore, the reaction of vrp to such shocks is informative about how market participant adjust their views on future interest rates level and volatility after unforeseen developments in macroeconomic variables. Ultimately, this analysis sheds light on the size of the changes in risk aversion induced by economic surprises.

We expect differences in the way surprises affect interest rates, realised and implied volatilities. Previous literature has found that the impact of surprises on interest rates depends on the sign of the surprise and that positive news for the economy (say, a positive surprise to the industrial production index) broadly leads to higher yields. Always according to this literature, realised volatility tends to rise after a shock, quite independently of the sign of the latter, while implied volatility tends to fall, again independently on the sign of the surprises, since uncertainty is resolved after economic announcements are made (Ederington and Lee, 1993; Beber and Brandt, 2006). However, resulting from both the revision to the future path of interest rates and the revision in implied and expected realised volatilities, compensation for volatility risk is not expected to move in any specific direction after the release of macroeconomic data, although one may expect good news for the economy to lower risk aversion and bad news to raise it.

Starting with the reaction of dollar vrp to US economic releases, we find that a small number of surprises significantly affect vrp over the full sample, although the Nonfarm payrolls and the Chicago PMI are the strongest. Table 7 reports the regression of changes in vrp between day t and day t+1 on the Nonfarm payrolls, the Chicago PMI and indus-

<sup>&</sup>lt;sup>13</sup>All surprises are defined as the difference between the actual release of a variable and the corresponding consensus forecast. Such differences are then standardised to allow comparison across types of news. For brevity the list of macroeconomic variables considered is not given in the main text, although Figure 13 displays the variables that influence dollar and euro vrp.

trial production (the full set of significant variables is reported in Figure 13). Regressions are corrected for heteroskedasticity and autocorrelated residuals and are run separately for selected rates and maturities, for the full sample and for three subsamples (October 98 - September 2001; September 2001 - June 2004 and after June 2004). Over the whole sample, vrp is negatively related to nonfarm payrolls, although such a significance comes mainly from the second and especially the third subperiods, both for the 2- and the 5-year rates, across all maturities. We get a similar pattern for the Chicago PMI although in this case the link between surprises and changes in vrp seems to be significant in the second subsample only for the 2-year rate and in the third one for the 5-year rate. By contrast, the industrial production surprise had the strongest influence on vrp before September 2001, while its effect was lower between September 2001 and June 2004 and nearly negligible in the last subperiod. This finding would suggest that while 'hard data' concerning industrial production influenced the perception of risk between 1998 and 2001, labour market concerns and survey data on activity became more relevant after the recession and throughout the deflation scare of 2003 and the deceleration in activity in 2006.

# [insert Table 7 about here]

Looking at the figures in Table 7, after September 2001 a 2 standard deviation surprise in nonfarm payrolls reduced vrp on average by about 250 basis points and 160 basis points at the 3- and 12-month horizons, respectively, for the 2-year rate; for the 5-year rate the comparable figures are about 90 and 50 points, at the same forecast horizons. For the Chicago PMI the size of the impact of a 2 standard deviation surprise on volatility risk premiums was on average only one fourth of that.

In assessing these results it has to be acknowledged that the regressions in Table 7 rest on a few data points and possibly suffer from the presence of noise in economic releases. A more accurate estimate of the intensity of the relation between economic surprises and vrp can be obtained by adopting a methodology developed in Rigobon and Sack (2006). In this framework, financial variables are allowed to react too much to economic surprises simply because the latter convey to market participants more information than what simply related to the release 'per se'. It is easy to see this by considering that the nonfarm payrolls bring information on the most recent change in payrolls but also contains the revisions to previously released payrolls. The overreaction can be isolated

by looking simultaneously at how a number of financial variables move in a given time interval around the economic release, under the hypothesis that the surprise at hand is the only information item that may have influenced the financial variables in the interval analysed.

Supposing to have observed  $N_A$  changes in vrp on day t, when a given news item was released (in our case we observe 20 contemporaneous changes in vrp, obtained from 4 interest rates and 5 maturities) we calculate the principal components of those  $N_A$ changes. Typically the first component is able to explain far more than 90% of the overall variability of the 20 individual vrp series. This first component can be seen as the true effect of the surprise on the vrp and, as said before, will represent the effect of the economic surprise and other related information that such surprise conveys, released simultaneously or within a short period of time after the release itself. However, as we are interested in the effect played by the economic surprise itself we use a second step to regress 'back' such first component on the macroeconomic surprise. At this point the 'clean' reaction of the individual vrp to the economic surprise is recovered by multiplying the slope coefficient in this regression<sup>14</sup> by, in turn, each element of the eigenvector which defines the principal component. This different way of calculating the impact of surprises on vrp will also allow to evidence if variables whose effect was negligible in the ols estimation, turn out to be significant, and viceversa. Figure 13, top panel, shows for the 2- and the 5-year dollar rates at the 6-, 12- and 24-month horizon the results of applying this alternative methodology over the full sample and in the post September 2001 period only. The analysis confirms the existence of a much stronger influence of surprises in the post-01 sample, with nonfarm payrolls and Chicago PMI playing the strongest effect. In the post-2001 period a 1 standard deviation surprise in the nonfarm payrolls reduces the vrp by about 150 basis points (hence it reduces risk aversion by 120 points) for the 2-year rate and by 125 points for the 5-year rate (hence risk aversion falls by 100 basis points). Signs seem to be in accordance with expectation, with the noticeable exception of the CPI, as a rise in consumer price inflation is found to decrease risk aversion, with a much marked effect on the post 2001 sample with reference for the 5-year dollar swap rate.

The lower two panels of the Figure shows instead the reaction of dollar vrp to 1 standard

<sup>&</sup>lt;sup>14</sup>This coefficint typically happened to be high but lower than unity, indicating the existence of a lower association between the economic surprises and changes in risk aversion - as captured by the principal component analysis - than can be gathered from individual regressions based on individual vrp series.

deviation surprise in the euro area macroeconomic variables. Only the 12 surprises that yielded a significant change (5 from Germany, 5 from Italy, 2 from France) have been reported in the Figure. Their effect is overall much more limited than found for the nonfarm payrolls or the Chicago PMI and on average -20 basis points for the release of the Italian ppi (on the 2-year rate vrp) and about +25 basis points for the release of news on the German trade balance (histograms in the Figure are stacked). When judging the effect of euro area news on dollar vrp it is more complex to assess the plausibility of the sign, as the overall impact depends on the movement of the euro and dollar pricing kernels.

Rather than looking at the average reaction of vrp to economic surprises, the top panel of Figure 14 evidences the time series of the US nonfarm payrolls surprise together with the corresponding changes in implied volatility and the vrp of the 2-year rate at the 6-month horizon. The two charts in the upper panel show the changes in implied and vrp that followed, respectively, the release of positive surprises and of negative surprises. As was evidenced by the regressions analysis, it is straightforward to see that positive surprises to nonfarm payrolls reduce risk aversion while the opposite occurs for negative surprises. However the Figures also confirms that the reaction in both implied and vrp to surprises has been much stronger in 2002-2004 than in other years, with the negative slope of the regression deriving almost entirely from this period while in the remainder of the sample surprises appear to induce small positive or negative changes in vrp rather irrespective of their sign. The bottom panel of Figure 14 shows the absolute values of the changes reported in the two charts above and evidences more clearly that before 2002 surprises were overall quite sizeable but the change in implieds and vrp that they induced was very limited. Between 2002 and 2004, however, susrprises have been smaller than in previous years but the reaction of market participants has been extremely higher than before, as highlighted by the huge increase in the black area (which represents the changes in vrp after the occurrence of surprises) in these three years.

# [insert Figures 13 - 14 about here]

Coming to other currencies, the compensation requested on euro rates is affected by a small number of european economic surprises. Among these, the UK manufacturing wages (5-year rate, positive sign, post-2001), German trade balance (5-year rate, positive sign, post-2001), Italian Ppi (2-year rate, positive sign), Italian Ip index (2-year rate, negative

sign), French Cpi (5-year rate, positive sign), euro area IP index (2-year rate, pre-2001, 5-year rate, post-2001, always negative sign). Overall the effect is rather limited, always well below 1 percentage point in absolute value. Some US surprises also affect compensation for volatility risk on euro rates. Among these the IP index (2-year rate, post-2001 and 5-year rate, pre-2001, always negative sign), the nonfarm payrolls (2-year and 5-year rate, post-2001, negative sign), the ism index (all rates, negative sign). The impact of US variables on euro vrp is similar to the impact of European news on euro vrp, with the exception of the nonfarm payrolls, whose 1 standard deviation positive release decreases vrp by about 100 basis points for the 5-year rate, although its effect is rather limited (20 basis points) for the 2-year rate.

# 5 Conclusions

Starting from levels similar to those of euro and pound sterling rates, implied volatilities of dollar rates rose significantly between September 2001 and June 2004. We have investigated whether rising dollar implied volatilities were reflecting higher expected volatilities or increased compensation demanded for bearing volatility risk. To this aim we have compared implied swaptions volatilities to forecasts of realised volatilities, the latter simulated out of a conditional variance model, for various swap rates and swaptions maturities. Results clearly evidence that volatility risk was perceived to be extremely high for all currencies between September 2001 and mid-2003, when implied volatilities were persistently near or above the upper side of the simulated confidence interval for the expected realised volatility. Over such period rates embodied a sizeable - negative - compensation for volatility risk, significantly larger - in absolute terms - for dollar rates than for the remaining currencies. Since June 2004 this component has gradually receded and at present the compensation for volatility risk appears to be small and smilar in size across currencies. On average compensation for volatility risk was 5 percentage points across all currencies, a figure which is in line with estimates of risk aversion derived from other types of econometric estimates.

Compensation for volatility risk is mainly related to the interest rate level as well to its physical and risk neutral volatility. Other variables, as the slope of the term structure, the slope of the volatility term structure and indicators of corporate probability of default also affect the compensation. Among economic announcements, positive surprises for three US variables (the nonfarm payrolls, the Chicago PMI and industrial production) lead to a significant decrease in compensation for volatility risk on dollar rates. They also affect compensation on euro and pound rates, but to a much lower extent. Surprises coming from euro area and UK economic variables also affect the compensation for volatility risk demanded on dollar rates.

# References

- [1] Andersen, T., T. Bollerslev and F. X. Diebold, 2002, Parametric and Nonparametric Volatility Measurement, in Ait-Sahalia, Y. and L. P. Hansen (eds.), Handbook of Financial Econometrics, North Holland, forthcoming.
- [2] Andersen, T., T. Bollerslev, F. Diebold and P. Labys, 2003, Modeling and Forecasting Realised Volatility, Econometrica, 71, 579-625.
- [3] Bakshi, G. S., C. Cao and Z. Chen, 2000, Do Call Prices and the Underlying Stock Always Move in the Same Direction?, Review of Financial Studies, 13, 549-84.
- [4] Bakshi, G S and N Kapadia, 2003, Delta-hedged gains and the negative market volatility risk premium, Review of Financial Studies, 16, 527-66.
- [5] Bakshi, G. S. and D. Madan, 2006, A Theory of Volatility Spreads, Management Science, 52, 1945-56.
- [6] Bates, D., 2000, Post-87 Crash Fears in the S&P500 Futures Options Market, Journal of Econometrics, 94, 181-238.
- [7] Beber, A. and M. Brandt, 2006, Resolving Macroeconomic Uncertainty in Stock and Bond Markets, NBER, Working Paper 12270.
- [8] Benzoni, L., 2002, Pricing Options under Stochastic Volatility: An Empirical Investigation, Working Paper, University of Minnesota.
- [9] Bollerslev, T, M Gibson and H Zhou, 2007, Dynamic estimation of volatility risk premia and investor risk aversion from option-implied and realized volatilities, Duke University, mimeo.

- [10] Branger, N. and C. Schlag, 2003, Is Volatility Risk Priced? Properties of Tests Based on Option Hedging Errors, Goethe University, Department of Finance, Working Paper No. 136.
- [11] Britten-Jones, M. and A. Neuberger, 2000, Option Prices, Implied Price Processes and Stochastic Volatility, Journal of Finance, 55, 839-866.
- [12] Buraschi, A. and J. Jackwerth, 2001, The Price of A Smile: Hedging and Spanning in Options Markets, Review of Financial Studies, 14, 495-527.
- [13] Carr, P. and L. Wu, 2007, Variance Risk Premia, Review of Financial Studies, forthcoming.
- [14] Chernov, M., 2003, On the role of risk premia in volatility forecasting, Journal of Business and Economic Statistics, forthcoming.
- [15] Chernov, M. and E. Ghysels, 2000, A Study Towards A Unified Approach to the Joint Estimation of Objective and Risk Neutral Measures for the Purpose of Option Valuation, Journal of Financial Economics, 56, 407-58.
- [16] Coval, J. and T. Shumway, 2001, Expected Options Returns, Journal of Finance, 56, 983-1009.
- [17] Ding, Z., R. Engle and C. Granger, 1993, A Long Memory Property of Stock Market Returns and A New Model, Journal of Empirical Finance, 1, 83-106
- [18] Drost, F. and B. Werker, 1995, Temporal Aggregation of Garch Processes, in Engle, R. F. (ed.), ARCH: Selected Readings, Oxford University Press.
- [19] Ederington, L H and J H Lee, 2003, The creation and resolution of market uncertainty: The impact of information releases on implied volatility, Journal of Financial and Quantitative Analysis, 31, 513-
- [20] Engle, R. and G. G. L. Lee, 1999, A Long-Run and Short-Run Component Model of Stock Return Volatility, in Cointegration, Causality and Forecasting, by R. F. Engle and H. White (eds.), Oxford University Press.
- [21] Eraker, B., 2004, Do stock prices and volatility jump? Reconciling the evidence from Spot and Option Prices, Journal of Finance, 59, 1367-1403.

- [22] E. F. Fama and K. R. French, 1993, The Cross Section of Expected Stock Returns, Journal of Finance, 47.
- [23] Fornari, F. and A. Mele, 1997, Weak Convergence and Distributional Assumptions for a General Class of Nonlinear Garch Models, Econometric Reviews, 16, 205-27.
- [24] Fornari, F. and A. Mele, 2001, Recovering the Probability Density Function of Asset Prices using Garch Models as Diffusion Approximations, Journal of Empirical Finance, 8, 83-110.
- [25] Guo, D., 1998, The Risk Premium of Volatility Implicit in Currency Options, Journal of Business and Economic Statisitics, 16, 498-507.
- [26] Guo, H. and C. J. Neely, 2004, Is Foreign Exchange Delta Hedging Risk Priced?, Federeal Reserve Bank of St. Louis, WP 2004-029A
- [27] Heston, S, 1993, Closed form solution for options with stochastic volatility, with applications to bond and currency options, Review of Financial Studies, 6, 327-43.
- [28] Jiang, G. and T. Tian, 2005, Model-Free implied volatility and its information content, Review of Financial Studies, 18, 1305-1342.
- [29] Jones, C. S., 2003, The dynamics of stochastic volatility: Evidence from underlying and options markets, Journal of Econometrics, 116, 181-224.
- [30] Pan, J., 2002, The jump-risk premia implicit in options: Evidence from an integrated time series study, Journal of Financial Economics, 63, 3-50.
- [31] Rigobon, R. and B. Sack, 2006, Noisy Macroeconomic Announcements, Monetary Policy, and Asset Prices, NBER Working Paper 12420.

Table 1: Regression of (monthly) delta-hedged gains on dollar and euro swaptions on lagged gains and lagged log-volatility

The state of the s											
	United Stat	es			euro						
	c	$dhg_{t-1}$	$\log(\sigma_{t-1}^2)$	$R^2$	c	$dhg_{t-1}$	$\log(\sigma_{t-1}^2)$	$R^2$			
$1m, 1-year\ rate$	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	$\underset{(0.24)}{0.033}$	$-6.98 \cdot 10^{-4}$ $(-3.30)$	0.09	$1.75 \cdot 10^{-3}_{(0.28)}$	$0.223 \ (2.49)$	$-4.77 \cdot 10^{-4}$ $(-1.25)$	0.05			
$1m, 2-year\ rate$	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	$\underset{(1.22)}{0.121}$	$-1.12 \cdot 10^{-4}$ $(-0.53)$	0.00	$4.51 \cdot 10^{-3} \\ {}_{(0.64)}$	$0.062 \\ (0.82)$	$-4.16 \cdot 10^{-4}$ $(-1.08)$	0.00			
$1m,5-year\ rate$	$\begin{array}{ c c c c c } 1.80 \cdot 10^{-2} \\ & (2.46) \end{array}$	$0.029 \ (0.22)$	$-1.72 \cdot 10^{-3}$ $(-3.88)$	0.19	$5.71 \cdot 10^{-3} $ $_{(0.64)}$	$0.211 \atop (2.42)$	$-1.11 \cdot 10^{-3}$ $(-1.98)$	0.11			
$3m, 1-year\ rate$	$3.19 \cdot 10^{-2}$ $(3.30)$	$0.633 \ (7.69)$	$-1.97 \cdot 10^{-3}$ $(-3.51)$	0.68	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	$0.762 \\ (11.70)$	$-2.60 \cdot 10^{-4}$ $(-0.36)$	0.58			
$3m, 2-year\ rate$	$1.76 \cdot 10^{-2} $ (2.36)	$0.540 \\ (5.60)$	$-1.55 \cdot 10^{-3}$ $(-3.67)$	0.58	$3.20 \cdot 10^{-3} \\ {}_{(0.31)}$	$0.691 \\ (9.69)$	$-6.83 \cdot 10^{-4}$ $(-1.14)$	0.51			
$3m,5-year\ rate$	$\begin{array}{ c c c c c }\hline 2.45 \cdot 10^{-2} \\ (2.89) \\ \hline \end{array}$	$0.228 \ (1.82)$	$-1.84 \cdot 10^{-3}$ $(-4.17)$	0.29	$1.05 \cdot 10^{-2} \\ _{(1.28)}$	$0.523 \ (7.14)$	$-9.59 \cdot 10^{-4}$ $(-2.13)$	0.34			
$6m, 1-year\ rate$	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	$0.799 \atop (10.25)$	$-1.06 \cdot 10^{-3}$ $(-1.35)$	0.78	$1.92 \cdot 10^{-2} \atop (1.51)$	0.724 (11.3)	$-1.29 \cdot 10^{-3}$ $(-1.52)$	0.59			
$6m, 2-year\ rate$	$\begin{array}{ c c c c c } 1.12 \cdot 10^{-2} \\ & (1.16) \end{array}$	$\underset{(10.25)}{0.764}$	$-9.32 \cdot 10^{-4}$ $(-1.56)$	0.73	$9.58 \cdot 10^{-3} \\ {}_{(0.72)}$	$0.739 \atop (11.9)$	$-7.66 \cdot 10^{-4}$ $(-1.07)$	0.59			
$6m, 5-year\ rate$	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	$0.770 \\ (11.54)$	$-7.17 \cdot 10^{-4}$ $(-1.10)$	0.68	$\begin{array}{ c c c c c c }\hline 1.36 \cdot 10^{-2} \\ & (1.11) \end{array}$	$0.695 \\ (10.54)$	$-9.47 \cdot 10^{-4}$ $(-1.34)$	0.54			
$12m, 1-year\ rate$	$2.98 \cdot 10^{-3} $ (1.88)	$0.910 \\ (24.23)$	$-1.53 \cdot 10^{-3}$ $(-1.87)$	0.92	$\begin{array}{ c c c c c }\hline 1.49 \cdot 10^{-3} \\ {}_{(0.09)} \end{array}$	0.897 $(18.57)$	$-2.16 \cdot 10^{-4}$ $(-0.19)$	0.81			
$12m, 2-year\ rate$	$\begin{array}{ c c c c c }\hline 4.28 \cdot 10^{-2} \\ & (2.36) \\ \hline \end{array}$	$0.818 \ (12.76)$	$-2.07 \cdot 10^{-3} $ (-2.18)	0.89	$1.01 \cdot 10^{-2} \atop (0.59)$	0.852 $(13.91)$	$-6.32 \cdot 10^{-4}$ $(-0.66)$	0.76			
$12m,5-year\ rate$	$\begin{array}{ c c c c c }\hline 4.39 \cdot 10^{-2} \\ & (2.55) \\ \hline \end{array}$	$\underset{(8.94)}{0.736}$	$-2.35 \cdot 10^{-3} \\ (-2.44)$	0.79	$2.21 \cdot 10^{-3}_{(0.15)}$	0.829 $(12.62)$	$-1.12 \cdot 10^{-4} \\ _{(-0.14)}$	0.70			

Notes: The Table reports the results of a regression of the delta-hedged gains on US and euro swaptions on three rates and four maturities on a constant (c) their first lag  $(dhg_{t-1})$  and on the first lag of the conditional garch log-volatility of the corresponding rate  $(log(\sigma_{t-1}^2))$ . T-stats are reported in brackets. All standard errors have been corrected with Newey-West covariance matrix, with 6 lags. Residuals exhibited no significant autocorrelation. The regression is run at a monthly frequency between October 1998 and September 2006. The data on delta-hedged gains, originally available at a daily frequency, have been re-sampled as of end-month.

Table 2: Percentage of days that the vrp is negative, US, euro and pound rates, across maturities

	1-year	2-year	5-year	10-year
dollar rates				
1-month	68.2	79.7	79.8	87.9
3-month	64.4	87.9	87.1	90.7
6-month	66.5	88.9	90.1	91.8
12-month	79.6	88.9	91.4	92.4
24-month	80.1	86.1	90.2	92.8
euro rates				
1-month	93.2	92.7	87.6	77.0
3-month	94.2	93.9	84.6	77.5
6-month	96.4	93.8	80.5	76.9
12-month	97.3	93.0	72.2	73.5
24-month	97.6	90.9	47.9	60.2
pound rates				
1-month	84.3	94.3	91.9	65.2
3-month	85.2	95.8	85.3	71.6
6-month	89.6	92.2	86.6	75.9
12-month	97.2	97.6	89.5	78.6
24-month	97.9	97.9	96.5	92.6

Notes: The Table reports the ratio, as a percentage, between the numer of days in which the volatility risk premium is negative, and the total number of days in the analysed sample.

Table 3: Mean and standard deviations of vrp; dollar, euro and pound rates

	1-year	2-year	5-year	10-year
dollar rates				
1-month	4.46 (8.8)	2.36 (5.5)	2.60 (5.2)	4.19 (3.5)
3-month	5.11 (8.3)	6.39(6.7)	4.37(4.4)	4.95(4.0)
6-month	6.78 (9.4)	6.78(7.0)	5.10(4.7)	5.12(4.1)
12-month	6.79 (8.8)	6.24(6.2)	4.84(4.3)	4.74(3.6)
24-month	4.10 (5.2)	4.42(4.3)	3.76(3.3)	3.85(2.8)
euro rates				
1-month	6.42 (4.8)	4.62 (3.6)	4.24 (3.3)	1.65 (2.1)
3-month	6.92 (4.8)	5.25(3.7)	3.66(2.9)	1.72(1.9)
6-month	7.72 (4.9)	5.29(3.4)	2.79(2.5)	1.54(1.8)
12-month	7.76 (4.5)	4.62(2.9)	1.62(2.0)	1.15(1.6)
24-month	5.96 (2.9)	2.92(2.1)	0.09(1.5)	0.45(1.5)
pound rates				
1-month	4.50 (4.5)	4.52 (3.3)	2.95 (2.5)	1.45 (2.6)
3-month	4.29 (4.1)	5.05(3.4)	2.99(2.8)	1.72(2.5)
6-month	4.73 (3.9)	5.17(3.8)	3.03(2.9)	1.74(2.3)
12-month	5.04 (3.3)	5.60(3.1)	3.05(2.6)	1.65(2.1)
24-month	5.05(2.8)	5.61(3.0)	2.92(2.1)	3.96(2.2)

Notes: The Table reports the mean and in parentheses the standard deviation of the compensation for volatility risk for the three currencies, for the four interest rates and the five maturities analysed. The statistics are referred to daily data observed between October 1998 and August 2006.

Table 4: Correlation matrix of dollar and euro compensation for volatility risk

	3m1y_eu	3m2y_eu	3m5y_eu	$24 \mathrm{m1y}$ eu	24m2yeu	24m5y_eu
$3m1y \longrightarrow$	0.62	0.53	0.25	0.27	0.07	-0.06
$3\text{m2y} \longrightarrow$	0.68	0.67	0.24	0.27	0.19	0.00
$3m5y \longrightarrow$	0.52	0.51	0.38	0.36	0.26	0.20
$24 \text{m1y} \longrightarrow$	0.63	0.60	0.35	0.56	0.44	0.18
$24\text{m2y} \longrightarrow$	0.59	0.61	0.19	0.37	0.45	0.14
$24 \text{m5y} \longrightarrow$	0.66	0.63	0.27	0.41	0.40	0.19

Notes: The Table reports the coefficients of correlation between volatility risk compensation on a given rate and over a given horizon in one country with corresponding volatility risk compensation in another country. The first column identifies dollar compensations while the remaining columns refer to euro (\_eu) rates. The statisitics are referred to daily data observed between October 1998 and August 2006.

Table 5: Correlation matrix of dollar and pound compensation for volatility risk

	3m1v uk	3m2v uk	3m5v uk	24m1v uk	24m2y uk	24m5v 11k
0 1	· – –					
$3m1y \longrightarrow$	0.57	0.48	0.34	0.16	0.45	0.40
$3\text{m2y}\longrightarrow$	0.57	0.31	0.45	0.12	0.27	0.33
$3m5y \longrightarrow$	0.46	0.25	0.04	0.26	0.22	0.11
$24\text{m1y}\longrightarrow$	0.58	0.42	0.31	0.44	0.20	0.20
$24\text{m}2\text{y}\longrightarrow$	0.45	0.23	0.37	0.20	-0.03	0.05
$24 \text{m5y} \longrightarrow$	0.52	0.39	0.41	0.24	0.16	0.15

Notes: The Table reports the coefficients of correlation between volatility risk compensation on a given rate and over a given horizon in one country with corresponding volatility risk compensation in another country. The first column identifies dollar compensations while the remaining columns refer to pound (\_uk) rates. The statistics are referred to daily data observed between October 1998 and August 2006.

Table 6: Regression of US vrp referred to various rates and various swaptions' maturities on financial variables

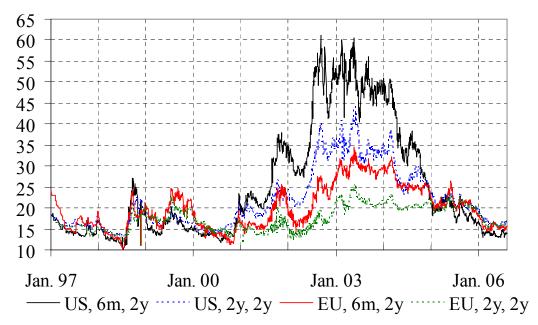
	$\alpha_1$ $(r)$	$\begin{pmatrix} \alpha_2 \\ (\sigma^{rn}) \end{pmatrix}$	$\alpha_3 \atop (\sigma^P)$	$\alpha_4$ $(S)$	$\alpha_5 \ (S^{\sigma})$	$\alpha_6$ (CR)	$\alpha_7$ $(CR^1)$	$\gamma_1 \atop (\Delta r)$	$\begin{array}{c} \gamma_2 \\ (\Delta\sigma^{rn}) \end{array}$	$\gamma_3 \ (\Delta \sigma^P)$	$\gamma_4 \atop (\Delta S)$	$\gamma_6 \ (\Delta CR)$	$R^2$
1m,1y	$0.10 \\ (0.55)$	-0.09 $(-4.00)$	$0.07 \\ (4.58)$	$0.10 \\ (0.70)$	-0.04 $(-1.28)$	-0.73 $(-1.09)$	0.45 $(1.98)$	55.07 (2.92)	-0.20 $(-3.41)$	-0.07 $(-3.03)$	$0.26 \atop (0.15)$	$0.95 \\ (0.34)$	0.09
3m,1y	$\underset{(1.72)}{0.25}$	$-0.03 \ (-1.28)$	$\underset{(4.27)}{0.07}$	$\underset{(2.54)}{0.25}$	$0.04 \\ (1.49)$	$-0.93 \ (-1.85)$	$0.54 \\ (2.84)$	39.44 $(2.99)$	$-0.16 \ (-1.26)$	$-0.05 \ (-2.00)$	$-0.71 \ (-0.51)$	$\underset{(1.46)}{3.34}$	0.11
6m,1y	$0.19 \\ (1.65)$	$-0.03 \ (-2.33)$	$0.07 \\ (3.90)$	$0.19 \ (2.15)$	$0.02 \\ (1.46)$	$-0.67 \\ (-1.84)$	$0.55 \\ (4.24)$	$27.44 \ (2.27)$	$-0.28 \ (-4.98)$	$-0.03 \\ (-0.83)$	$-2.51 \ (-2.04)$	$4.49 \ (2.33)$	0.10
$^{12m,1y}$	$0.19 \ (2.10)$	-0.01 $(-1.79)$	$0.06 \\ (3.44)$	0.17 $(2.40)$	$\underset{(2.70)}{0.02}$	-0.54 $(-2.01)$	$0.48 \\ (4.84)$	$\underset{(1.36)}{10.22}$	-0.36 $(-5.13)$	-0.08 $(-2.16)$	-4.17 $(-4.60)$	$4.61 \\ (3.20)$	0.13
24m,1y	$0.08 \\ (0.99)$	-0.01 $(-2.16)$	$0.05 \\ (2.83)$	0.04 $(0.55)$	$0.01 \\ (1.16)$	-0.08 $(-0.38)$	$0.26 \ (2.98)$	0.01 $(0.00)$	-0.46 $(-5.44)$	$-0.05 \ (-1.03)$	-3.41 $(-5.75)$	$\frac{2.86}{(3.02)}$	0.16
1m,2y	$-0.05 \ (-0.39)$	$-0.08 \ (-3.51)$	$0.07 \\ (5.19)$	$-0.12 \ (-0.97)$	$0.01 \\ (0.54)$	$-0.22 \ (-0.45)$	0.17 $(0.90)$	$-19.30 \atop (-1.11)$	$-0.16 \ (-1.36)$	$-0.06 \ (-2.10)$	-3.79 $(-2.30)$	$8.25 \\ (3.00)$	0.07
3m,2y	$0.10 \\ (0.98)$	-0.09 $(-4.89)$	$0.10 \\ (6.27)$	$0.14 \\ (1.38)$	$-0.00 \\ (-0.16)$	-0.80 $(-1.92)$	$0.78 \\ (3.97)$	-9.52 $(-0.59)$	-0.23 $(-3.27)$	$-0.03 \atop (-1.14)$	$-2.49 \ (-1.55)$	$\underset{(2.92)}{6.38}$	0.09
6m,2y	$0.10 \\ (0.99)$	-0.08 $(-4.53)$	$0.10 \\ (5.76)$	$\underset{(0.76)}{0.07}$	$-0.00 \\ (-0.06)$	$-0.70 \\ (-1.83)$	$0.79 \\ (4.44)$	$-12.65 \ (-0.98)$	-0.18 $(-3.24)$	$-0.05 \ (-1.54)$	$-2.95 \ (-2.38)$	$6.28 \ (3.80)$	0.08
12m,2y	$0.10 \\ (1.19)$	-0.05 $(-3.98)$	$0.08 \\ (4.93)$	$0.05 \\ (0.59)$	$0.00 \\ (0.49)$	-0.44 $(-1.50)$	$0.58 \\ (4.24)$	-17.32 $(-2.05)$	-0.17 $(-2.96)$	-0.03 $(-0.85)$	-3.78 $(-4.17)$	$5.71 \\ (4.31)$	0.09
24m,2y	$0.08 \\ (1.20)$	-0.04 $(-3.87)$	$0.06 \\ (4.42)$	$0.03 \\ (0.54)$	$0.00 \\ (0.65)$	-0.19 $(-0.90)$	$0.38 \\ (3.58)$	$-12.90 \ (-2.44)$	-0.33 $(-4.44)$	-0.03 $(-0.75)$	$-3.05 \ (-5.26)$	$\frac{3.38}{(4.42)}$	0.12
1m,5y	$\underset{(0.56)}{0.07}$	$-0.06 \\ (-3.44)$	$0.10 \\ (6.12)$	$0.05 \\ (0.49)$	$0.04 \\ (2.96)$	$-0.38 \ (-0.77)$	$0.14 \atop (0.64)$	$15.46 \atop (1.11)$	$-0.12 \\ (-1.41)$	$0.18 \ (5.24)$	$-1.28 \\ (-0.85)$	$0.73 \\ (0.36)$	0.09
3m,5y	$0.13 \\ (1.56)$	$-0.06 \ (-3.31)$	$0.11 \\ (5.90)$	$0.09 \\ (1.28)$	$0.03 \\ (2.97)$	$-0.64 \ (-2.01)$	$0.44 \ (2.67)$	$6.15 \\ (0.63)$	-0.17 $(-1.99)$	$0.24 \atop (6.26)$	$-1.80 \\ (-1.68)$	$\frac{2.03}{(1.59)}$	0.12
6m,5y	$0.13 \\ (1.97)$	$-0.05 \\ (-3.48)$	$0.10 \\ (5.67)$	0.09 $(1.57)$	$0.02 \ (2.72)$	-0.57 $(-2.26)$	$0.52 \\ (3.91)$	-0.36 $(-0.05)$	-0.21 $(-2.62)$	$0.28 \atop (6.77)$	-2.52 $(-3.15)$	$\frac{2.99}{(3.02)}$	0.14
12m,5y	0.11 (1.84)	-0.04 $(-3.49)$	$0.09 \\ (5.26)$	0.08 $(1.50)$	$0.01 \\ (2.32)$	-0.32 $(-1.68)$	0.43 $(4.62)$	-5.75 $(-1.33)$	-0.26 $(-3.82)$	$0.29 \atop (6.91)$	-3.04 $(-4.97)$	$\frac{2.78}{(3.89)}$	0.15
$^{24m,5y}$	$0.05 \\ (0.89)$	-0.03 $(-3.61)$	$0.06 \\ (4.34)$	0.04 $(0.89)$	$0.01 \\ (1.98)$	-0.07 $(-0.54)$	0.27 $(4.19)$	-6.64 $(-2.15)$	-0.23 $(-4.36)$	$0.32 \\ (6.91)$	-2.53 $(-6.48)$	$\frac{2.19}{(4.52)}$	0.16
1m,10y	-0.11 $(-1.43)$	-0.12 $(-6.99)$	$0.12 \\ (8.55)$	-0.02 $(-0.35)$	$0.00 \\ (0.74)$	-0.07 $(-0.26)$	0.43 $(4.09)$	-1.55 $(-0.12)$	-0.07 $(-1.75)$	-0.09 $(-2.25)$	-0.88 $(-0.80)$	$0.53 \\ (0.46)$	0.08
3m,10y	-0.01 $(-0.24)$	-0.07 $(-6.32)$	0.10 (5.89)	$0.01 \atop (0.24)$	0.00 (0.42)	-0.14 $(-0.78)$	0.36 (4.89)	-3.03 $(-0.37)$	-0.10 $(-2.05)$	-0.04 $(-1.14)$	-1.28 $(-1.77)$	0.76 $(0.99)$	0.07
6m,10y	$0.01 \\ (0.15)$	-0.04 $(-5.42)$	$0.08 \\ (4.39)$	0.02 $(0.39)$	0.00 $(0.39)$	-0.08 $(-0.55)$	$0.28 \\ (4.58)$	-6.69 $(-1.24)$	-0.16 $(-3.42)$	-0.04 $(-0.94)$	-1.79 $(-3.69)$	1.78 (3.13)	0.09
12m, 10y	-0.03 $(-0.81)$	-0.03 $(-4.91)$	0.05 (3.48)	-0.02 (0.56)	0.00 (0.31)	0.03 $(0.24)$	0.19 (4.17)	-6.31 $(-1.66)$	-0.20 $(-4.84)$	-0.03 $(-0.57)$	-2.07 $(-5.91)$	1.68 (3.59)	0.12
24m,10y	-0.08 $(-2.41)$	-0.03 $(-4.76)$	0.02 $(1.53)$	-0.06 $(-2.17)$	$0.00 \\ (0.17)$	0.08 (0.88)	0.10 (2.68)	-6.00 $(-2.14)$	-0.19 $(-4.74)$	-0.04 $(-0.56)$	-1.93 $(-7.53)$	1.68 (4.42)	0.14

Notes: The Table reports the result of a regression of the first difference of vrp on a set of determinants considered both in levels (lagged by one period) and in first differences (lagged by one period). The coefficients  $\gamma_5$  which were referred to the slope of the volatility term structure have not been reported, as not significant, with the only aim of saving space. T-ratios are reported in parentheses. Regressions are run from October 1998 to end-August 2006.for daily observations.

	all sam	ple	10/98 -	9/01	9/01 -	6/04	6/04 -	8/06
	c	β	c	β	c	β	c	β
nonfarm payrolls								
rate = 2-year								
3-month	-0.24 $(-1.55)$	-0.63 $(-3.96)$	0.02 $(0.27)$	-0.10 $(-1.07)$	-0.26 $(-0.72)$	-1.14 $(-2.54)$	-0.63 $(-3.47)$	-1.26 $(-5.52)$
12-month	-0.22 $(-2.24)$	-0.41 $(-3.56)$	-0.05 $(-0.76)$	-0.05 $(-0.77)$	-0.30 $(-1.41)$	-0.78 $(-2.45)$	-0.39 $(-2.78)$	-0.78 $(-4.45)$
rate=5-year								
3-month	-0.25 $(-1.97)$	-0.41 $(-2.95)$	0.09 (1.06)	-0.03 $(-0.35)$	-0.56 $(-2.10)$	-0.55 (-1.80)	-0.38 $(-1.76)$	-1.28 $(-4.67)$
12-month	-0.12 $(-1.66)$	-0.24 $(-2.77)$	$0.00 \\ (-0.03)$	-0.02 $(-0.46)$	-0.29 $(-1.79)$	-0.31 $(-1.64)$	-0.11 $(-0.95)$	-0.76 $(-4.41)$
Chicago PMI								
rate = 2-year								
3-month	0.17 $(0.91)$	-0.42 $(-2.46)$	-0.14 $(-1.00)$	-0.26 $(-1.67)$	0.46 $(0.95)$	-1.13 $(-2.04)$	0.30 $(2.22)$	-0.18 $(-2.39)$
12-month	0.10 $(0.77)$	-0.22 $(-2.17)$	-0.05 $(-0.79)$	-0.09 $(-1.23)$	$\underset{(0.72)}{0.23}$	-0.54 $(-1.60)$	0.19 $(-1.76)$	-0.17 $(-2.32)$
rate=5-year								
3-month	0.04 $(0.27)$	-0.12 $(-0.94)$	-0.05 $(-0.50)$	-0.17 $(-1.50)$	0.02 $(0.05)$	0.00 (0.00)	0.17 $(1.46)$	-0.20 $(-2.17)$
12-month	0.06 $(0.81)$	-0.10 $(-1.43)$	-0.01 $(-0.24)$	-0.05 $(-0.98)$	0.12 $(0.57)$	-0.05 $(-0.20)$	0.12 (1.45)	-0.18 $(-2.96)$
IP								
rate = 2-year								
3-month	-0.22 (-1.42)	-0.20 $(-1.42)$	$0.00 \\ (-0.04)$	-0.19 $(-3.02)$	-0.52 $(-1.32)$	-0.47 $(-1.19)$	-0.08 $(-0.43)$	0.05 $(0.57)$
12-month	-0.15 $(-1.45)$	-0.12 $(-1.34)$	0.06 $(0.90)$	-0.14 $(-2.08)$	-0.42 $(-1.69)$	-0.25 (-1.06)	-0.04 $(-0.28)$	0.27 $(0.36)$
rate=5-year								
3-month	-0.21 (-1.33)	-0.22 $(-2.22)$	$0.00 \\ (0.12)$	-0.14 $(-2.76)$	-0.72 $(-1.96)$	-0.40 $(-1.69)$	0.16 $(0.59)$	-0.09 $(-0.61)$
12-month	-0.12 $(-1.36)$	-0.10 $(-1.96)$	0.03 $(0.47)$	-0.09 (-1.68)	-0.35 $(-1.72)$	-0.19 $(-1.63)$	$0.00 \\ (0.02)$	-0.02 $(-0.27)$

Notes: The Table reports the result of regressions of the volatility risk premium on dollar 2- and 5-year rates, at the 3-month and the 12-month maturity on three standardised US surprises, across four time intervals, including the full sample. T-stats are reported in parentheses.

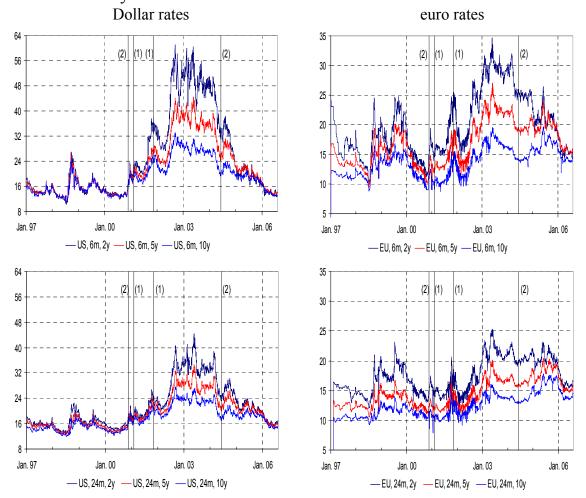
Figure 1: Implied volatilities of US and euro 2-year swap rates at selected horizons (daily data, percentages per year)



Source: Bloomberg.

Notes: The Figure shows daily implied volatilities taken from swaptions on the 2-year dollar and euro swap rates with life to maturity of 6 months and 2 years.

Figure 2: Swaption volatilities of dollar and euro 2-, 5- and 10-year rates at the 6-month and 2-year horizons



Source: Bloomberg.

Notes: The area between the symbols (2) denotes the period of monetary policy easing carried out by the Federal Reserve. The area between the symbols (1) denotes instead the US recession. The implied volatilities are taken from swaptions on dollar and euro 2-, 5- and 10-year rates with life to maturity of 6 months and 2 years. Data are daily and expressed as percentages per year.

Figure 3: Delta-hedged gains for dollar-denominated swaptions at selected horizons

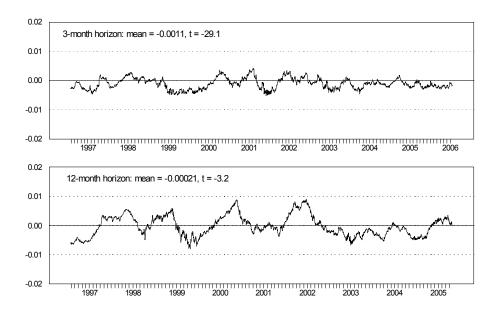
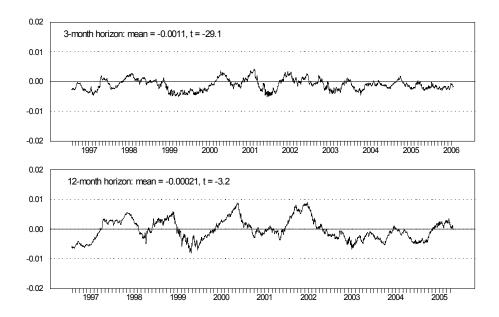
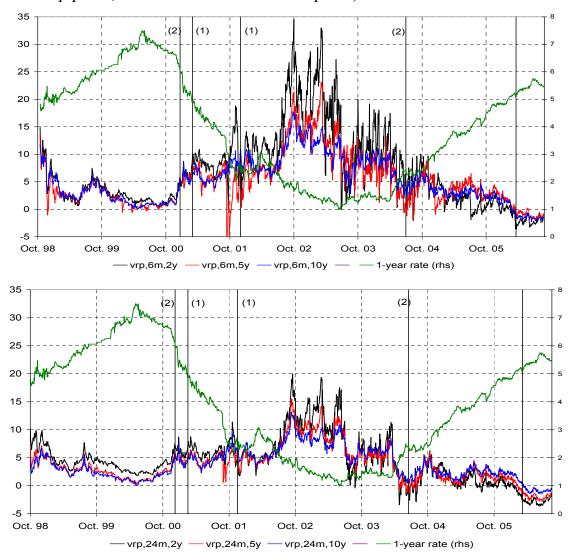


Figure 4: Delta-hedged gains for euro-denominated swaptions at selected horizons



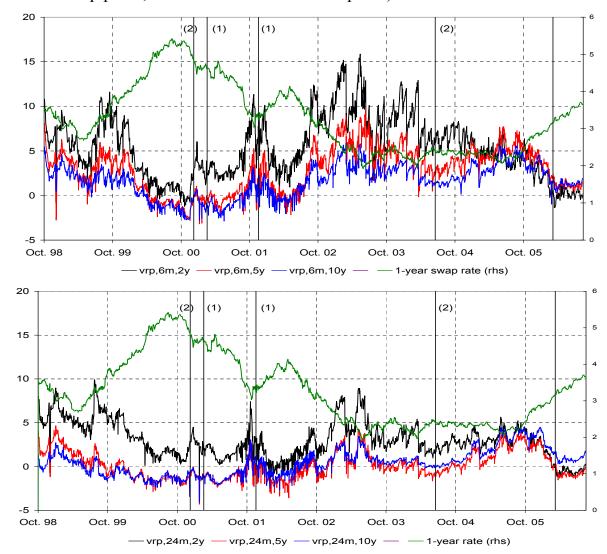
Notes: The two figures show the time series of delta-hedged gains on swaptions written on the dollar and euro 2-year swap rate with time to maturity of 3 and 12 months. In the calculations of delta-hedged gains, swaptions are bought and sold after one day. The delta (derivative of call price with respect to underlying price) used in the construction of the gains is based on Black 1976 model, but the volatility comes from a garch(1,1) estimate.

Figure 5: Volatility risk compensation, dollar 2-, 5- and 10-year swap rates (6-month horizon: top panel; 24-month horizon: bottom panel)



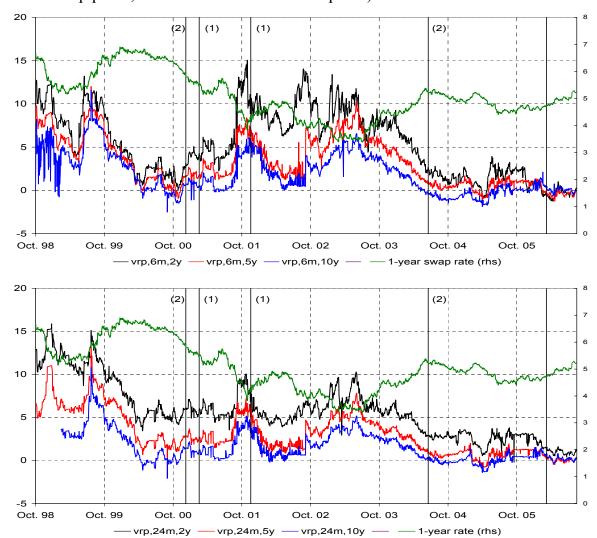
Notes: The Figure shows (minus) the compensation for volatility risk (vrp) calculated as the difference between implied swaptions volatility and the corresponding simulated expected realised volatility. As an example, vrp,6m,2y indicates the compensation required to sell a swaption on the 2-year swap rate, 6 months forward. The 1-year spot rate is also reported. The area between the symbols (2) denotes the period of monetary policy easing carried out by the Federal Reserve. The area between the symbols (1) denotes instead the US recession.

Figure 6: Volatility risk compensation, euro 2-, 5- and 10-year swap rates (6-month horizon: top panel; 24-month horizon: bottom panel)



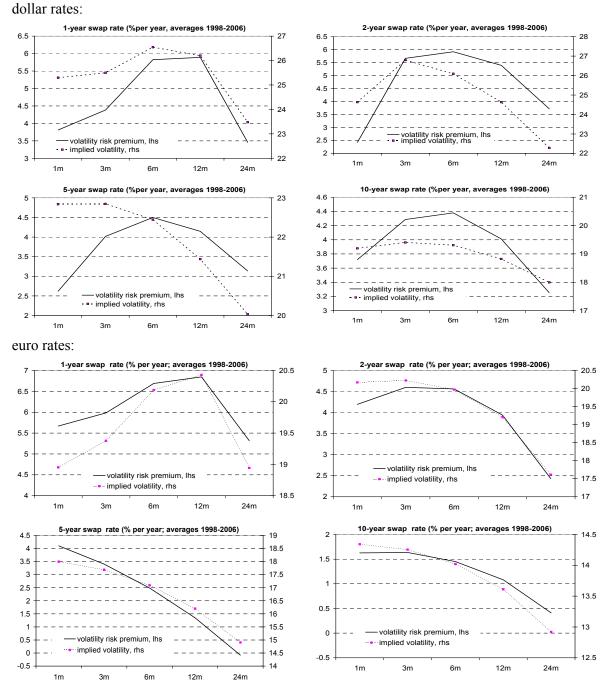
Notes: The Figure shows (minus) the compensation for volatility risk (vrp) calculated as the difference between implied swaptions volatility and the corresponding simulated expected realised volatility. As an example, vrp,6m,2y indicates the compensation required to sell a swaption on the 2-year swap rate, 6 months forward. The 1-year spot rate is also reported. The area between the symbols (2) denotes the period of monetary policy easing carried out by the Federal Reserve. The area between the symbols (1) denotes instead the US recession.

Figure 7: Volatility risk compensation, pound 2-, 5- and 10-year swap rates (6-month horizon: top panel; 24-month horizon: bottom panel)



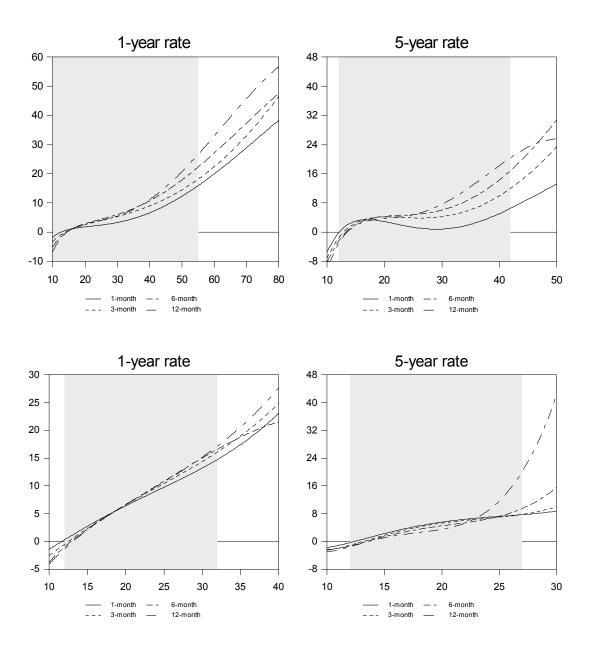
Notes: The Figure shows (minus) the compensation for volatility risk (vrp) calculated as the difference between implied swaptions volatility and the corresponding simulated expected realised volatility. As an example, vrp,6m,2y indicates the compensation required to sell a swaption on the 2-year swap rate, 6 months forward. The 1-year spot rate is also reported. The area between the symbols (2) denotes the period of monetary policy easing carried out by the Federal Reserve. The area between the symbols (1) denotes instead the US recession.

Figure 8: Term structure of the volatility risk premia and the implied volatilities of dollar and euro interest rates



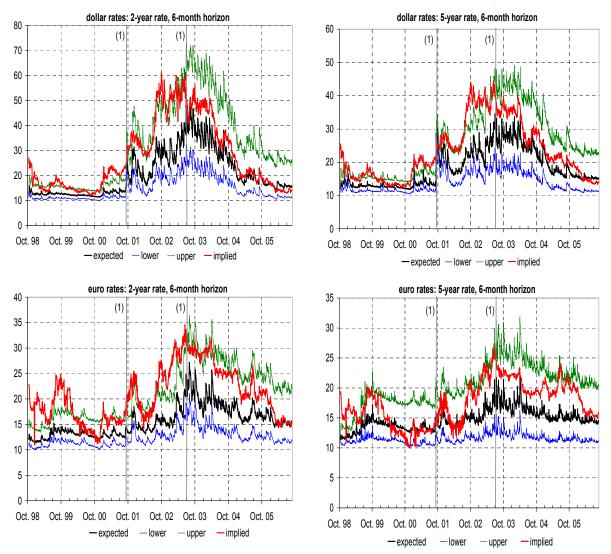
Notes: The two panels of the figure report – for dollar and euro rates – at selected horizons, the term structure of the implied volatilities on the 1-, 2-, 5- and 10-year swap rates along with the term structure of the corresponding volatility risk premium. Data are expressed as percentages per annum and are averages between October 1998 and August 2006.

Figure 9: Estimated functional form linking implied volatilities to volatility risk compensation



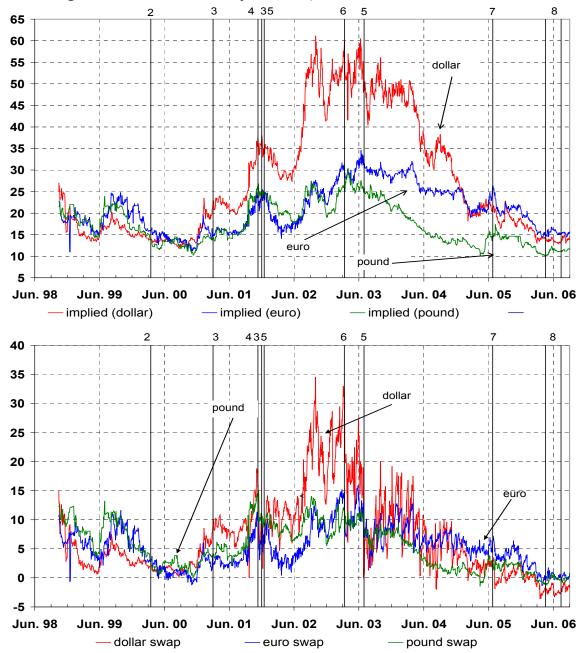
Notes: The upper panel refers to dollar rates., the lower panel to euro rates. The shaded areas represent the historical range in which implied volatilities have oscillated. The curves are obtained by regressing volatility risk compensations on a polynomial in implied volatilities where current and lagged values are considered. The polynomial includes a linear, quadratic, cubic and logarithmic term.

Figure 10: Implied swaptions volatilities and confidence interval for expected volatilities (dollar and euro rates, selected rates and forecast horizons)



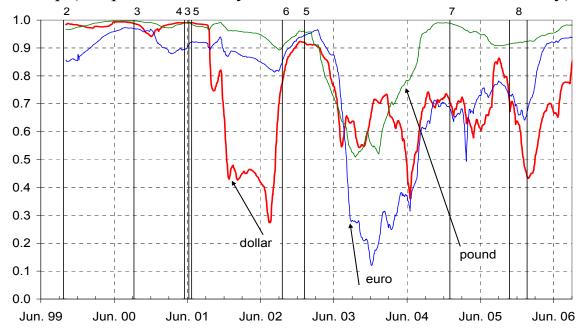
Notes: The figure reports, for dollar and euro rates only, the implied volatilities from swaptions on the 2- and the 5-year rate with maturity of 6 months, along with the simulated realised volatility and its confidence interval. Simulated volatility and confidence intervals are based on 5000 replications of a asymmetric garch(1,1) fitted to logarithmic rates of change of swap rates. Daily figures in percentages, annualised. The area delimited by the (1) symbols identifies the period between September 2001 and June 2003, when implied volatilities were often outside or close to the upper side of the confidence bands.

Figure 11: Implied swaptions volatility and vrp around selected events (swaptions and vrp on the 2-year rate at the 6-month maturity; in percentages, annualised; daily values)



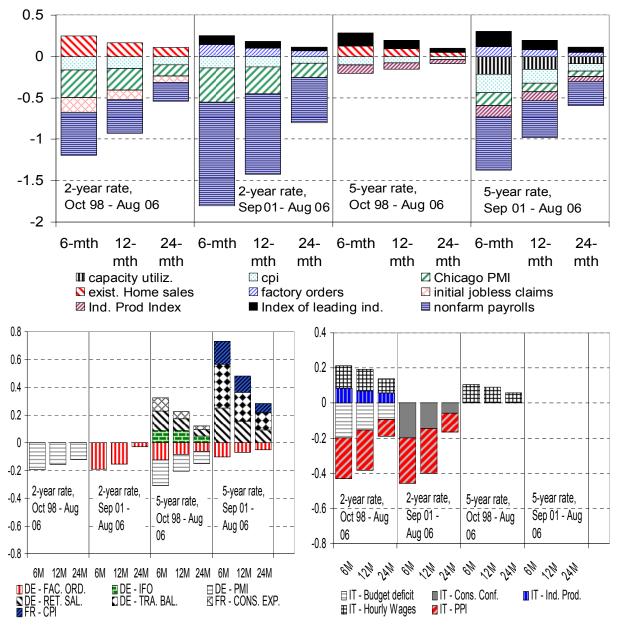
Notes: The upper panel reports implied volatilities, the lower panel the compensation for volatility risk (vrp). The numbered vertical bars identify the following events: 2 = Nasdaq starts to fall; 3 = US recession; 4 = 9/11; 5 = jobless recovery / deflation scare; 6 = Madrid train bombing / start of Fed tightening; 7 = Ford and GM debt downgraded to junk status; 8 = Global stock markets drop.

Figure 12: Rolling correlations between implied swaptions volatilities and vrp (swaption on the 2-year rate with 6-month life to maturity)



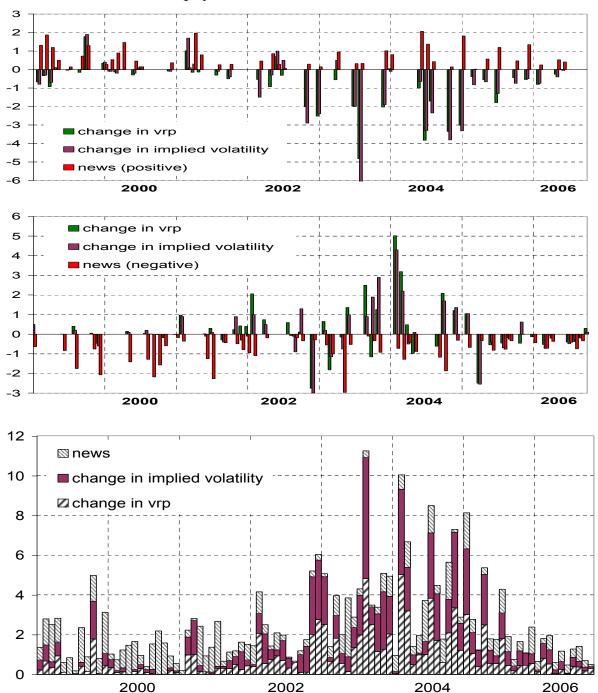
Notes: The figure reports 1-year rolling correlations between implied volatilities and compensations for volatility risk on the 2-year swap rate, at the 6-month horizon, for dollar, euro and pound (the series reported in the upper panel of Figure 11). Daily figures in percentages, annualised. The number vertical bars identify the following events: 2 = Nasdaq starts to fall; 3 = US recession; 4 = 9/11; 5 = jobless recovery / deflation scare; 6 = Madrid train bombing / start of Fed tightening; 7 = Ford and GM debt downgraded to junk status; 8 = Global stock markets drop.

Figure 13: Principal component-based reaction of dollar volatility risk compensation to US news (top panel) and euro area news (bottom panels), in percentage points, averages October 1998 – August 2006.



Notes: The figure reports the reaction of dollar volatility risk premium to various types of news, calculated through principal components. In the labels, DE stands for Germany, FR for France, IT for Italy. Fac. Ord. is factory orders, PM is purchasing managers' index, Ret. Sal. is retail sales, Tra. Bal. is trade balance, Cons. Exp. is consumer expenditures, CPI is consumer price index,. Cons. Conf. is consumer confidence, Ind. Prod. Is industrial production, PPI is producer price index.

Figure 14: Reaction of implied volatilities and compensation for volatility risk to the US Nonfarm payroll announcements



Notes: The upper two panels of the figure reports the change in the implied volatility and the volatility risk compensation on the 2-year dollar swap rate at the 6-month horizon on days characterised by the release of the nonfarm payrolls. Such releases have been standardised and named surprised. The first of the two panels reports the changes in the two variables following positive surprises while the second reports their change after negative surprises. The bottom panel shows the absolute values of the figures reported in the two panels at the top of the figure. Histograms in the bottom panel are stacked, therefore the scale of the y-axis in this panel does not correspond to the scale reported in the two top panels. Values for changes in implied volatilities and vrp are expressed in percentages annualised, news are standardised values.

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